Variational Methods and Structural Optimization
Course Project

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Project Proposal

Robot navigation in a 2D world with obstacles using Artificial Potential Fields

Back Ground

The problem of Robot navigation, can be defined as one of finding a path for a Robot starting from a fixed location 's' towards a target 'f' such that, it does not collide with any of a given set of obstacles which are placed in a certain dimensional space. The simplest form the problem would be a navigation of a point in a 2D space. However the method applied, using Artificial Potential Fields (APF) is claimed [1] to be applicable for any higher dimensional space.

A complete problem of an object moving in a 3 dimensional space, with rotation also included can be solved, by the construction of a Configuration Space [2], which can be used to reduce the finite sized object to a point by suitably deriving a space from both the geometry of the object and the surrounding. Inclusion of freedom of rotation will make the configuration space to be of 6 dimensions, to which the same method can be extended.

Here, the problem of navigation of a point sized robot in a 2D space filled with obstacles has been taken up. The robot is restricted from colliding with the obstacles by defining a potential field [3], around the obstacle. These fields are constructed such that they have peak at around the obstacle boundary and reduce farther away. The destination is modeled in [3] as a depression in the potential field which attracts the robot towards it. Then a suitable path is found out that will minimize the potential and thus move towards the robot.

Statement of the problem

The most suitable path of motion may be the 2D projection of the geodesic obtained on the potential surface [3]. Such work can be found in [4]. In [3] the concept of a quasi geodesic has been used to obtain the shortest path. The idea is applied when the problem is solved numerically, where from a point the direction of further movement is obtained from the direction to maximize the decrease of the straight line distance from this point to the final destination point.

A simpler method will be applied here. There are basically two quantities the path taken by the robot has to minimize. One is the peaks climbed by the point and the other the length of the 2D projected curve. To minimize both together they can be multiplied and we get is an area of the surface obtained by slicing the potential field with the path curve.

Let each of the obstacles be given by a curves C_1,C_2,..C_n. The potential field should be defined such that it is very large in the vicinity of the obstacles and reduces farther away. Some [3] use a Newtonian inverse square distance function, while some [5] use a Gaussian distributions placed over the center of each obstacle. Here the method suggested in [3] will be used.

The potential density can be given by

$$D_i(X,Y,x,y) = \frac{K_i}{d(X,Y,x,y)^{N/2}} \qquad \text{-------------(1)}$$

where, *i* – refers to the i^{th} obstacle (say ‘n’ obstacles)

X,Y – coordinates of a point on the 2D domain

x,y – coordinates of a point lying on the boundary of the obstacle.

K_i – is a constant

d(X,Y,x,y) – distance between (X,Y) and (x,y)

N – factor that can be chosen

The complete potential function is given by

$$V(X,Y) = Max\left[\int_{C_i} \frac{1}{\left[(X-x)^2 + (Y-y)^2\right]^{N/2}} ds\right] \qquad \text{------------(2)}$$

where *i*, ranges from 1 to n, n being the number of obstacles

Here ‘*Max*’ means, that at a point *(X,Y)* where more than one obstacle contributes to ‘*V*’, instead of summing them, we chose the one which is greatest. This will make sure that fake hills are not created at points which are close to many obstacles.

If we were to represent *(x,y)* in their parametric form, as

$$x = x(u)$$

$$y = y(u)$$

where u varies from u_i to u_f, we can write equation (2) as

$$V(X,Y) = Max\left[\int_{u_s}^{u_f} \frac{\sqrt{x_u^{\,2} + y_u^{\,2}}}{\left[(X-x)^2 + (Y-y)^2\right]^{N/2}} du\right] \qquad \text{------------(3)}$$

The solution curve, should satisfy, the equation (3) at every point.

As mentioned previously, the functional to be minimized will be the area of the surface obtained from the slicing of the potential field by the solution curve.

Hence the objective function is given by,

$$\underset{X(u),Y(u),Z(u)}{Min} \int_0^1 Z(u)\sqrt{X_u^{\,2} + Y_u^{\,2}}\, du \qquad \text{-----------(4)}$$

Subjected to constraint of

$$Z(u) - V(X(u),Y(u)) = 0 \qquad : \quad \lambda$$

At boundaries, if starting point is (X_s,Y_s) and target (X_f,Y_f), we get

$$X(u=0) = X_s, Y(u=0) = Y_s, Z(u=0) = V(X_s,Y_s)$$

$$X(u=1) = X_f, Y(u=1) = Y_f, Z(u=1) = V(X_f,Y_f)$$

Analytical Solution for a circular obstacle

The equation of the circular obstacle is given by,

$x = r\cos(u), y = r\sin(u)$, where $u = 0 .. 2\pi$

Hence, the Potential field is given by, (setting N=2)

$$V(X,Y) = \int_0^{2\pi} \frac{r}{(X - r\cos(u))^2 - (Y - r\sin(u))^2} du$$

Substituting $t = \tan(u/2)$, we get

$$V(X,Y) = \int_0^{\infty} \frac{r}{\left[(X-r)^2 + Y^2\right] - 4rYt + \left[(X+r)^2 + Y^2\right]t^2} dt$$

The Lagrangian is given by,

$$L = \int_0^1 Z(u)\sqrt{X_u{}^2 + Y_u{}^2}\,du + \int_0^1 \lambda\left\{Z - \int_0^{\infty} \frac{r}{\left[(X-r)^2 + Y^2\right] - 4rYt + \left[(X+r)^2 + Y^2\right]t^2} dt\right\} du$$

We get the equation to solve, X(u), Y(u), Z(u), from the variation of L, with respect to X, Y and Z.

As we see, even for a simple case of a circle, the Lagrangian is difficult to solve, analytically, hence a numerical method will be more suitable.

Further work

The problem will be tried numerical by minimizing the functional given in equation (4). (Though the solution will be a possible path for the robot avoiding the obstacles, it is not sure that it will be the minimum path or not, since N has been chosen arbitrarily)

The problem will also be tried in the method suggested in [3].

The ultimate problem to solve will be one of moving obstacles and moving target. This will lead to a problem of a 3D domain, where the third axis is ‘t’.

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[3] Josu Agirrebeitia , Rafael Avile´s, Igor F. de Bustos, Goizalde Ajuria, A new APF strategy for path planning in environments with obstacles, Mechanism and Machine Theory 40 (2005) 645–658

[4] R. Kimmel, Curve evolution on surfaces, Ph.D. Thesis, Israel Institute of Technology, Iyar 5745, Haifa, May 1995.

[5] D.E. Koditschek, Exact robot navigation by means of potential functions: some topological considerations, in: Proceedings of the IEEE International conference on robotics and automation, Raleigh, NC, 1987, pp.1–6.