Lecture 1

An Overview of Structural Optimization: Size, Shape, and Topology

ME260 Indian Institute of Science

Structural Optimization: Size, Shape, and Topology

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Outline of the lecture

Understanding the spirit of optimization

What makes up an optimization problem?

What is structural optimization?

Topology, shape, and size

What we will learn:

What is optimization? Philosophically and mathematically?

What is structural optimization? Hierarchy in structural optimization.

How do we distinguish among topology, shape, and size?

Optimization and its spirit

Optimization is achieving the best with the available resources while satisfying the constraints.

- We optimize in our daily lives. Nature seems to have optimized almost everything.
  - It is about surviving with what one has and getting the best out of it.
  - Optimization is a way of life!
- Optimization has a lot to do with optimism.
  - Optimists view the proverbial glass half full and not half empty.
  - Given any number of obstacles (i.e., constraints), optimists try to make the best out of the situation.

Optimization hinders evolution!

- It is a witty way of saying the essence of optimization.
- Eventually, everything gets optimized.
  - The fittest and the best survive. The fittest would have used resources in the most effective manner and would have surmounted or circumvented the obstacles.
- But when we optimize at this instant, evolution is not necessary as we have already achieved the best already.
  - So, evolution is hindered.

Everything can be optimized.

- It is not an exaggeration; indeed everything can be optimized.
- It is simply a question of knowing what is the best, what the issues are, and how we can achieve it.

What do we need to optimize?

Objective function

- This is the most important thing in optimization; we need to know what we want to improve to the extreme.
- Extreme can be a maximum or minimum, depending on the identified objective.
  - If it is cost, we minimize; and if it is profit, we maximize.
  - In structures, if it weight, we minimize; and if it is strength we maximize, most often.
- We call it an objective function because it must depend on some variables in order to optimize.
  - The objective function should be **a function of optimization variables**.

Optimization variables

- These are the variables to which we try to assign suitable values to optimize the objective function.

Constraints

- Quantities that should be within bounds while we optimize the objective function.
- Constraints too are functions of optimization variables.

Formulae or methods to quantify and compute objective function and constraints

- We should have either mathematical expressions (formulae) or numerical methods to compute the values of the objective function and constraints once we know the values of the optimization variables.

There will be subsidiary variables too. (more later: see Slide 9)

There will also be constraints that govern those subsidiary variables.

Subsidiary variables are called state variables in the context of structural optimization.

There must be conflict in order to optimize!

It is no fun to maximize $f = 2x$!

- You would simply say that x should be ∞.
- This objective function is unbounded: you can make x as large as you want to maximize the objective function.

It is more fun to maximize $f = 2x$ subject to $x^2 + y^2 - 4 \le 0$.

- Now there is an upper limit on x.
- Even if you have $y = 0$, x is bounded from above at 2. So, $f = 2 * 2 = 4$ at best.
- It will be more fun if y is also constrained in some other way to prevent $2x$ from becoming too large.

Basically, there must be conflict between the objective function and at least one constraint.

- This should be so with respect to each optimization variable.

Conflicts can be between two (or more) constraints too.

Conflict can be within a single function, i.e., the objective function.

- Consider $f = 2x - x^2$.
- Now, at different points (i.e., at different values of x), as you move to the right or left, you have no conflict; you simply in the direction of increasing f.
- But then you come to a point ($x = 1$) at which you have a conflict; there, f does not increase whether you move to the right or left. Then, you stop; you have reached your maximum.
- **Here, the conflict is in the two terms within f.**
  - $2x$ says you should make x as large as possible.
  - x^2 says that you should make x as small as possible because it is subtracted.

