Gustave Eiffel and his Optimal Structures

M Meenakshi Sundaram and G K Ananthasuresh

The eponymous tower, as noted by its chief architect Alexandre Gustave Eiffel himself, overshadows his other structural marvels. It is not just the beauty of the Tower that ought to impress an onlooker but also its optimality. Eiffel, as we explain in this article, excelled in using the material in the most optimal way in many of his structures that include more than 42 bridges and many buildings built in various parts of the world. We also draw an analogy between Nature's designs and Eiffel's designs in the way the material is arranged hierarchically with different shapes at different scales. We consider the Maria Pia Bridge and the Eiffel Tower to illustrate this point and highlight their optimal characteristics. We argue that the structural designs of Eiffel & Co. have an aesthetic appeal that emerges from the economic use of material with their roots in rigorous engineering principles.

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Introduction

In a Telugu science fiction novel (*Anandobramha*, 1984) written by Yandamuri Virendranath, a teenager breaks down upon hearing that the Eiffel Tower is destroyed in a terrorist attack. Hollywood movies too have not spared the Tower from fictional attacks. From *The War of the Worlds* (1953), *The Great Race* (1965), *Mars Attacks* (1996) to the recent *GI Joe: the Rise of the Cobra* (2009), we come across heart-rending scenes of the collapse of the Tower or the Tower under siege. An emotional attachment to an inanimate structure may seem irrational. But, even for an engineer who might see the Tower as merely a wrought iron structure, it is difficult not to have an emotional affinity to it when one appreciates its mathematical elegance and structural ingenuity. The *300-metre tower*, as it was called when Eiffel & Co. built it in 1889, is adorned with optimality all through: in its skyline profile, the shape, the size, its foundation, and in its internal structure.

The Eiffel Tower has more holes than the material. It would not be an exaggeration to say that every beam and rivet in this structure was put to good use by placing it just where it should be to serve the overall purpose. The Tower is not the only creation of Alexandre Gustave Eiffel that shows this optimality and elegance. This famous French engineer's bridges and buildings too were trend-setters in his time and they continue to be inspirational for structural designers who want to achieve aesthetics and optimality simultaneously. In this article, we explain the notion of optimality found in Eiffel's structures by taking two of his famous designs, the Maria Pia Bridge built on the river Douro in Porto, Portugal (*Figure* 1 on p.842), and the 300-metre tower in Paris (*Figure* 5 on p.846). Before that, we present two fundamental concepts: the importance of shape at different levels of detail and the role of optimisation in structural design.

Hierarchy of Shape in Structural Design

Macroscopic Shape

If we look around, we find beams and columns of different cross-sections. Some have rectangular cross-section, some have I-sections, while the rest have other cross-sectional shapes. Similarly, rotating shafts have circular solid or tubular cross-sections. We rarely find rectangular cross-sections for rotating shafts. Why? Engineers have found that certain cross-sections are better than others for certain loads: stretching/contracting, bending, and twisting [1]. Take for instance, the twisting of a shaft. It does not make sense to put material at the centre of a large circular cross-section when much of the shear stress occurs away from the centre of the circle. Material, if it is put in the central portion, adds more load than it contributes to bearing the stress. The best structure is one in which all the material is more or less equally stressed. A shape that makes this possible can be considered optimal.

Shape specificity is not limited to cross-sections of beams, columns and shafts but is true in general. Let us ask why ornate temple towers of southern India have truncated pyramidal shape as they rise to great heights with masonry material. One can also ask why the great Egyptian pyramids have the pyramidal shape. They all have those specific shapes perhaps because it is the optimal use of material. A vertical structure such as a temple tower or a pyramid has more stress at the bottom than at the top. So, making it a prism makes little sense. More material is needed at the bottom and less as we go up the structure. But how should the cross-section size vary as we move from the bottom to the top? Temple towers and pyramids follow linearly tapering cross-section profile. The Eiffel Tower, on the other hand, has a curved profile. The arch in the Maria Pia Bridge has nonlinearly varying depth and width. There is a reason for these shapes as we shall see later in this article.

Nature's columns and beams are no different. Tree trunks have nearly circular solid or tubular cross-sections. They too taper down as we move up from the bottom to the top. Nature's other columns – the legs of large animals such as elephants – have circular cross-section but they do not taper. The legs of smaller animals including humans have a reverse taper. These 'columns' are mobile and their loading and end conditions are different from those of the trees; they are attached to something on both the sides. Hence, the stress pattern is different and so is the optimal shape.

If we look beyond just the external shape, we find specific shapes inside some structures. Nature has many examples of this. A tree has a trunk from which branches emerge. Each branch has sub-branches and then twigs. Leaves are attached to the twigs in the same as way as branches to the trunk, sub-branches to a branch, and twigs to a sub-branch. There is a *'fractal-like'* hierarchy here. There is a structural arrangement at different levels of detail within a structure. We indeed call it a *tree structure*. The root system of plants and trees is no different. Sub-roots emerge from the main root in much the same way. This particular divisional structure is dictated by the need to let the water and nutrients diffuse into the roots with a large surface area and get transported up the roots. Vasculature that provides for the blood flow in our bodies is similar. This is a sign of hierarchy of shape in Nature's structures that is easily visible at the macroscopic level. And there is also shape-hierarchy at the microscopic level.

