ME 256 : Variational methods and structural optimization

Project proposal on
"Optimisation of profile for rotating disk"

by
Vinay Vaze
M.E. (Mechanical)
6332-410-041-03180

==

- ***Background to the problem***

High speed rotating disks are very commonly used as flywheels, gears, rotors in turbines and compressors. In the design of these rotating disks, the magnitude and distribution of the stresses constitute a major constraining factor. This problem has attracted the attention of many investigators. While designing a rotor for a given duty, the objective will be to arrive at an optimum decision for the dimension of the disc under the various constraints of space, cost and other physical limitations. A rational design procedure requires the formulation of the problem in mathematical terms and development of logic and decision steps to facilitate the search for optimum solutions.

It is well established that an efficient design of a rotating disk calls for a variable section, being thicker at the hub and tapering to a smaller thickness towards the periphery.

When the thickness of a disc is relatively small compared to its radius, the axial stress (i.e. stress along the shaft) would be negligible and hence the disc can be considered in a state of plane stress. However, the material could be assumed to be isotropic, homogeneous in the radial direction. Thus the member and loading are both axisymmetric and may contain internal and external pressures, i.e. due to centrifugal loading and temperature gradients. The boundary conditions are essentially arbitrary.

- ***A clear statement of the optimization problem*** (with objective function, constraints, design variables, state variables, and data.)

In the design of rotating disk the best objective function is not well-defined. De Silve has taken minimization of weight as objective function. Ah Seirag and Surana have carried out investigation with six objective functions and finally selected minimization of difference between maximum and minimum tangential stress as objective function for thier study.

I plan to consider one of the following ***objective function***s:

(1) Minimization of difference between maximum and minimum tangential stress, for maximum utilization of disc material.

Minimize $F = \sigma_{\theta \max} - \sigma_{\theta \min}$

(2) Stress leveling i.e. to get uniform stress.

Minimize $F = \int (\sigma - \sigma_a)^2 d\Omega$

σ is the maximum principal stress and σ_a, is the average stress.

(3) Minimization of the maximum level of tangential stress in the disc.
In the case of rotating disc, the maximum tangential stress invariably becomes the maximum principal stress that determines the failure criterion.

Minimize $F = \sigma_{\theta \max}$

(4) Minimization of volume or weight of the disk.

Minimize $F = \int \pi r^2 h(r) dr$

(5) A weighted objective function having equal or otherwise weightage for volume minimization and stress leveling can also be used. I will not consider this.

Thickness h(r) is design variable here and displacement u(r) is state variable.

Constraints:

- Side constraints can be imposed that as the maximum (if any) and minimum thickness at any point. (at any point the minimum thickness can be taken as say 1% of the outer radius.)
  $h(r) \geq h_{min}$
- For a rotating disc of uniform thickness, the magnitude of the maximum tangential stress or hoop stress at any point is always observed to be greater than that of the maximum radial stress. The same physical condition may thus be used as an implicit constraint as given below for the present problem to avoid an unrealistic configuration of the disc during optimization
  i.e. $(\sigma_r)_{\max} \leq (\sigma_\theta)_{\max}$
- Governing differential equation can be written as follows.

$$\frac{d}{dr}(hr\sigma_r) - h\sigma_\theta + h\rho\omega^2 r^2 = 0$$

where

σ_r, σ_θ are the radial and tangential stresses respectively,
ω is the angular velocity of the disc,
$\rho = \rho(r)$ the density of the disc material i.e. mass per unit volume.
h = h(r) the thickness of the disc at distance r.
u= u(r) the displacement at distance r.

Radial stresses

$$\sigma_r = \frac{E}{1-\upsilon^2}\left(\frac{du}{dr} + \upsilon \frac{u}{r}\right)$$

Tangential stresses

$$\sigma_\theta = \frac{E}{1-\upsilon^2}\left(\frac{u}{r} + \upsilon \frac{du}{dr}\right)$$

Material properties are E = Young's modulus and υ = Poisson's ratio .

- Using the radial and tangential stresses, equivalent stresses defined by von Mises or Tresca or any other criterion can be calculated and employed in the stress constraints for protection against failure in the optimization process. The von Mises criterion in an axisymmetric plane stress case is described as follows

$$\sigma_e = (\sigma_r^2 + \sigma_\theta^2 - \sigma_r \sigma_\theta)^{\frac{1}{2}}$$

The numerical data about properties and dimensions is not given here.

- ***Analytical solution*** (as much as possible) for the ***simplified setup of the problem***.

Statement of the problem with objective function, constraints, design variable, state variable etc. is explained in the previous section.

There will be three constraints viz. governing equation, stress limits, bounds on the thickness. All are local constraints (to be satisfied point wise in the domain) and will have corresponding Lagrange multipliers. We can obtain Euler- Lagrange equation by taking variation of F and equating to zero.

Boundary conditions for solid disk are zero displacement at the center of the disk and zero radial stress at the free periphery.

- ***Plan for further work***.

Optimization procedure can be used to get a profile of the disk thickness based on the objective function. The stress analysis and the calculation of sensitivities of stresses to design variables can be carried out. The stress distributions in the disc can be calculated using a suitable FEM tool. Optimization of the profile may require starting values of the variables and physical & geometrical constraints specified as an input. On evaluating the objective functions, search procedure will locate a new feasible profile design with the improved values. The search can be then continued until a point is reached where from no further improvement in the objective function is obtained. The corresponding geometrical parameters will provide an optimum solution.

References:

1. "Optimum Shape Design Of Rotating Disks"; S. S. Bhavikatti, C.V. Ramakrishnan; Computers and structures Vol.11, pp. 397-40l; Pergamon Press Ltd. 1980. Printed in Great Britain.
.
2. "Minimum weight design of inhomogeneous rotating discs"; Hamid Jahed, Behrooz Farshi, Jalal Bidabadi International Journal of Pressure Vessels and Piping 82 (2005) 35–41