Conflict within a function: non-monotonicity

Conflict of the terms within an objective function

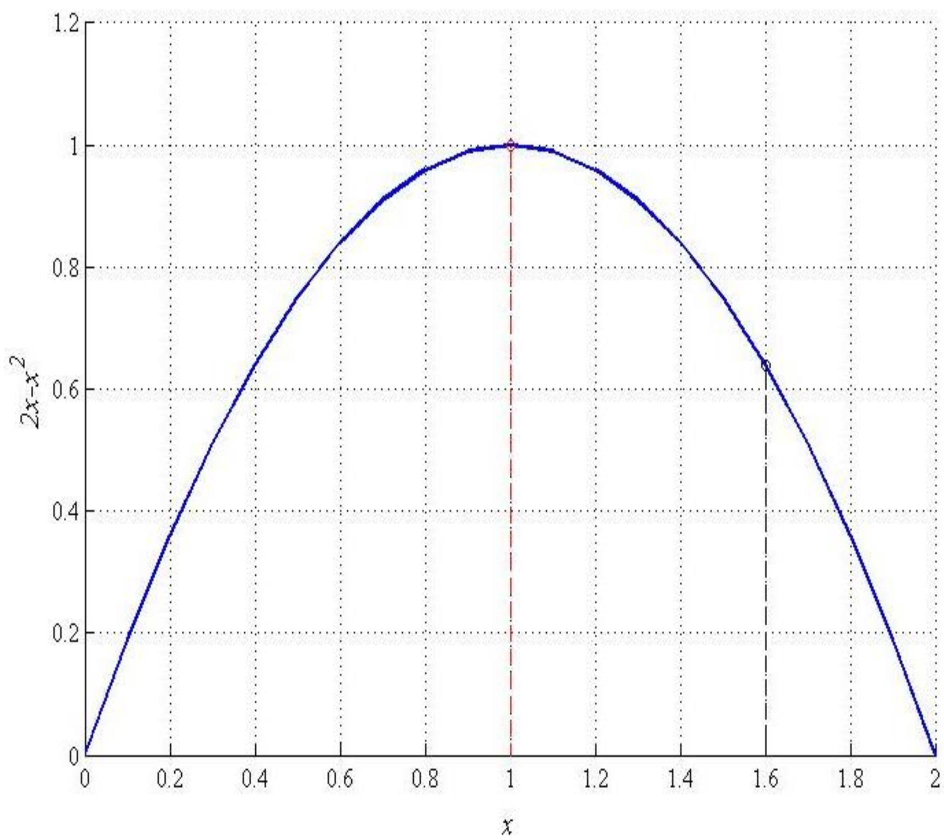
$$2x - x^2$$

- While $2x$ increases with x and makes f larger, $(-x^2)$ decreases f with increasing x.
- Such as function is called non-monotonous.
- Monotonicity, i.e., a situation when a function increases with increasing x and decreases with decreasing x, does not give an optimum.
  - As we say, in real life, that monotonicity is boring? So, it is in optimization.

Conflicting monotonicities in objective and constraints

- If the objective function is monotonous and a constraint is also monotonous, then their monotonicities should be opposite.
- That is, when x increases, one should increase and the other decrease, and vice versa.
- Conflicting monotonicites can be in two different constraints also.

Anyway, the main point is that conflict is crucial to optimization.



Is there conflict in structural optimization?

Sure, there is.

- If you want to make a stiff structure for given loading, you need more material; more material increases the weight and cost.
- So, there is conflict if you want to design the stiffest structure with least amount of material.

What if we want to make a lightest structure with high natural frequency?

- Light structures have low inertia and low stiffness too, at least in general. This will mean that their frequencies will be low.
- So, there is conflict.

Suppose that you want to make a flexible structure that is very strong.

- Flexible structures deform and it may seem that they are weak when strains are large in them.
- So, there is conflict too.

Imagine a structure that is subject to multiple loading conditions.

- Making a structure stiff under one loading may cause it less stiff in another loading.
- So, there will be conflict.

Imagine more situations of designing structures. There will be enough conflict!

Optimization problem statement

Minimize Objective function

Optimization variables

Subject to

Constraints

Limits on variables

Minimize $f(x_1, x_2)$

x_1, x_2

Subject to

$$h(x_1, x_2) = 0$$

$$g(x_1, x_2) \le 0$$

$$x_1^l \le x_1 \le x_1^u$$

$$x_2^l \le x_2 \le x_2^u$$

This is a typical constrained optimization problem statement. Make it a habit to write in this format.

Structural optimization problem statement

Minimize Objective(optimization variables, state variables)

Optimization variables — Related to the geometrical features

State variables — Displacements, temperature, electric field, etc.

Subject to

Constraints on state variables — Governing differential equations

Constraints on resources — Weight, cost, size, etc.

Constraints on performance — Stiffness, strength, frequency, etc.