Microscopic Shape

When it comes to trees, there is more to it than meets the blinking eye. If we begin to look at the internal structure of a tree trunk's cross-section, we find more organization in terms of shape. We see rings – a result of the seasonal growth pattern of the tree trunk. They are concentric shells of increasing size with foam-like material in between. If we look at a vertical section with a magnifying glass, we see fibres along the vertical axis of the trunk. Under a microscope, we see wood cells of prismatic and other shapes. There is a pattern to the shape of the cells too. The wood cells formed in the early part of the growing season have large cavities and thin walls while those formed later have small cavities and thick walls. This is very different from engineering materials we use. Metals and ceramics are homogeneous in the sense that they have uniform distribution of material and possess no discernible shape or pattern unless we go to much smaller scales. There are, on the other hand, a hierarchy of shapes and inhomogeneous distribution in Nature's materials.

Let us consider the internal structure of the materials that our bones are made of. There is the outer part, the *cortical* or *compact* bone; and the inner part, the *trabecular* or *cancellous* bone. The left side of *Figure* 1 shows the hierarchical structure of the compact bone. The basic units

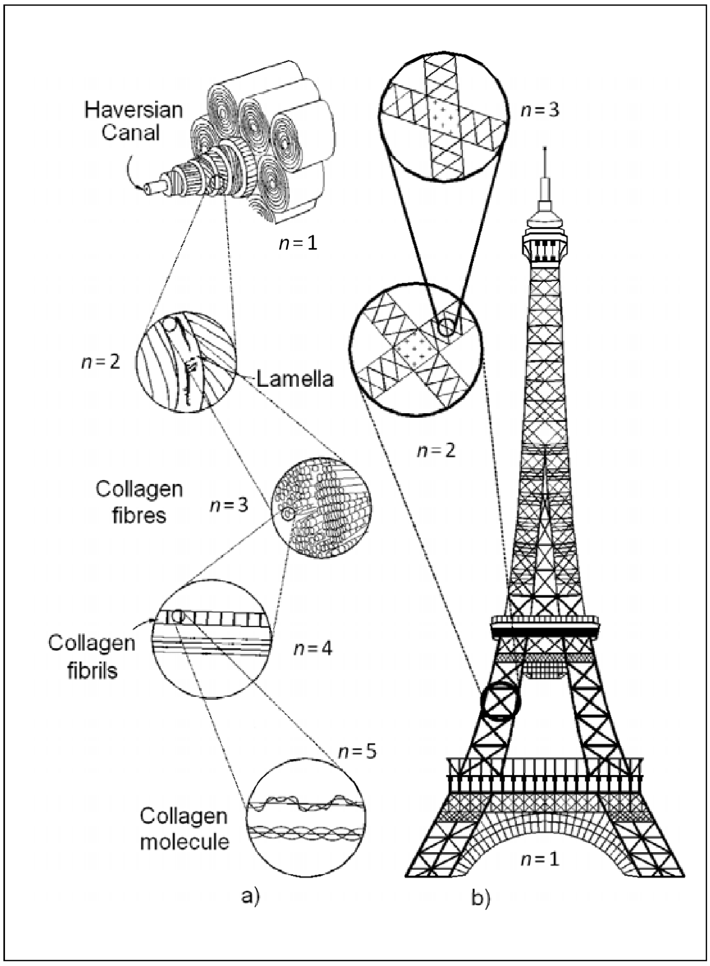


Figure 1. Shape-hierarchy in a compact bone and the Eiffel Tower: (a) Five hierarchical levels of shape in compact bone (adapted from [2]), (b) three levels of shape-hierarchy in the Eiffel Tower.

of this part of the bone are *lamellae*, which are layered rings surrounding Haversian canals that carry the blood vessels. A lamella is made of collagen fibres, which are in turn made of fibrils. A fibril comprises collagen molecules and minerals. As can be seen in the left column of *Figure* 1, there is a shape inside a shape at five different levels within the compact bone. If we denote the number of levels of detailed shape by *n*, the hierarchy is said to be of level-*n*. Hence, compact bone has level-5 shape-hierarchy. The shape and underlying material at each level has a purpose and it imparts special properties to the overall structure [2]. What can an engineer do when forced to work with a single homogenous material? Is shape-hierarchy a prerogative of only Nature? Eiffel's structures tell us that engineered structures too can have the shape-hierarchy.

Let us take another look at the Maria Pia Bridge and the Eiffel Tower. We should look beyond the basic external shape to notice the criss-crossing beams. A closer look at a single beam reveals that there are smaller criss-crossing elements within it. Eiffel's structures have level-3 hierarchy of shape. *Figure* 2, which is a close-up of an arch in Eiffel's famous viaduct – the Garabit Viaduct – makes this point; so does the right column of illustrations in *Figure* 1 where the shape-hierarchy of the Eiffel Tower is juxtaposed with that of the compact bone.

It appears that Eiffel was the first structural designer to exploit the shape-hierarchy. Today, it

Figure 2. A close-up view of Garabit Viaduct shows level-3 shape-hierarchy. At level 1, we have the arch shape and we see criss-crossing beams at next two levels with smaller sizes just like in the Eiffel Tower.
Adapted from www.panoramio.com.

is not an uncommon feature. See, for example, the Howrah Bridge located in Kolkata (*Figure* 3). This too has a level-3 shape-hierarchy. In fact, it has been observed that in the design of light-weight structures as well as engineered materials with voids, significant enhancement in stiffness for a given mass is achieved because of shape-hierarchy [2]. Thus, there is a correlation between optimality and shape-hierarchy.

Next, we discuss the optimality of shape in Eiffel's structures after introducing a few mathematical concepts of optimal structural design.

