

DETC2006-99684

TOPOLOGY OPTIMIZATION OF ELECTROSTATICALLY ACTUATED MICROMECHANICAL STRUCTURES WITH ACCURATE ELECTROSTATIC MODELING OF THE INTERPOLATED MATERIAL MODEL

Aravind Alwan

Mechanical Engineering
Indian Institute of Science
Bangalore 560 012, India
aravindalwan@gmail.com

G K Ananthasuresh

Mechanical Engineering
Indian Institute of Science
Bangalore 560 012, India
suresh@mecheng.iisc.ernet.in

ABSTRACT

In this paper, we present a novel formulation for performing topology optimization of electrostatically actuated constrained elastic structures. We propose a new electrostatic-elastic formulation that uses the *leaky capacitor model* and material interpolation to define the material state at every point of a given design domain continuously between conductor and void states. The new formulation accurately captures the physical behavior when the material is between a conductor and a void is present during the iterative process of topology optimization. The method then uses the optimality criteria method to solve the optimization problem by iteratively pushing the state of the domain towards that of a conductor or a void in the appropriate regions. We present examples to illustrate the ability of the method in creating the stiffest structure under electrostatic force for different boundary conditions.

INTRODUCTION

The use of electrostatic force for actuation in microsystems is desirable because of the large amplitudes that are achieved at the micron scale as well as the ease of manufacturing and integration along with electronic components. In microelectromechanical systems (MEMS), an electrostatic force that is attractive in nature deforms the mechanical structure. The potential difference between different conductors determines the magnitude of force, which in turn controls the equilibrium positions of the constrained elastic structures of the conductors. The fact that silicon can be used for generating both the electrostatic force as well as the mechanical restoring force makes it ideal for fabricating low cost devices using any of the standard micromachining methods.

To meet the growing demand for electrostatic actuators in microsystems, it is important that synthesis methods are

developed in order to automate or aid the process of generating new designs. Synthesis techniques are also important from the point of view of generating complex designs that are not easily visualized through intuition. Among the many popular design methods, topology optimization is one such technique that is gaining popularity due to its ability to adapt to situations that involve many different physical phenomena [1].

Topology optimization refers to the synthesis of structures in a given domain so as to optimize an objective function subject to one or more constraints. For instance, a volume constraint is usually used to limit the amount of material available to the optimization algorithm while generating structures. Topology optimization is quite powerful because it requires only essential information from the user and is able to generate optimal designs that are often readily manufacturable. This is particularly true in the case of planar designs (such as those used in microsystems) which may be easily fabricated by choosing appropriate mask layouts without too much additional cost.

There are a number of methods for solving problems of topology optimization. One popular approach among these that we shall be using here is the SIMP (Simple Isotropic Material with Penalization) approach [2]. In this method, we discretize the domain into a set of finite elements and define a material interpolation parameter for each of these elements. This parameter, which takes values between zero and one, is raised to some power and is multiplied with the material property values to interpolate the material properties throughout the domain in a continuous manner. This continuous interpolation between material and no-material calls for accurate modeling of the physics of the problem under intermediate, interpolated state of material(s). This paper deals with one such problem in topology optimization.

Topology optimization has been applied to design situations that involve many diverse physical phenomena like those present in mechanical structures, electrothermal actuators, piezoelectric actuators and even optical media like photonic crystals [3]. However, for electrostatically actuated structures, topology optimization has been implemented only recently [4], possibly due to the lack of a physical model to smoothly interpolate the material state from a conductor to a dielectric or a void. In this paper, we propose an accurate formulation for a material interpolation model that uses the so-called *leaky capacitor model* to provide a physical basis for this interpolation. In the following sections, we shall explain this model and discuss how it may be applied to topology optimization to yield optimal structures.

In the next section, we begin with a brief description of the electrostatic analysis and force-computation when a material is in an intermediate state between a conductor and a void, which was discussed in our recent past work [5]. This analysis method is combined with the new material interpolation model of this paper to lead to topology optimization of electrostatically actuated structures.

