ME256 Variational Methods and Structural Optimization Jan. 2005

Final Examination

Points: 30 Time: 120 minutes

Question 1 (10 points)

A beam is to be optimally designed to for a given volume of material V^* so that $\int_0^L \{w(x)\}^2\, dx$ is minimized where $w(x)$ is the transverse displacement of the beam under a given load $q(x)$. Assume a rectangular cross-section with a fixed width, b , but variable depth, $t(x)$.

a) Write the complete statement of the problem as a constrained variational problem. Use the weak form of the governing equation.
b) Ignoring the upper and lower bounds on the design variable, write the necessary conditions for the problem.
c) Identify the adjoint equation.
d) Write the update formula for the design variable if we want to use the optimality criteria method.
e) Outline the procedure for the numerical implementation of the optimality criteria method based on the update formula. Be sure to keep in mind the upper and lower bounds on the design variables.
f) How does the update formula change if the width of the beam is made variable by keeping the depth constant throughout the beam?

Question 2 (10 points)

Verify if $\left\{ 4/3 \quad 4/3 \right\}^T$ and $\left\{ 1.2 \quad 1.4 \right\}^T$ are local minima for the following constrained optimization problem.

$$\underset{x_1, x_2}{\text{Minimize}} \; f = x_1^2 + x_2^2 - 2x_1 - 2x_2 + 2$$

Subject to

$$-2x_1 - x_2 + 4 \le 0$$
$$-x_1 - 2x_2 + 4 \le 0$$

Question 3 (10 points)

Solve the following problem using the dual method. Show all the steps clearly.

$$\underset{x_1, x_2}{\text{Minimize}} \; f = (x_1 - 3)^2 + (x_2 - 3)^2$$

Subject to

$$2x_1 + x_2 - 2 \le 0$$
$$-x_1 \le 0$$
$$-x_2 \le 0$$