Optimal Structural Design

Philosophically, optimisation is a word that is said to have originated from optimism. The Merriam–Webster dictionary defines optimism as "*an inclination to put the most favourable construction upon actions and events or to anticipate the best possible outcome*". The process

Figure 3. Shape-hierarchy in the Howrah Bridge, Kolkata. The overall iconic shape of the bridge is one level. Within that we see cross-beams, which is the second level. Some of those cross-beams have smaller cross-beams within them, which is the third level.
Adapted from http://www.fotothing.com.

Box 1. An Optimisation Problem

$$\min_{\mathbf{x}} f(\mathbf{x}) \quad \rightarrow \text{ Objective criterion}$$

$$\begin{aligned} \text{subject to:} \quad & g_i(\mathbf{x}) \le 0 \quad i = 1..\,n \quad \rightarrow \text{ Inequality constraints} \\ & h_j(\mathbf{x}) = 0 \quad j = 1..\,m \quad \rightarrow \text{ Equality constraints} \\ & \mathbf{x}_l \le \mathbf{x} \le \mathbf{x}_u \quad \rightarrow \text{ Bounds on variables} \end{aligned}$$

An optimization problem in general has the above form and aims at minimizing an objective criterion subject to a set of constraints by selecting a proper design variable. The optimal value of the design variable **x** is the solution of the optimization problem.

of optimisation aims for the same. Let us see this in the context of optimal structural design. We aim to find the best design among all those that satisfy certain requirements. Let us say that we want to design a bridge over a river with the following requirements. It has to span a certain length between the two banks of the river. It needs to be of a certain height above the river. It should be strong enough to bear, in addition to its own weight, the load that will act on it due to vehicles that pass over it and the load due to wind. It should be of certain stiffness. One might also stipulate that it be made of a certain material, say iron. Many bridges can be conceived to meet these requirements. In order to choose the best among them, we need to define a criterion. Let that be the weight of the bridge. Now, we can pose an optimal design problem wherein we want to minimize the weight of the bridge subject to constraints as per the aforementioned requirements. See *Box* 1 where such a problem is written as a general optimisation problem.

The symbol f in *Box* 1 defines the objective criterion. In the case of the bridge, it is the weight. Equality constraints may come from the span of the bridge and the height of the bridge. Inequality constraints come from the stiffness and strength considerations. We want the maximum stress to be lower than the strength of the material and we want the bridge to be stiff enough to have its deflection less than a stipulated value. The objective criterion and constraints are shown to be functions of $\mathbf{x} = \{x_1, x_2, x_3, \ldots, x_p\}$, which we call the *design variables*. An optimisation algorithm would find the numerical values of these variables so as to minimize f while satisfying the constraints. But what are these variables? They decide the geometric form of the structure, the bridge in our case.

The geometric form of a structure can be looked at three different levels: topology, shape, and size. Let us consider a hypothetical bridge design problem shown in *Figures* 4a. It has two supports and two loads. Let us denote the support points as S_1 and S_2, and the loading points as L_1 and L_2. These can be interconnected in many ways to transfer the load to the supports.

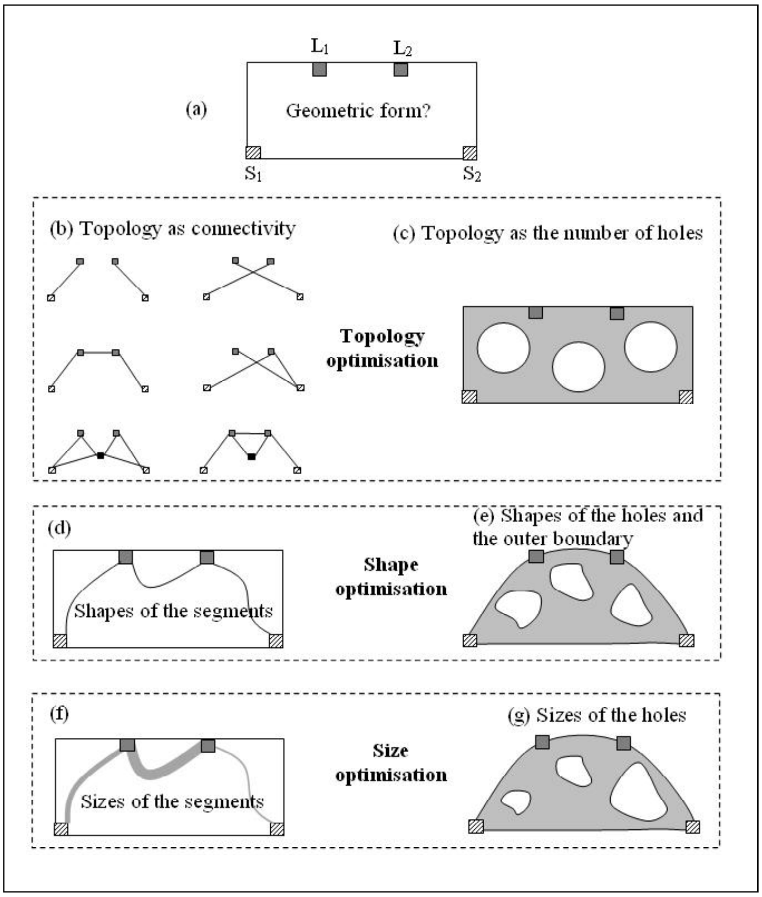
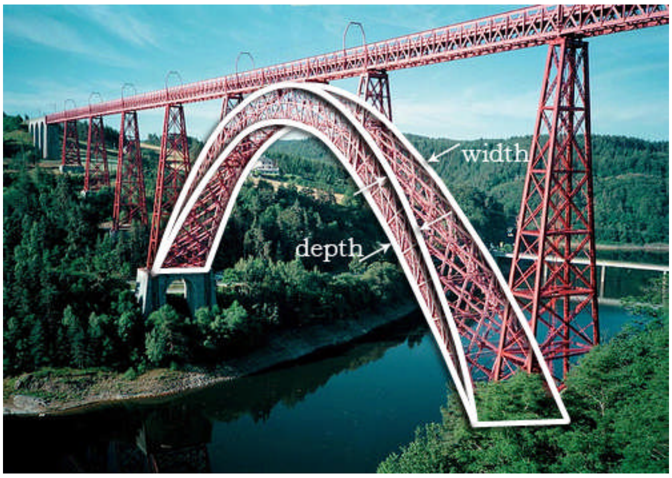