BRIEF OVERVIEW OF THE ANALYSIS METHOD

Consider the domain to be a region that has some finite and spatially varying value of conductivity. In topology optimization, the changing material properties of the domain are defined in terms of a material interpolation scheme. Let γ be such a parameter used for material interpolation. In other words, the value of γ varies spatially in order to interpolate material properties between those of a conductor and a void in the case of electrostatic analysis. Upon the application of electrostatic boundary conditions, when there is “intermediate” material we observe a distribution of current flowing through the domain due to the finite value of its conductivity. The flow of current through this inhomogeneous domain under steady state is given by

$$\nabla \cdot (\mathbf{J}) = \nabla \cdot (\sigma \nabla V) = 0 \quad (1)$$

where \mathbf{J} is the current density, V the electrostatic potential and $\sigma(x, y, z)$ the spatially varying value of conductivity that ideally varies between infinity (for a conductor) and zero (for an insulating dielectric or void).

In order to model the electrostatic force that is generated in actuators, we note that when current flows through an inhomogeneous domain, electric charge accumulates at the regions of discontinuity giving rise to an electrostatic force. This will be a body force at these regions in contrast to familiar surface force of electrostatics. In this case, the electrostatic force is localized to regions wherever there is a discontinuity in either conductivity or permittivity of the medium (see Eq. (2)). Using the generalized electrostatic stress tensor [6], we write the expression for this force per unit volume \mathbf{F}_{es} as follows.

$$\mathbf{F}_{es} = \rho_e \mathbf{E} - \frac{1}{2} E^2 \nabla \varepsilon + \frac{1}{2} \nabla \left(E^2 \frac{\partial \varepsilon}{\partial \rho_m} \rho_m \right) \quad (2)$$

Here ρ_e is the free electric charge density, \mathbf{E} the electric field, ε the permittivity and ρ_m the mass-density of the material.

The third term in expression for the body force of electrostatics is like a hydrostatic force that is the same in all directions inside a dielectric medium. Since we are considering only resultant forces on the domain, the third term in Eq. (2) may be neglected. Neglecting the third term, the expression becomes identical to the force predicted from Maxwell’s electrostatic stress tensor. For a detailed discussion of the above, please see [6].

The electrostatic body force is applied on the mechanical structure. The same material interpolation that is used to interpolate conductivity is also used to perform the same task on the mechanical moduli of the material (e.g., Young’s modulus and Poisson’s ratio in isotropic materials). The deformation in the mechanical structure is computed using the elastostatic governing equation:

$$\nabla \cdot (\mathbf{S}) + \mathbf{F}_{es} = \mathbf{0} \text{ and boundary conditions.} \quad (3)$$

Here, \mathbf{S} is the stress tensor, which is the product of the constitutive elastic modulus tensor and the strain tensor.

When the material interpolation parameter takes values in between its two extreme limits in certain regions, we see that these parts partially conduct current and store electrostatic energy as well. In lumped modeling, this is known as a *leaky capacitor model* and is represented by a resistor and a capacitor in parallel. As the conductivity values in the domain are pushed towards the limits (i.e., for piece-wise homogeneous conductor-void combinations), in the absence of a conducting path across the potential difference, we see that the structure resembles an ideal capacitive configuration. In this situation, the electrostatic force is localized to the interface between the conducting and void regions and becomes identical to the electrostatic surface force that is found in electrostatic actuators. Thus this model allows for the continuous interpolation of electrostatic material state between the limits of a conductor and a void. An example is shown in Fig. 1. More analysis results are in [5].

MATERIAL INTERPOLATION

In this paper, we interpolate the material state only between two cases, i.e. a conductor and a void, while assuming permittivity to be unity everywhere¹. It must be noted here, that the physical model allows for the independent interpolation of dielectric permittivity too [5], though we do not make use of that in our optimization in this paper. The material interpolation is done in the following manner to ensure that regions of high

¹ The dielectric permittivity is exactly unity only in vacuum; however, the value in air and most metallic conductors is only slightly greater than unity.

conductivity (i.e. conductors) also comprise the mechanical structure with higher elastic moduli when compared with void regions.

$$\begin{aligned}\sigma &= \sigma_{\min} + (\sigma_{\max} - \sigma_{\min})\gamma^p \\ \mathbf{Y} &= \mathbf{Y}_{\min} + (\mathbf{Y}_{\max} - \mathbf{Y}_{\min})\gamma^p\end{aligned}\quad (4)$$