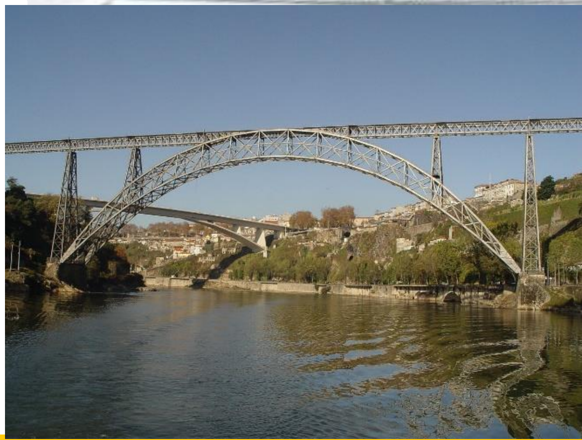
Limits on variables

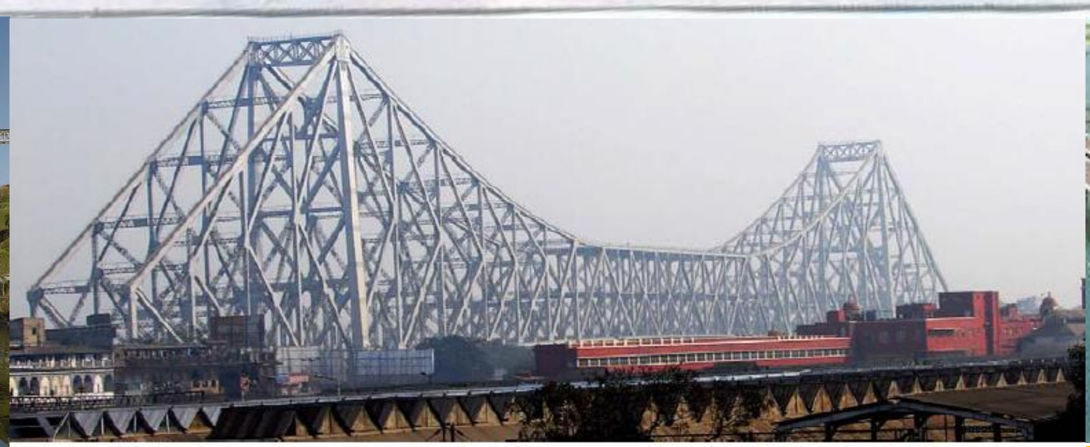
Data — Material properties, loads, etc.

This is a typical structural optimization problem statement. Make it a habit to write in this format, including the data.

Structural optimization

Is this how we design a bridge?





Structural optimization

Optimizing a structure for

- Stiffness
- Strength
- Flexibility, desired motion
- Natural frequency, mode shapes, dynamic response
- Stability, preventing buckling
- Weight reduction
- Cost reduction
- Manufacturability
- Reliability
- Controllability
- Safety
- Aesthetics