Figure 4. Hierarchy in structural optimisation: (a) A hypothetical specification of a structural optimisation problem, (b) connectivity among different portions of interest in a structure; one view of topology, (c) another view of topology as the number of holes, (d) shapes of the segments, (e) shapes of the holes, (f) sizes of the segments' cross-section profiles, and (g) sizes of the holes.

Figure 4b shows a few possibilities. The last two possibilities also consider an intermediate point not specified in the problem but a bridge designer is free to do it if that helps. If there are more intermediate points, there will be many more connections. Among all those, which connection is optimal? The process of finding this is called *topology optimisation*. Another view of topology optimisation is offered in *Figure* 4c. Here, we ask how many holes there can be in the geometric form of the bridge.

After the topology is known, we want to give optimal shapes to the connections (*Figure* 4d) or we want to determine the optimal shapes of the holes (*Figure* 4e). We call this *shape optimisation*. The next step after this is to optimise the sizes. *Figure* 4f shows that the size along the shapes can be varied in numerous ways. *Figure* 4g shows this in a different way by varying the sizes of the holes whose shapes are now determined. Here lies the *size optimisation*. In this, we define the structural design problem hierarchically. But this is only qualitative hierarchy and not quantitative hierarchy that we discussed in the last section.

Figure 5. Shape variation overlaid in solid white curves in the arch found in Garabit Viaduct. Both the width and depth are varied but differently. Widest width is at the fixed supports while the widest depth is at the centre.
Adapted from www.panoramio.com.



Based on the type of structural optimisation we want to do (i.e., topology, shape, or size), we choose the design variables, $\boldsymbol{x} = \{x_1, x_2, x_3, \ldots, x_p\}$, accordingly. Let us consider the simple one first, the size optimisation. Here, we designate some parameters (widths, thicknesses, lengths, etc.) as the design variables assuming that we know the shape and topology. In shape optimisation, we need to use a different set of variables to vary shapes to find the optimum shape. Let us consider the shape of Eiffel's arch in his Garabit Viaduct or the Maria Pia Bridge. As can be seen in *Figure* 5, he varied it in two ways: the depth of the arch is zero at the fixed supports and widens gradually to the centre where it is the largest. The reverse is true of the width of the arch. The width is most narrow at the centre and widens to a maximum value at the fixed supports. Such shapes can be computed by posing the shape optimisation as a 'calculus of variations' problem.

Defining variables for topology optimisation is tricky. If we think of it as finding the optimal connectivity as in *Figure* 4b, it becomes a combinatorial optimisation problem. Instead, it can be posed as a continuous optimisation problem. The clue for this lies in the way we interpret materials with structural hierarchy as new materials with homogeneous properties. In general, even if there is a microscopic structure for a material, we can write a material property for a structure (whose size is much larger than the microscopic shape) using a simple approximate relationship. Let us take Young's modulus, the basic material property that relates stress and strain for an elastic material. Let us denote this for the base material by E_0. Now, for the nth-level hierarchy, the homogenous Young's modulus E_n can be written as [2]

$$E_n = k^p \rho^r,$$

where k and r depend on the shape of the microstructure and ρ is the volume fraction of the

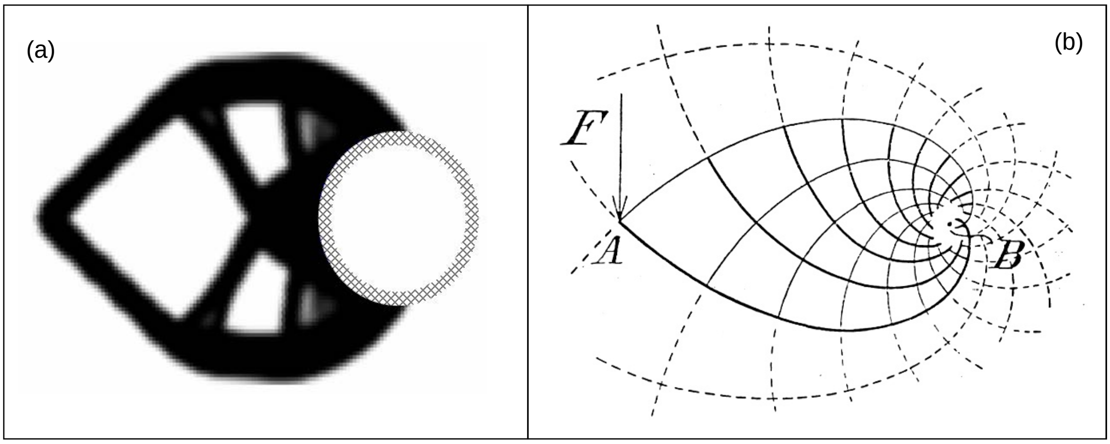


Figure 6. (a) Optimal geometric form given by an optimisation algorithm. (b) Optimal distribution of a framed structure for a benchmark problem as given by A G M Michell in 1904 [4].

material at the microscopic shape that has holes in it. By following this, we can choose ρ as the design variable for topology optimisation. We will then interpolate homogeneous Young's modulus E as follows by choosing $k = 1$.

$$E = E_0 \rho^r.$$

We assume that when $\rho = 1$, there will be material at that point in the structure; and when $\rho = 0$, there will be a hole. By letting ρ vary between 0 and 1 at every point in the structure during the process of optimisation, we can get any geometric form – not just topology, shape, or size one at a time but all of them at the same time. It is possible that ρ may stay in between 0 and 1. Then, there are numerical tricks to push them to 0 or 1. Usually, when r is greater than or equal to three, we get 0–1 designs without the 'intermediate' material [3].