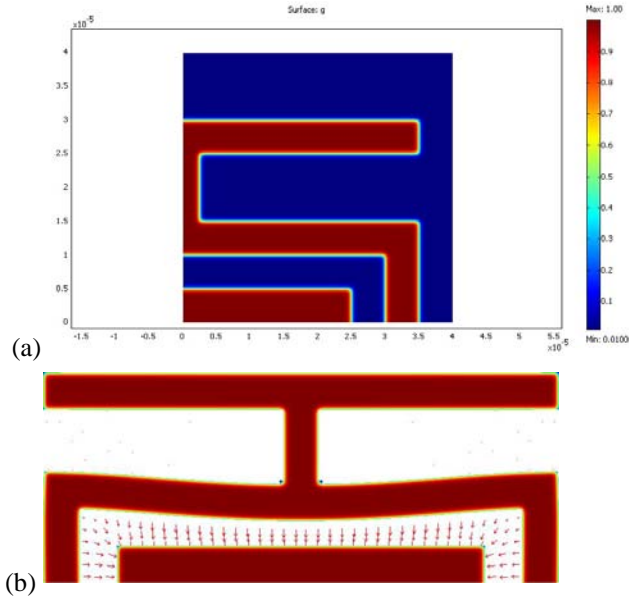


Figure 1. An analysis result from [5]. Analysis of an electrostatically actuated structure in which the electrostatic force is used to move a mirrored surface vertically without deformation [7] (a) Plot of γ showing undeformed structure modeled using symmetry boundary conditions on the left vertical edge (b) Complete deformed structure. Arrows indicate the direction and magnitude of the electric field. Only mesh elements with γ greater than 0.5 have been shown for clarity.

The quantity \mathbf{Y} in Eq. (4) refers to the constitutive elastic modulus tensor, while p is the penalizing exponent that is used to drive the value of γ towards zero or one. The value of p is normally chosen to be greater than or equal to three in order to ensure existence of physical materials with elastic moduli that satisfy this power law interpolation (see [8] for a detailed discussion). Appropriate lower bounds are chosen for the conductivity and the elastic moduli to avoid singularities in the stiffness matrices during finite element calculation.

In practice, the interpolation of conductivity in the manner as shown in Eq. (4) leads to improper computation of electrostatic force due to numerical errors. We observed that at every conductor-void interface, the localized electrostatic force slightly shifts towards the side of the void. This results in the force acting on the void material of low stiffness rather than the

conducting material that has high stiffness. This leads to poor computation of deformations. In order to avoid this, we rewrite the interpolation of conductivity as follows.

$$\begin{aligned}\rho &= \rho_{\min} + (\rho_{\max} - \rho_{\min})(1 - \gamma^p) \\ \sigma &= 1/\rho \quad \rho_{\max} = 1/\sigma_{\min} \quad \rho_{\min} = 1/\sigma_{\max}\end{aligned}\quad (4)$$

where ρ is the resistivity of the medium and σ_{\min} and σ_{\max} are the bounds of conductivity as defined in Eq. (4). It is easily seen that when γ approaches zero or one, the value of conductivity becomes equal to the corresponding bounds. By using this alternative interpolation scheme, we circumvent the problem of improper calculation of the electrostatic forces.

The material interpolation model described in this section and the analysis method of the previous section will be used in the topology optimization as discussed next.

THE TOPOLOGY OPTIMIZATION PROBLEM

The topology optimization problem is stated in terms of the new formulation as follows.

$$\begin{aligned}\text{minimize } & \phi \\ \text{subject to } & \\ \mu : & \quad \nabla \cdot (\sigma \nabla V) = 0 \quad \text{on } \Omega \\ \lambda : & \quad \nabla \cdot (\mathbf{Y} : \mathbf{e}(\mathbf{u})) = \mathbf{F}_{es} \quad \text{on } \Omega \\ \Lambda : & \quad \int_{\Omega} V(\gamma) d\Omega \leq V^*\end{aligned}\quad (5)$$

Here ϕ is the given objective function to be minimized over the domain Ω , \mathbf{e} is the strain tensor, \mathbf{u} is the displacement vector, $V(\gamma)$ is the volume density function, V^* is the volume allowed by the volume constraint while μ , λ and Λ are Lagrange multipliers corresponding to the three constraints.

In this paper, we have solved only for the objective function that minimizes the strain energy of a given domain due to electrostatic force. Thus, the objective function is given by

$$\phi = \int_{\Omega} \mathbf{e}^T(\mathbf{u}) : \mathbf{Y} : \mathbf{e}(\mathbf{u}) d\Omega \quad (6)$$

The optimization problem is solved using the optimality criteria method in which the widely used heuristic updating scheme [9] is used to iteratively modify the design variables.

$$\gamma_i^{new} = \begin{cases} \max(\gamma_{\min}, \gamma_i - m) \\ \quad \text{if } \gamma_i B_i^\eta \leq \max(\gamma_{\min}, \gamma_i - m) \\ \min(1, \gamma_i + m) \\ \quad \text{if } \min(1, \gamma_i + m) \leq \gamma_i B_i^\eta \\ \gamma_i B_i^\eta \\ \quad \text{otherwise} \end{cases} \quad (7)$$

where $m(>0)$ is the move-limit, $\eta(=1/2)$ is the damping factor and B_i is determined from the optimality criteria condition as explained ahead.