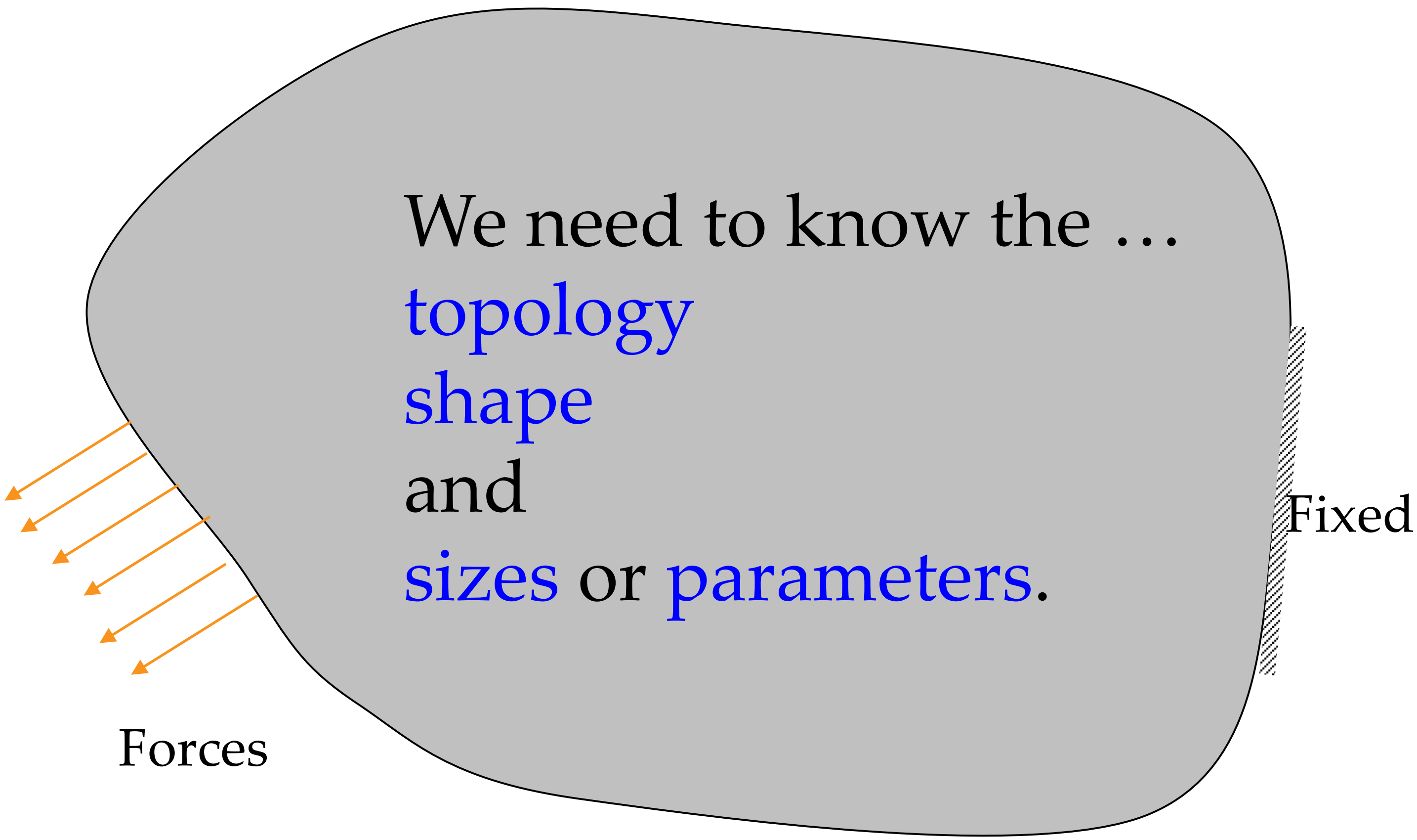
Keep also in mind:

Hierarchy
Modularity
Complementarity

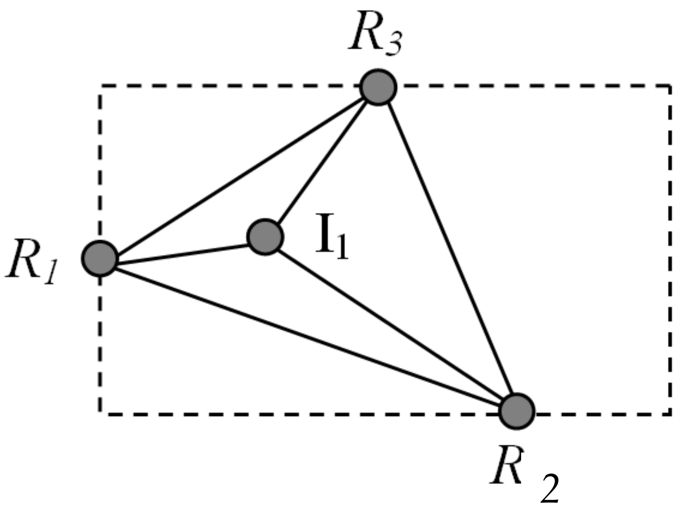
The more you think, the less material you need.