Figure 6a shows a problem solved in the manner described above wherein material can be optimally distributed within a given space. It shows the result of the optimum geometric form of a structure that ought to be the stiffest under a given load with a given amount of material. The structure is attached to a fixed circle on the right and the load acts at a distance away from the circle. In the black and white image of *Figure* 6a, $\rho = 1$ where black colour (material present) is shown and $\rho = 0$ where white (empty; material absent) is shown. This is the optimal structural form predicted by the optimisation algorithm for ρ at every point in the rectangular space. *Figure* 6b shows an analytical solution given by A G M Michell in 1904 in a classic paper entitled 'On the Economy of Material in Frame Structures' [4]. One can see the similarities in topology between *Figures* 6a and 6b. But there are also differences. *Figure* 6b indicates that there can be many criss-crossing spiraling beams, in fact, as many as one can manufacture in as thin a width as possible. The optimal form shown in *Figure* 6a is a practical solution that can be realized.

Armed with a technique to compute the optimal structural form, we will now proceed to examine optimality in the Maria Pia Bridge designed and built by Eiffel during 1875–1877.

Optimality of the Maria Pia Bridge

In 1875, the Royal Portuguese Railroad Company invited proposals to build a bridge over the river Douro between Lisbon and Porto. It entailed many challenges. The river was 20 m deep and the river bed had loose soil and the currents were rapid. So, no pier could be constructed in the river. It meant that the central span had to be 160 m long, the longest at that time for a non-suspension bridge. Eiffel's company was still young at that time. Yet, among the four proposals submitted, Eiffel's company quoted the least. To be precise, Eiffel quoted 0.965 million francs while the next closest bid was 1.41 million francs. The other bigger companies had quoted nearly twice and thrice of Eiffel's quote. Among the four designs submitted, Eiffel's design was also clearly the most beautiful. It was selected by the Portuguese government.

Eiffel got it built in less than two years. As shown in *Figure* 1 on p.842, it had an arch in the middle that supported the railroad with piers that supported it away from the river banks. It had a total length of 352.75 m and a height of 62.40 m. When we examine the other three design proposals [5], we see that Eiffel's Bridge used the least amount of material. In order to investigate this, we ran an optimisation program that optimally determines the geometric form of a structure using the method described in the previous section.

In *Figure* 7a, we show the specifications of the problem in two dimensions even though a bridge is a three-dimensional structure. We traced the shape of the land available for fixing the bridge on either side of the banks of the river on a photograph and took its size as shown in *Figure* 7a. The size we took is not exactly the same as what Eiffel had considered but the proportions are preserved. We considered uniform loading on the bridge throughout its span. We used properties of steel for Young's modulus (210 GPa) and Poisson's ratio (0.3). The available space was divided into 6,764 quadrilateral elements and a design variable was assigned to each of them. The algorithm determines the ρ value for them. The solution given by the optimisation algorithm is shown in *Figure* 7b for 35 % material constraint to fill the space below the straight railroad plate and up to the landmass on the banks. Its likeness to Eiffel's design (*Figure* 3c) is striking, some minor differences notwithstanding. The other three proposed designs were quite different in their form. One can only wonder how Eiffel and his engineers thought of an optimum form more than a century before a mathematical method could be developed to predict the optimal geometric form for the stiffest structure with a given amount of material. The small

