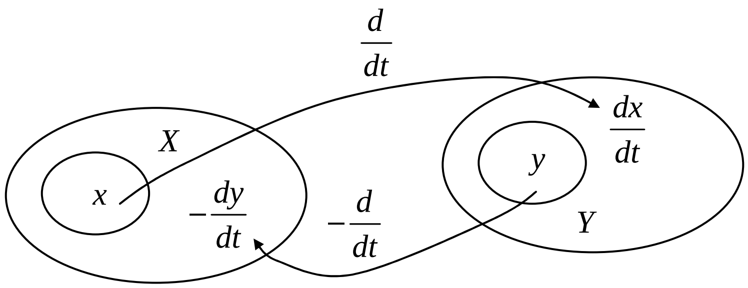
Mid-term Examination

Marks: 30 Open notes and open books **Time: 60 minutes**

Question 1 *(10 marks)*

Consider two inner products defined on vector spaces X and Y as mappings $X \times X$ and $Y \times Y$ respectively into their common scalar field K and denoted as $\langle x_1, x_2 \rangle_1$, $x_1, x_2 \in X$ and $\langle y_1, y_2 \rangle_1$, $y_1, y_2 \in Y$. Note also that there are the following relationships between X and Y.



Now, consider $J = \int_a^b x \frac{dy}{dt} dt$ with $x(t) \in X$ and $y(t) \in Y$.

If the result of the integration by parts of J can be expressed as $\left\langle x, -\frac{dy}{dt} \right\rangle_1 = \left\langle y, \frac{dx}{dt} \right\rangle_2$, write down the formulae for the two inner products used here.

- Comment on the validity of your definitions of inner products for extra marks of up to 5.

Question 2 *(10 marks)*

Write the Euler-Lagrange equation of the following problem of minimizing a functional of two independent variables.

$$\underset{u(x,y)}{\text{Minimize}} \quad J = \iint_R \left[D\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)^2 - 2D(1-\nu)\left\{ \frac{\partial^2 u}{\partial x^2}\frac{\partial^2 u}{\partial y^2} - \left(\frac{\partial^2 u}{\partial x \partial y} \right)^2 \right\} - f(x,y)u \right] dxdy$$

Simplify the Euler-Lagrange equation to the extent possible.

- Extra points will be given for writing the boundary conditions. (*Up to 10 extra marks.*)

Question 3 *(10 marks)*

The figure below shows a horizontal beam of length L, Young's modulus E, moment of inertia I, and loaded with a force P at its mid point and constrained as follows. The left end is attached to a rigid crank of radius r_1 with a pin joint and a torsional spring of spring constant κ. The right end is fixed to another rigid crank of radius r_2. The vertical separation between the grounded pivots of the two cranks is shown to be e. Obtain a functional the minimization of which gives the static equilibrium equation to solve for the deformed configuration of this problem. And then, write the Euler-Lagrange equations along with the boundary conditions in terms of the symbols noted here.