Optimal Design of Electro-Thermal Compliant Mechanisms

Background to the problem:

Motivation: Actuators for MEMS applications uses different kinds of energy such as electrostatic, magnetic, thermal etc.. which are capable of providing small displacements. Actuators based on the Joule heating–induced thermal expansion provide large forces with moderate input of power. When this actuators coupled with the compliant mechanisms deliver large deflections. *Electro-Thermal-Compliant* actuation generates much larger forces over large distances compared with electrostatic actuation. Hence it is desired to optimally design the topology of the Electro-Thermal-Compliant (ETC) actuator to provide large displacement and force.

Working principle of the ETC actuator:

A typical ETC actuator is shown in the fugue below.

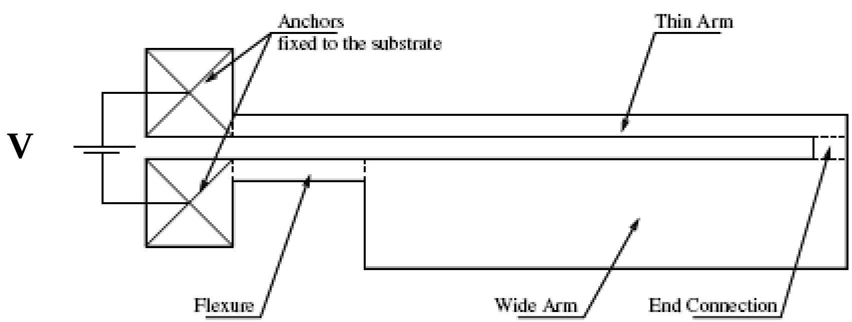


figure: Basic ETC actuator

ETC actuator is U-shaped folded beam consisting of a thin arm, a wide arm, end-connector and flexure for the wide beam with two anchors fixed to the substrate. When voltage is applied between the two anchors, same current will pass through thin arm and wide arm which are in series resistance from electrical analogy. So current density in the thin arm will be more compared with wide arm, which results in more joule heating per unit volume in the thin arm thereby experiencing more thermal expansion as compared with the wide arm, then the structure achieves equilibrium by bending towards the wide arm to accommodate the large expansion of the narrow arm.

Modeling of ETC actuator:

For simulating the behavior of an ETC actuator, it is required to do the following analysis steps:

1). Electrical analysis

This analysis has to be carried to determine the current distribution in the actuator

2). Thermal analysis

This analysis has to carried to determine the temperature distribution in the presence of Joule heating

3). Elastic analysis

This analysis has to be carried to determine the elastic deformation of the structure under the thermal loads.

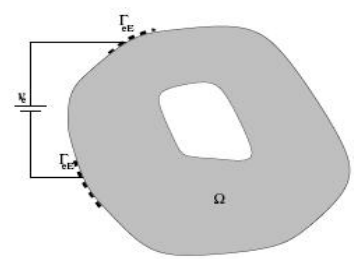
Electrical analysis, thermal analysis and elastic analysis are not independent, they are coupled due to various phenomena depending on the material properties. One significant source of coupling comes from the temperature dependence of the material properties which can be taken in to account by considering the temperature dependence of the electrical and thermal properties such as electrical conductivity, thermal conductivity, coefficient of thermal expansion etc…

Electrical analysis:

Electrical analysis of the device has to be carried out to determine the current density distribution $k_e \nabla V$ with in the structure for specified value of the voltage V on the boundary for the give value of electrical conductivity k_e in general it will vary spatially, but it is constant for single material when there is no temperature dependence. The governing equations for steady –state equilibrium analysis is given below

$$\nabla \bullet (k_e \nabla V(r)) = 0 \ in \ \Omega$$

$$V = V_{specified} \ on \ \Gamma_{eE}$$



Where Ω is the region of interest and Γ_{eE} the portion of the boundary where voltage is specified.

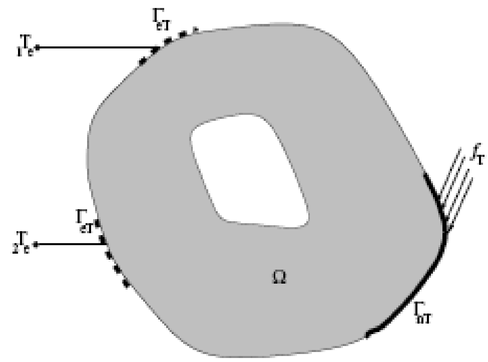
The above equations in weak form for the 2-D domain is given by

$$\int_{\Omega} \nabla^T V \quad (k_e \nabla V) \ \nabla V_v \ t \ d\Omega = 0$$

Where V_v denotes the admissible trial function for V and t is thickness of the structure which is assumed to be constant. When temperature dependence of the k_e is considered electrical analysis has to be carried along with thermal analysis.

Thermal analysis:

Thermal analysis of the device has to be carried to determine the temperature distribution with in the structure due to Joule heating, taking conductive, convective and radiation effects in to consideration. In ETC devices the operating temperatures are low, so the radiation effects are not pronounced. The governing equations for steady state thermal analysis is given below.



$$\nabla \bullet (k_t \nabla T) + \nabla^T V\, k_e \nabla V = 0 \ in\ \Omega$$

$$T = T_{specified} \ \ on\ \Gamma_{eT}$$

$$n \bullet (k_t \nabla T) = f_{nt} \ \ on\, \Gamma_{nT}$$

where

k_t is thermal conductivity

T is the temperature

Γ_{eT} is the boundary where temperature is specified

Γ_{nT} is the boundary where heat flux specified(i.e.. convection boundary etc..)

n is the unit normal to the surface

f_{nt} is the temperature dependent boundary heat flux that accounts for convection, radiation, and conduction in some cases.

