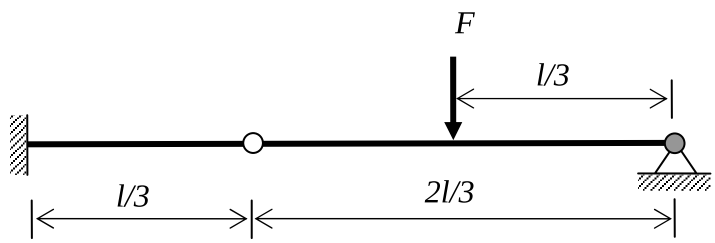
Mid-term Examination

Marks: 30 Open notes and open books **Time: 90 minutes**

Question 1 *(10 marks)*

The figure below shows a segmented beam of length l with uniform beam modulus EI along its length. There is a pin joint at one third the length from the left and a downward point load F at one third the length from the right. The left end is fixed and the right end pinned as shown. Use the principle of minimum potential energy to write down the differential equation, the end conditions, and the conditions at the intermediate pin joint. Sketch the deformed shape qualitatively.



Question 2 *(10 marks)*

Consider the functional $\int_{-2}^{2} y^2 \sqrt{1+y'^2}\, dx$. We want to find the minimizing function $y^*(x)$ for the functional by starting with a given function, $y(x) = x+1$, and modifying it gradually. Assume that $y(x) = x+1$ satisfies the prescribed y values at both the ends. The modification procedure involves a slight change (positive or negative) in $y(x)$, say 0.01, at each point x such that the value of the functional decreases. Now answer the following.

(i) In the first step, how would you determine at which values of x you would increase $y(x)$ and at which values of x you would decrease $y(x)$.

(ii) Do you think such a procedure would converge to the minimizer of the functional? If yes, explain why. If not, what change would you suggest for making it converge?

Question 3 *(10 marks)*

We want to optimize $J = \dfrac{\int_0^a b y'^2\, dx}{\int_0^a y^2 dx}$ with respect to $y(x)$ such that $y(0) = y(a) = 0$.

(i) Derive the differential equation using which we can solve this problem.

(ii) Comment on the nature and the number of solutions for the differential equation.

Show your steps clearly.