Stiff structure design

Obtain the stiffest structure with a given amount of material

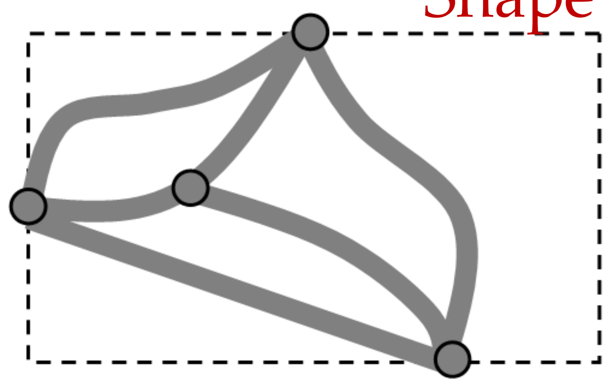


Hierarchy in SO

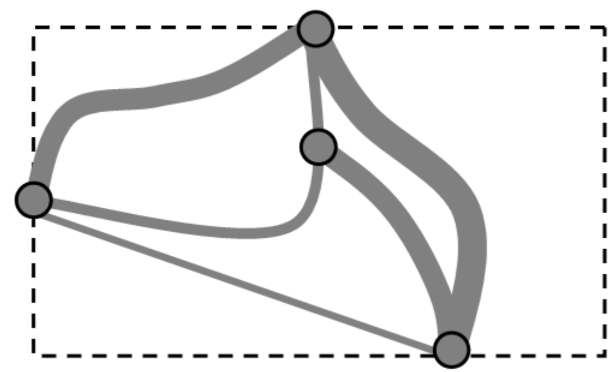


Topology (connectivity)

