Practice problems for ME 256

1. Find the extremizing function for the functional $J = \int_{x_1}^{x_2} (y'^2 + 2yy' - 16y^2)dx$ and then find another functional that has the same E-L equation as the one you found. Trivial multiplication or addition by a constant is not permitted in creating a new functional.

2. For which functional, $y = \frac{x^7}{7!} + \sum_{i=0}^{5} c_i x^i$ is an extremizing function?

3. Find an integrand functional $J = \int_{x_1}^{x_2} F(y, z, y', z')dx$ for which $y = (ax + b)\cos x + (cx + d)\sin x$ and $z = 2y + y''$ are the extremizing pair of functions.

4. Find the extremizing functions for the isoperimetric problem given below. Minimize $J = \int_0^1 (y'^2 + z'^2 - 4xz' - 4z)\,dx$

   subject to

   $$\int_0^1 (y'^2 - xy' - z'^2)\,dx = 2$$

   $$y(0) = 0,\ z(0) = 0,\ y(1) = 1,\ z(1) = 1.$$

5. It is desired that the absolute axial displacement of an axially loaded bar be less than Δ^*. Formulate an optimization problem and write the necessary conditions so as to minimize the volume of the bar.