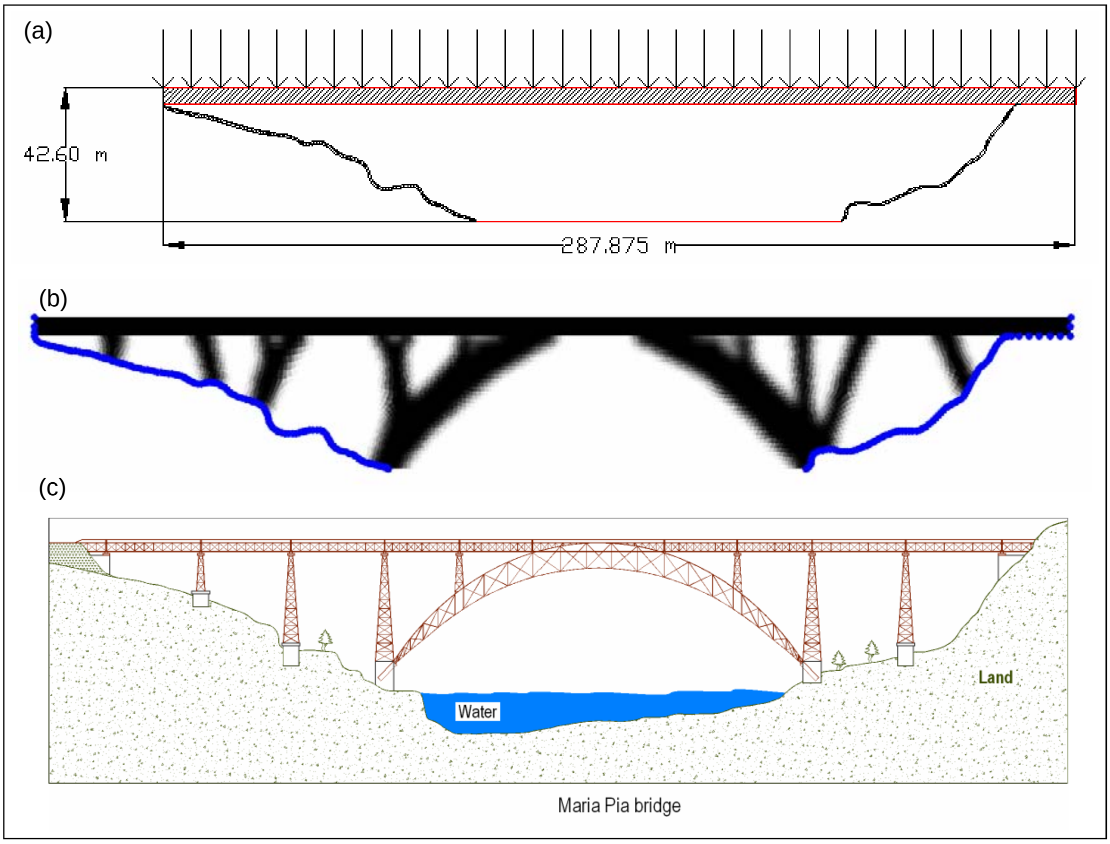


Figure 7. The Maria Pia Bridge problem in two dimensions: (a) The specifications for load, space available, and where the bridge structure could be fixed, (b) optimal geometric form given by the algorithm, (c) the design submitted by Eiffel's young company (redrawn from [5]).

difference in the number, location, and shape of the piers can be attributed to the different loads and manufacturing considerations Eiffel might have taken into account. Above all, the design of Eiffel company's engineers Theophile Seyrig and Henry de Dion also preserved the beauty of the landscape of the countryside with its elegant arch and mostly empty space inside and under the bridge. We have already discussed the level-3 shape-hierarchy in this bridge.

Next, we consider the optimality of shape of the Eiffel Tower itself.

The Optimal Shape of the Eiffel Tower

In the United States of America, Clarke, Reeves and Co. had proposed to build a tower for USA's Centennial. The tower was to be made of wrought iron and was to be 1000 ft (305 m) tall. But it never happened because of lack of funds. Perhaps inspired by this idea of building a tower that is twice as tall as the tallest at that time, the Washington Monument in USA's capital city, Eiffel took up this task [6]. Two engineers of Eiffel and Co., Emile Nouguier and Maurice Koechlin, were thinking of building a tall tower for some years by then. They came up with an idea for a 300 m tall tower when they were discussing possible attractions for the Centennial Exposition of Paris in 1889. It was in these minds that the 300-metre tower was envisioned. They came up with a design of a great pylon with four lattice girders standing apart at its base and gave it an architectural touch with the help of the company's chief architect Stephen Sauvestre. He gave the pylon its tower form with the arches at the bottom, and a platform at the first level. The year was 1884, and still only a tentative idea, but a good one to entice a great and daring builder like Gustave Eiffel. He had it patented in the same year.

Coincidentally or maybe because of the strong recommendations of Eiffel, the French government organised a competition for the construction of a 300 metre tower in Champ de Mars in 1886. Eiffel and Co. won this competition among more than 100 submissions. The tower won the competition for its optimality and many uses elaborated by Eiffel and Co. It was to be made of wrought iron that was known for its high strength to weight ratio. Its construction began by January 1887 and was completed by March 31, 1889, two months ahead of schedule with a few weeks to spare before the Centennial Exposition began in Paris.

Earlier in this article, we had commented on the shape of free-standing structures – human-made and those of Nature. Let us now discuss how the optimal shape was obtained by Eiffel's engineers. They were very familiar with the construction of piers in bridges wherein the wind loading was the major load on the structure. In the case of the piers they had to balance it against the load of the wind on the viaduct rather than on the pier itself, which is negligible. But in the case of the tower they had to take only the effect of the wind on the pier into account. Eiffel notes that the elaborate truss structure would hardly be efficient and hence one has to do away with the cross-beams (trellis elements) by shaping the four uprights linked with a few interspersed horizontal belts instead of the trellis bars [7].

In order to understand how they managed to remove the trellis elements among the four vertical arches, let us consider a simple truss structure shown in *Figure* 8. The wind loading is lumped at the vertices shown as P_1, P_2, and P_3. Let their resultant force be P_R whose line of action is shown in the figure. Members BE and CD are

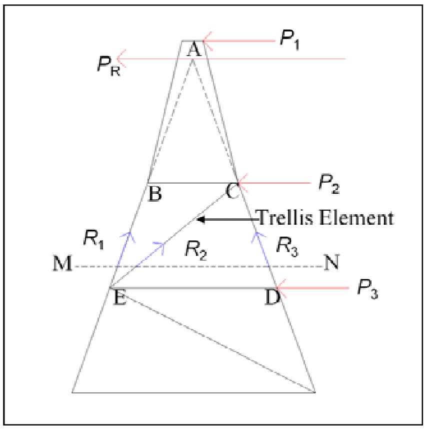


Figure 8. A simple truss example to illustrate how cross-beams (trellis elements) can be eliminated as per the reasoning used by Eiffel's engineers. (Redrawn from [7])

inclined such that they intersect the line of action of the resultant at A. Let the reaction forces at the section MN be R_1, R_2, and R_3. Since we have a structure here, the net moment at any point must be zero. So considering the moment about A, one can easily see that the moments due to R_1, R_3, and P_R must be zero as these pass through the point A. So, all that is left is the moment due to R_2, which must be zero. This implies that R_2 must be zero. Thereby one can do away with the trellis element EC. This way, it is possible to construct a tower that does not obstruct the view behind it.