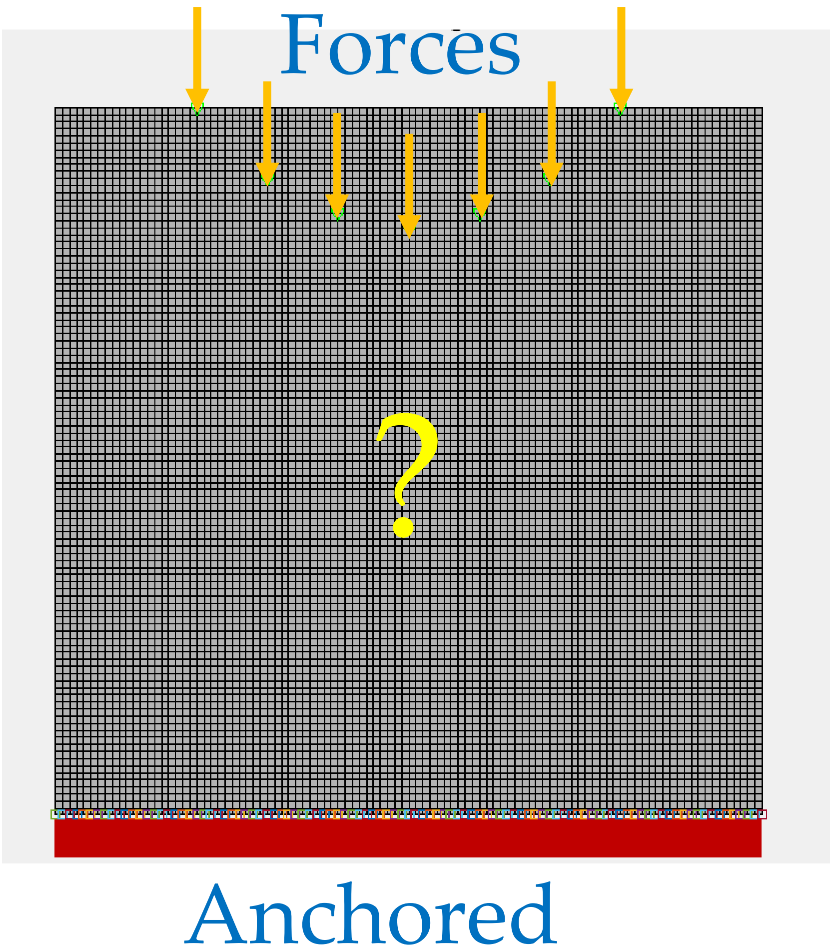
Shape

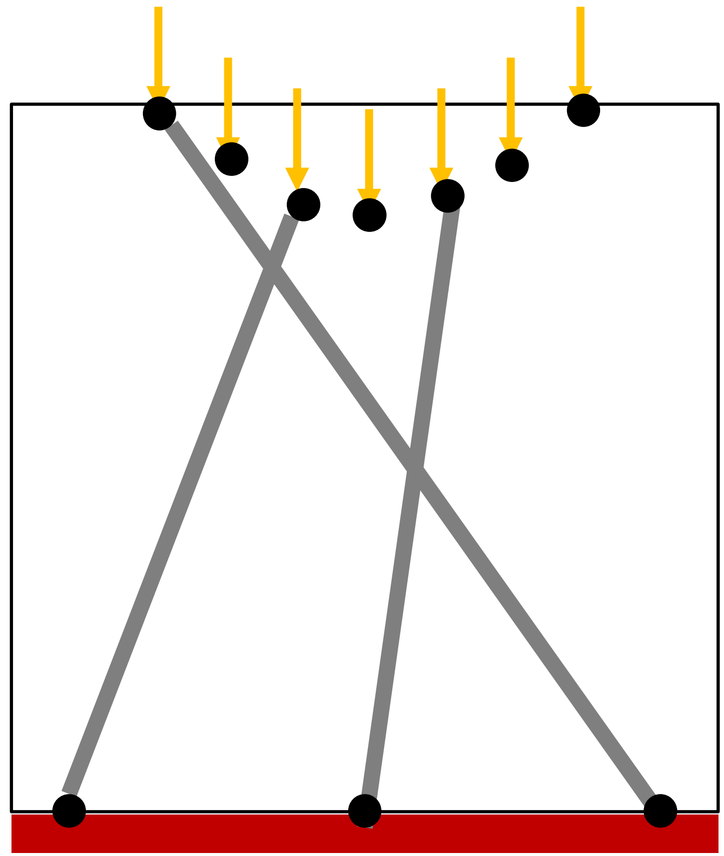


Size/parameter

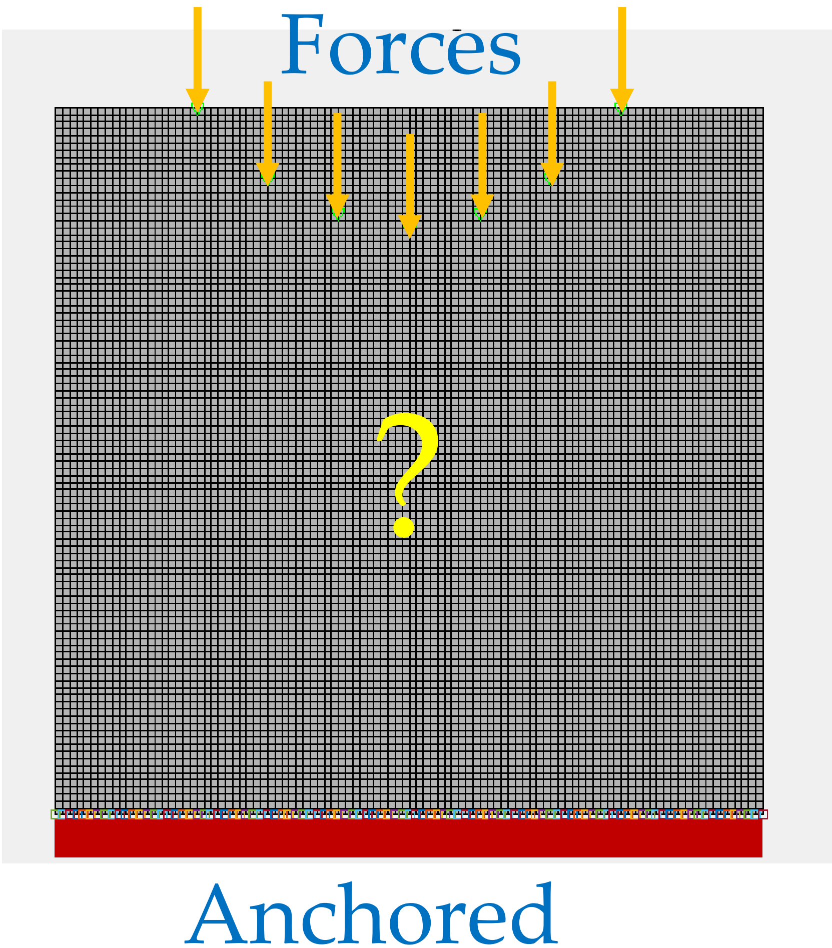