The optimality criterion is derived by performing the sensitivity analysis. In order to do this, we write the Lagrangian for the entire coupled system of partial differential equations given in Eq. (6) and take variation of the Lagrangian with respect to the state variables to obtain the adjoint equations (Eqs. (10) and (11)). The adjoint equations may be used to solve for the Lagrange multipliers μ and λ , which are then substituted in the optimality criteria condition. The entire procedure is given below.

$$L = \phi + \int_{\Omega} \mu \{ \nabla \cdot (\sigma \nabla V) \} d\Omega + \int_{\Omega} \lambda \{ \nabla \cdot (\mathbf{Y} : \mathbf{e}) - \mathbf{F}_{es} \} d\Omega + \Lambda \left\{ \int_{\Omega} V(\gamma) d\Omega - V^* \right\} \quad (8)$$

$$\delta L_u: \int_{\Omega} \nabla \cdot (\mathbf{Y} : \mathbf{e}(\lambda)) \delta \mathbf{u} d\Omega + \frac{\partial \phi}{\partial \mathbf{u}} = 0 \quad (9)$$

$$\delta L_v: \int_{\Omega} \{ \nabla \cdot (\sigma \nabla \mu) \} \delta V d\Omega - \int_{\Omega} \mathbf{F}_{es} \cdot \lambda d\Omega = 0 \quad (10)$$

$$\delta L_{\gamma_i}: \frac{\partial \phi}{\partial \gamma_i} + \int_{\Omega} \frac{\partial \sigma}{\partial \gamma_i} (\nabla \mu \cdot \nabla V) d\Omega + \int_{\Omega} \mathbf{e}^T(\lambda) : \frac{\partial \mathbf{Y}}{\partial \gamma_i} : \mathbf{e}(\mathbf{u}) d\Omega + \Lambda \int_{\Omega} \frac{\partial V(\gamma_i)}{\partial \gamma_i} d\Omega = 0 \quad (11)$$

The last equation above is the optimality criterion that must be satisfied by the converged solution. We may write Eq. (12) alternatively as

$$A_i + \Lambda D_i = 0 \quad \Rightarrow \quad \frac{-A_i}{\Lambda D_i} = 1 \quad (12)$$

The quantity on the left hand side of Eq. (13) is defined as B_i and is used in the heuristic updating scheme explained previously in Eq. (8). The numerical implementation details are presented in the next section.

NUMERICAL IMPLEMENTATION

The above optimization scheme was numerically implemented using COMSOL Multiphysics® [10], which was used to

perform the finite element analysis and do the computations related to the optimality criteria optimization algorithm. COMSOL Multiphysics® has the flexibility of a scripting language that is similar to the MATLAB® [11] programming language. This was used to combine the finite element solver with the optimality criteria algorithm in order to solve the topology optimization problem.

In all coupled electrostatic-elastic problems, the electrostatic boundary conditions are such that a potential difference is applied between two or more different parts of the domain boundary. Hence, there are two sets of electrodes such that one set is maintained at some finite potential, while the other set is grounded. Since the value of γ in the entire domain is initially given a uniform value, there is no discontinuity present in the domain and hence there is no electrostatic force. This leads to a trivial solution where the optimization algorithm does not proceed further, but instead instantly converges because the stiffness structure criterion is satisfied. In order to avoid this trivial solution, we manually remove a small portion from the design domain around all the electrodes from one of the two sets. Removing a portion from the design domain means that the optimization algorithm will not be allowed to change the value of γ in that region. Furthermore, we manually set the value of γ in these regions to form the electrodes of finite size surrounded by the void region on all the sides. This ensures that a meaningful solution is found by the algorithm.