With the above reasoning, Eiffel and Co. considered two worst cases of loading: (i) 300 kgf per square metre against the height of the tower, and (ii) linearly varying intensity of 200 kgf per square metre from the bottom to the 400 kgf per square metre at the summit. They had graphically obtained the profile of the tower. In a recent paper entitled 'Model equations for the Eiffel Tower profile: historical perspective and new results' [8], Weidman and Pinelis developed a nonlinear integro-differential equation modeled along the lines of Eiffel's discussion in his proposal and proved that the skyline profile is a solution to the equation.

To illustrate the optimality of the shape of the Eiffel Tower, we ran a shape optimisation program for maximizing the stiffness against the two loadings that were considered by Eiffel and Co. We modeled the entire structure as a continuum that has both the properties of a bar and a beam (a frame element as it is called in mechanics) and possessing square cross-section. We used the material properties of steel. The optimal shape profiles we obtained are shown in *Figure* 9.

Figure 9a corresponds to a linearly varying wind load and self-weight and *Figure* 9b to a steady wind load and self-weight. As remarked by Eiffel and Co. in their proposal, the shape is more

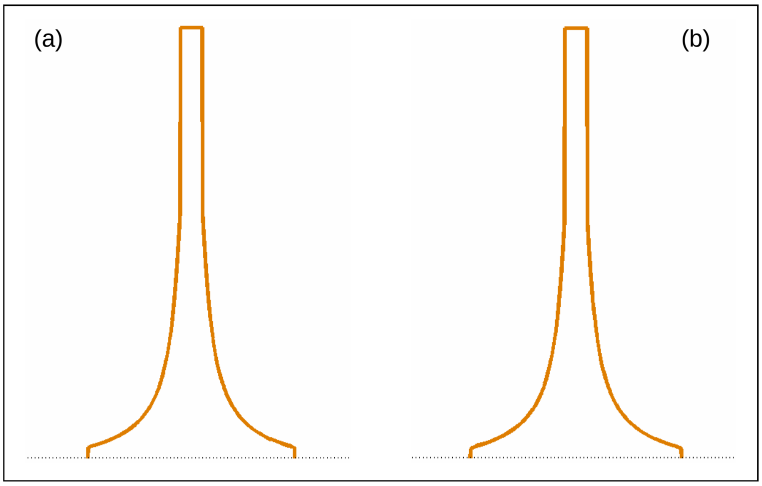


Figure 9. Optimal shapes for the skyline profile of the Eiffel Tower: (a) for linear variation of the wind load, (b) for uniform wind load. Both cases consider the self-weight.

or less the same for both the conditions. It should be remarked that Eiffel considered the worst possible wind condition and designed the entire structure according to it. Our shape optimisation illustrates the optimality of the gross outer shape and not the inner structure that Eiffel had to consider. As noted earlier, Eiffel Tower has level-3 shape-hierarchy and is indeed optimal in its overall shape and its material usage to create that shape. By creating holes, the wind load was reduced too.

Optimality and Aesthetics

Eiffel's optimal structures were not limited to bridges and his famous tower. His company had designed and built many buildings that include railway stations, schools, religious places of worship, houses, and observatories (see *Figure* 10). In all of them, he paid attention to the way materials were used. Most of his structures had the mark of shape-hierarchy and they all used as small an amount of material as was needed. Perhaps that is why his company won many competitive bids to build major structures. It is important to note that Eiffel's company used innovative techniques in building the structures. Most were completed within two years and were always in time or ahead of schedule and within the initially projected budget. Out of this optimality, Eiffel also achieved beauty.

Eiffel's Tower and his many bridges were appreciated for their aesthetic value. In a period that had mostly masonry structures, Eiffel exposed the ironwork whose elegant and optimal shapes evoked aesthetic appeal. Of course, there were critics who found his structures, including the Tower, ugly. Eiffel defended himself by saying that "*the intention of a building should be openly declared; the various materials should be used in a deliberate way; why should the*

Figure 10. Observatory in Nice, France, designed by Charles Garnier, the architect, and its dome designed by Gustave Eiffel. The right-side picture shows the inside view of the dome that could be turned by hand, even by a child, because of a floating ring designed by Eiffel. Level-2 shape-hierarchy is visible when we look at the semicircular arches used in the dome.

Photographs adapted from www.wikimedia.org.

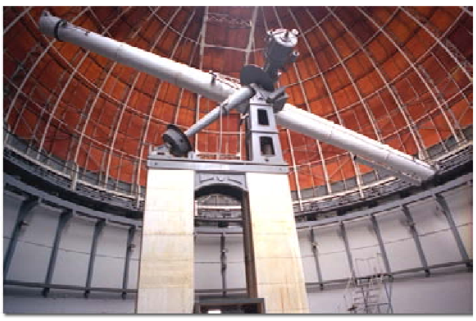


Figure 11. Pest Railway Station in Budapest, Hungary. Left: exterior, right: interior.
Adapted from www.dkimages.com.

industrial nature of a building be disguised, even in the middle of a city?" [5]. This defence was aimed at the Budapest railway station that Eiffel's company built between 1875-1877. Here, he used masonry, iron, and glass to achieve a grandiose structure (see *Figure* 11) that showed the interior through the glass-paneled gable that had the ironwork exposed to the outside.

