Problem of Flexible Membrane

Consider a perfectly flexible membrane simply supported at the boundary (See Figure 1 below) on the X-Y plane and with a tensile force T per unit length which is everywhere constant in the membrane. If a normal pressure distribution q(x,y) is applied to cause small deflection of the membrane, then using variational principle show that the static deflection w(x,y) of this membrane is solution of the following PDE

$$\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} = -\frac{q}{T}$$

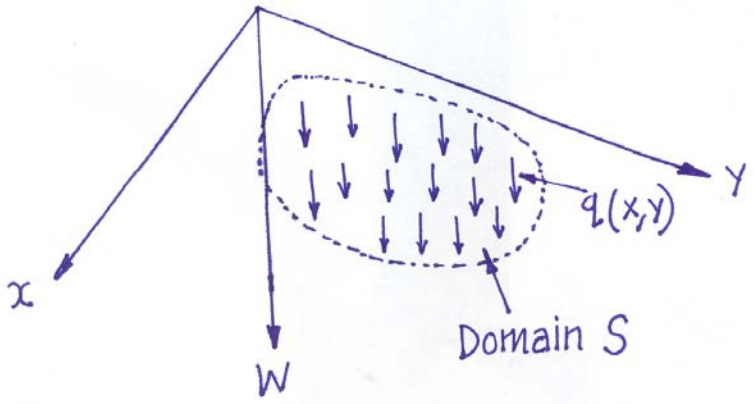

Figure: 1