Since the electrostatic analysis is at the core of the entire optimization algorithm, it is important to ensure that the electrostatic force is computed properly. Using the leaky capacitor model for electrostatic analysis poses some difficulties in terms of accurate calculation of electrostatic force. The main problem is due to the presence of spatial derivatives in the expression for this force (see Eq. (2)). As the algorithm progresses, the conductor and void regions become more and more well defined with distinct interfaces separating them. Consequently, the discontinuities at their interfaces become sharp. This leads to some loss of resolution in computing the force if the finite element discretization is not adequately fine. This problem may be partially resolved by using the standard mesh independency filter that is used in topology optimization [9]. This filter acts like a convolution filter on the sensitivity values and smoothens the interfaces between material and void regions. This helps in computing the forces to an acceptable degree of accuracy. The mesh independency filter is given as follows.

$$\hat{A}_i = \frac{\sum_{j=1}^N H_j \gamma_j A_j}{\gamma_i \sum_{j=1}^N H_j} \quad (13)$$

where H_j is the weight factor, which is defined as

$$H_j = r_{MF} - \text{dist}(i, j) \quad (14)$$

$$\text{s.t.} \{j \in N \mid \text{dist}(i, j) \leq r_{MF}\} \quad \forall i \in 1, \dots, N$$

where r_{MF} is the radius of the mesh independency filter and N is the number of mesh elements.

Although the problem of resolving the force accurately may be resolved throughout the domain using a mesh independency filter, it was observed that it still arises at the interfaces between the design and non-design regions. Since, by definition, the non-design regions are not part of the design domain, the mesh independency filter is not able to smooth the discontinuities at the interface. To rectify this problem, we introduce a secondary filter that acts on the values of γ and performs a local averaging in a small radius on either side of each of these interfaces. This has the effect of smudging out sharp discontinuities at the interfaces. The expression for the secondary filter is given below.

$$\hat{\gamma}_i = \frac{\sum_{j=1}^N H'_j \gamma_j}{\sum_{j=1}^N H'_j} \quad (15)$$

where H'_j is the weight factor for the secondary filter is defined as

$$H'_j = r_{SF} - \text{dist}(i, j) \quad (16)$$

$$\text{s.t.} \{j \in N \mid \text{dist}(i, j) \leq r_{SF}\}$$

for all i such that the distance of element i from its nearest such interface is less than or equal to r_{SF} . Here r_{SF} is the radius of the filter used in the secondary filter. The combination of these two filters ensures that the computation of the electrostatic force is accurate even at the interfaces between conductors and void regions irrespective of those interfaces being in contact with the non-design domain.

RESULTS AND DISCUSSION

We now present two examples to demonstrate the capability of the new formulation for performing topology optimization of electrostatically actuated structures. Although we shall deal only with simple examples here, the method is extendable to more complex scenarios with additional effort in implementation. The examples considered here serve as benchmark problems because intuitively we know the optimal solutions for these.

The first example problem was to get the stiffest structure that is held rigidly at two places on the bottom edge. The structure is maintained at a fixed potential of 10 V while a

grounded electrode is placed in between the two anchor points as shown in Fig. 2. Symmetry boundary conditions are used to simplify computation so that only the right symmetric half of the structure is designed. In this problem there is a natural conflict in deciding where to place material. Putting material close to the anchor points increases stiffness but increases the proximity between the two electrodes. This leads to higher electrostatic force which results in increased deformation of the structure. If the material is moved farther from the support and the bottom electrode, the electrostatic force decreases but the structures becomes less stiff. This is the nature of the problem where there are design-dependent loads. The optimal topology is expected to be one that obeys a compromise between the two extremes. The resulting optimal topology is shown in Fig. 3. As to be intuitively expected, we obtained an arch-like structure that is located near the middle of the design domain. This is the optimal position that is the stiffest under the action of distributed transverse loads, which electrostatic force becomes in the converged solution.

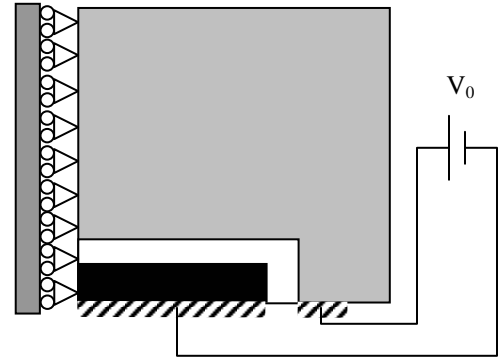


Figure 2. The stiffest structure optimization problem showing the symmetric right half of the design domain.