It is pertinent here to mention a comparison made between the Eiffel Tower and neo-impressionistic paintings of Georges-Pierre Seurat (1859–1891), a French painter. Seurat pioneered a technique of juxtaposing tiny colored dots which remain close to each other but separate and yet evoke a sense of continuity and elegance when perceived by human eyes. Seurat was influenced by the scientific principles of perception of color and achieved an amalgamation of science, emotion, and beauty. Therefore, the analogy between Seurat's paintings and the Eiffel Tower, and perhaps all his structures, is very apt. In the words of an art-historian, Meyer Schapiro [5], this analogy comes out most clearly: "*…the constructions of the (Eiffel's) immense monument out of small exposed parts, each designed for its place, and forming together out of the visible crisscross and multiplicity of elements a single airy whole of striking simplicity and elegance of shape, was not unlike his own (Seurat's) art with its summation of innumerable tiny units into a large clear form which retained the immaterial lightness evident in the smaller parts…*". Eiffel's designs continue to appeal to the aesthetic eye as much as to the engineer's eye that also cares about optimal use of material and resources.

Closure

Optimality runs through most of Eiffel's works that included not just his famous tower but also many bridges and buildings. The shapes of his structures, and more importantly, the internal shape-hierarchy of his structures remains his hallmark. By describing the shape hierarchy found in Nature's designs, we presented an argument that Eiffel's structures too are optimal. To further emphasize the point, we discussed the optimality of two famous structures of Eiffel. The Maria Pia Bridge's geometric form was nearly reproduced by an optimisation algorithm we described briefly in this article. We also included the result of shape optimisation of a vertical structure to show the resemblance between Eiffel Tower's skyline profile and the optimum shape solution.

Optimal traits of Eiffel were not limited to his structural designs. His manufacturing techniques, assembly of bridges, site-planning and management were also optimal in their own right. For instance, he found it economical and efficient to serve subsidised lunch to construction workers on the first level of the Tower rather than let them come down and then go up during the Tower's construction. His four-volume biography meticulously put together by himself projected his accomplishments (sometimes at the expense of not giving due credit to his collaborators) can also be thought of as an optimal exercise in ensuring his preeminence for posterity. He had unfulfilled dreams and a few failures too. His company could not build the Panama Canal for which he had a grand design of a flowing staircase. The other aborted projects included a fully submerged bridge across the English Channel, an observatory on Mont-Blanc, and the metro system for the city of Paris. Eiffel did not lose heart in such failures; like a true optimiser or an optimist he moved on to other endeavours. He invented new uses for his tower in terms of commercially rented space and, more importantly, for his scientific experiments on estimating aerodynamic loads, meteorology, and radio communications. His mind, perhaps, always found an optimal solution for the objective criterion no matter however severe the constraints that life brought to him. His contributions made in the *belle epoch* (beautiful era) of France are long-lasting. His bridges and buildings are either historic landmarks or are still in use across the world in South America, China, Philippines, and Vietnam in addition to the continental Europe.

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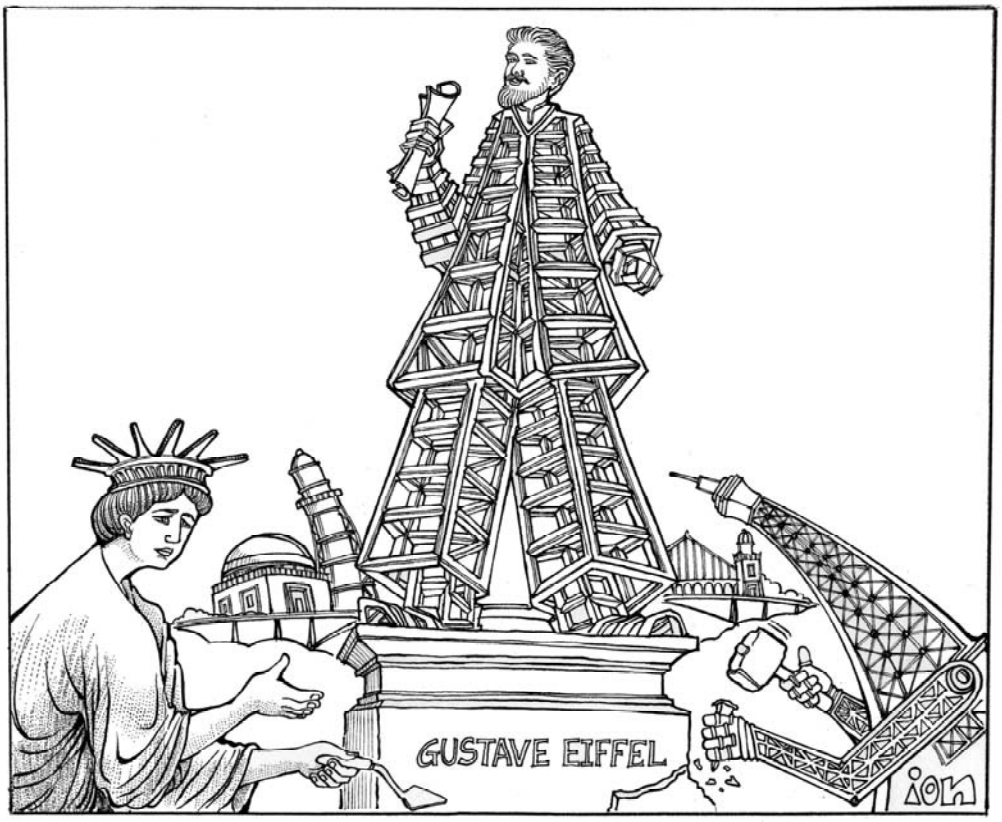

As a tribute to our creator we made an optimal statue of him ..