Above equations in weak formulation can be written as

$$\int_{\Omega} \nabla^T \phi\, k_t \nabla \phi_v\, td\Omega - \int_{\Omega} \nabla^T V\, k_e \nabla V\, \phi_v \ \ td\Omega \ + \int_{\Omega} 2h\phi\phi_v\, d\Omega \ + \sum_e \left(\sum_{k=1}^{N_n} \ \int_{lk} h(1-\gamma_k)\phi\phi_v\, t\, dl_k \right) = 0$$

where

$$\phi = T - T_{\infty}$$

T_{∞} is ambient temperature

ϕ_v is the trial function for ϕ

Elastic analysis:

Elastic analysis on the ETC structure is carried to determine the displacement field in the structure caused by thermal loading. In ETC actuator there is no mechanical actuating forces, but the actuation is due to thermal loading. The governing equations for the elastic analysis is given by the following equations.

$$\nabla \bullet \sigma = 0$$

Where

$$\sigma = E(\varepsilon - \alpha(T - T_\infty)I)$$

$$= E(\varepsilon - \alpha\phi I)$$

$$\varepsilon = \left(\nabla u + \nabla^T u\right)/2$$

$$u = u_{specified} \quad on \ \Gamma_{eM}$$

u is the displacement

u_v is the trial function for u

Above equations in the weak form can be written as

$$\int_\Omega \varepsilon^T(u)\, E\, \varepsilon(u_v)\ t\, d\Omega - \int_\Omega \varepsilon_{th}{}^T\ (u)\, E\, \varepsilon(u_v)\ t\, d\Omega = 0$$

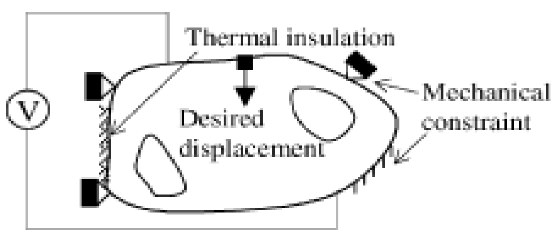
In the above equation

$$\varepsilon_{th} = \alpha\phi\{1\ 1\ 0\}^T$$

For accurate modeling of the device, large deformation analysis has to carried.

Synthesis of Electro Thermal Actuator:

A generalized version of the ETC design is shown below.



The objective of the present study is to determine the topology (i.e., the number of holes and connectivity) and shape of the structure that fits in the design region and deforms as desired when subjected to electrical, thermal, mechanical boundary conditions. This can be done by varying the design of the structure by trial and error and by using the designer's intuition. But this may not work always. Alternately, one can solve as design optimization problem, topology optimization in particular. In this case objective is to maximize the output displacement with the constraint on the amount of material to be used ,maximum temperature ,maximum stress, current drawn etc., Topology optimization is essentially a material distribution problem, with which we will optimally distribute the material in the design domain. This is done by discretising the design region and distributing the material on the fixed mesh ,which is done by defining a design variable to each element in the mesh. This variable decides whether material to be placed in that element or not. However in view of manufacturability this variable should reach either empty or full at the end of optimization procedure. The equation below shows how to interpolate a material property data smoothly for an element using *peak- function material interpolation* model.

$$E = E_1 e^{-(\rho-\mu_1)^2/2\sigma_1^2} + E_2\, e^{-(\rho-\mu_2)^2/2\sigma_2^2} + E_{void}$$

In the above equation

ρ is material selector parameter

σ_1 *and* σ_2 determines the sharpness of the material selection

μ_1 *and* μ_2 are the predefined parameters to locate the materials on the variable axis

Statement of the optimization problem:

For Electro-Thermal-Compliant actuator, optimization problem can be posed as follows

$$\text{Minimise}\,(-u_{out})$$

$w.r.t\ \rho_i, i = 1, 2....$ no.of elements

Subject to

$$\int_\Omega \nabla^T V \quad (k_e \nabla V)\ \nabla V_v \quad t\, d\Omega = 0 \qquad (1)$$

$$\int_\Omega \nabla^T \phi\, k_t \nabla \phi_v\, t d\Omega - \int_\Omega \nabla^T V\, k_e \nabla V\, \phi_v \quad t d\Omega \ + \int_\Omega 2h\phi\phi_v\, d\Omega + \sum_e \left(\sum_{k=1}^{N_n} \int_{lk} h(1-\gamma_k)\phi\phi_v\, t\, dl_k \right) = 0 \qquad (2)$$

$$\int_\Omega \varepsilon^T(u)\, E\, \varepsilon(u_v)\ t\, d\Omega - \int_\Omega \varepsilon_{th}{}^T\ (u)\, E\, \varepsilon(u_v)\ t\, d\Omega = 0 \qquad (3)$$

$$\int_\Omega \left(\sum_{m=1}^{2} w_m\ e^{-(\rho-\mu_m)^2/2\sigma_m} \right) d\Omega \ \le\ \Omega^* \qquad (4)$$

Here w_m are pre-specified weights

Ω^* is the upper limit on the amount of material to be used.

(1) is the electrical equilibrium equation in weak form

(2) is the thermal equilibrium equation in weak form

(3) is the elastic equilibrium equation in weak form

(4) is the constraint on the upper limit of the material

Design variables:

$$\rho_i, i = 1, 2 .... \text{no.of elements}$$

State Variables:

Displacement -- u
Temperature – T

Data:

Volume of the material
Thermal conductivity
Thermal expansion coefficient
Heat transfer coefficient
Young's modulus etc...

Plan for the future work:

The problem stated above has to be solved numerically with FEM programme to obtain the optimal topology.