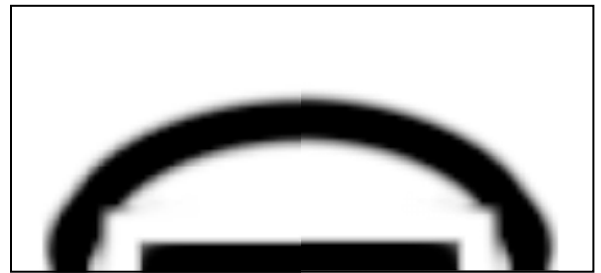


Figure 3. The optimal topology for the problem shown in Fig. 1 above.

The second example's problem specifications (Fig. 4) are given such that a conflict is created to place the material between two parallel electrodes. Since, when material is placed close to the electrodes the force increases, the algorithm has to resolve the tradeoff given that there are electrodes on both the sides. To make it more challenging, the anchors were specified asymmetrically. The design domain and boundary conditions

are shown in Fig. 4. We see that the presence of diagonal electrostatic boundary conditions results in an S-shaped beam that is fixed at both ends as shown in Fig. 5.

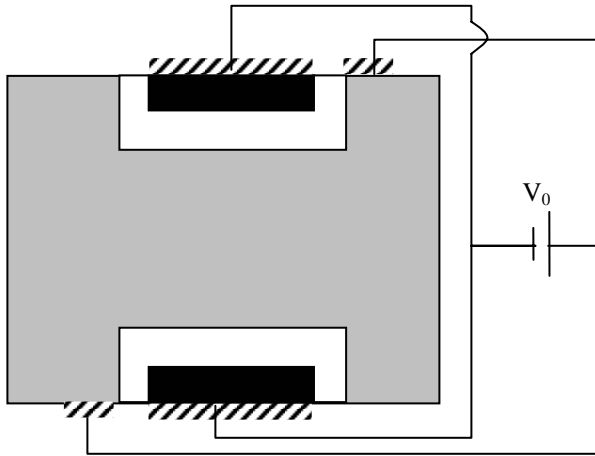


Figure 4. Strain energy minimization under diagonal electrostatic boundary conditions



Figure 5. The optimal topology for the problem shown in Fig. 4.

CLOSURE

With the miniaturization of devices, there is an increasing demand for systematizing the process of designing electrostatically actuated micromachined structures. Topology optimization offers an attractive method for achieving this goal due to the flexibility present in applying the same synthesis algorithm to a variety of situations. The key features of this paper are a material interpolation model and the accurate analysis model for the electrostatics problem even in the intermediate material state. The method is amenable for implementation on commercial finite element analysis platforms. Although in this paper we have chosen to demonstrate examples of stiffest structures for the sake of simplicity, the same scheme can be applied to any objective

function by making appropriate changes to the terms in the implementation in the COMSOL MultiPhysics platform. This is currently being pursued in our ongoing work.

REFERENCES

1. Bendsøe, M. P. and Sigmund, O., *Topology Optimization Theory, Methods and Applications*, 2003, Springer, Berlin.
2. Rozvany, G. I. N., Bendsøe, M. P., and Kikuchi, N., "Layout Optimization of Structures," *Applied Mechanics Reviews*, Vol. 48, 1995, pp. 41-119.
3. Ananthasuresh, G. K. (Ed.), *Optimal Synthesis Methods for MEMS*, 2003, Kluwer-Academic Publishers, Boston.
4. Rauli, M. and Maute, K., 2004 "Topology optimization of electrostatic MEMS," *Proceedings of 10th AIAA/ISSMO Conf. on Multidisciplinary Analysis and Optimization*, Albany, NY, 30 August – 1 September, 2004.
5. Alwan, A. and Ananthasuresh, G. K., "Coupled Electrostatic-elastic Analysis for Topology Optimization using Material Interpolation," *Journal of Physics: Conference Series*, Institute of Physics Publishing, 34 (2006), pp. 264-271.
6. Tamm, I. E., *Fundamentals of the Theory of Electricity*, 1976, Mir Publishers, Moscow, pp. 162-183.
7. Hung, E. and Senturia, S. D., "Extending the travel range of analog-tuned electrostatic actuators," *Journal of Microelectromechanical Systems*, Vol. 8, No. 4, 1999, pp. 407-505.
8. Bendsøe, M. P. and Sigmund, O., "Material interpolation schemes in topology optimization," *Archives of Applied Mechanics*, Vol. 69, No. 9-10, 1999, pp 635-654.
9. Sigmund, O., 2001 "A 99 line topology optimization code written in Matlab," *Structural Multidisciplinary Optimization*, Vol. 21, pp. 120-127
10. www.comsol.com, COMSOL MultiPhysics software.
11. www.mathworks.com, Matlab software.