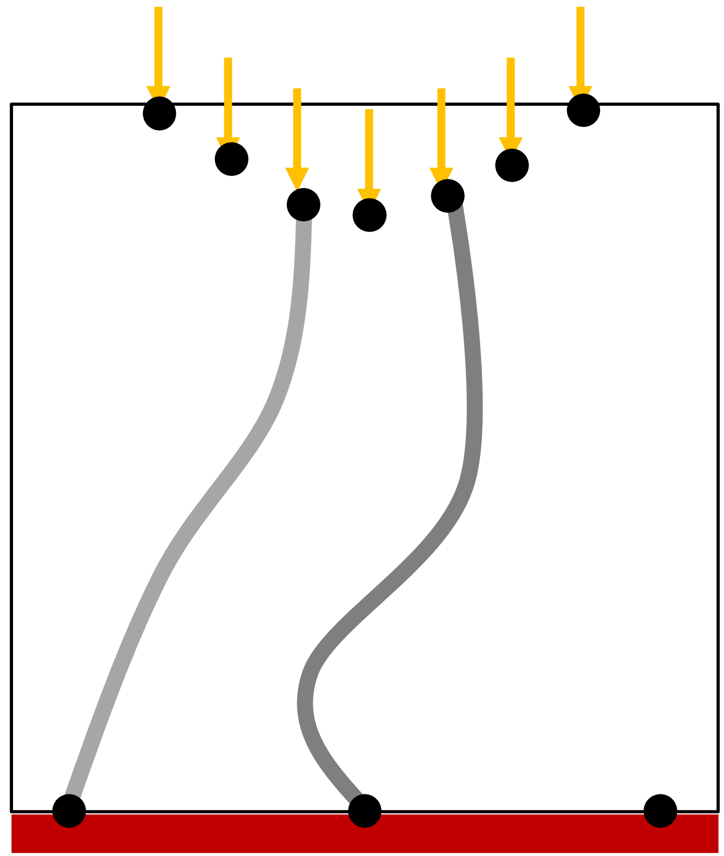
In how many ways should we connect?



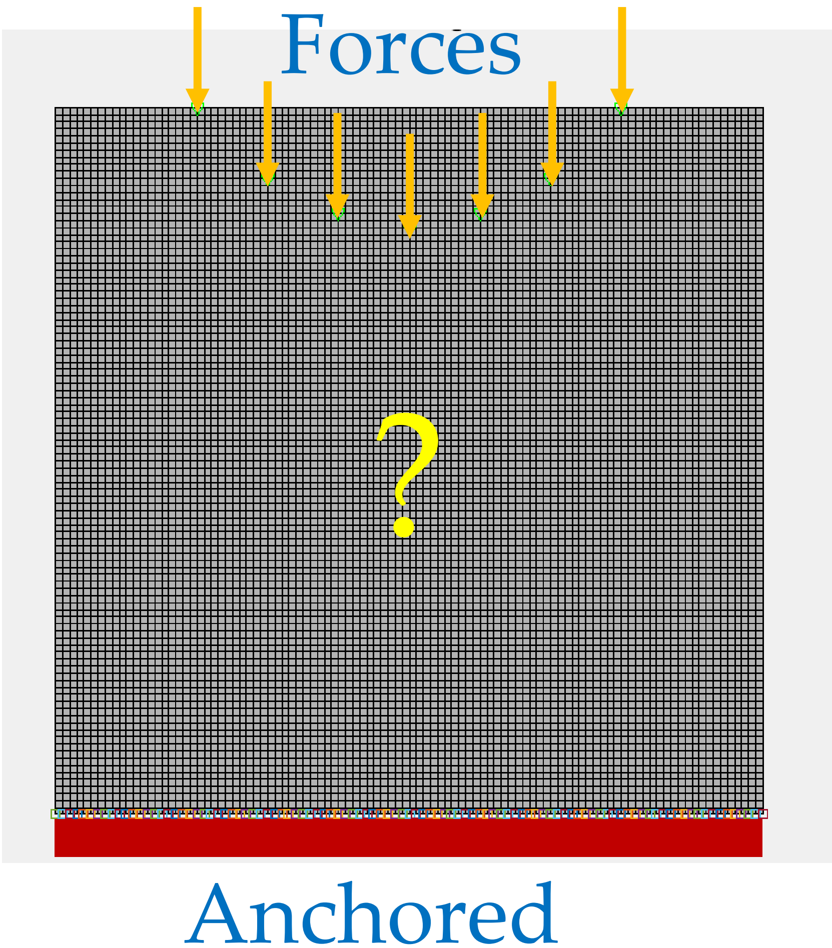


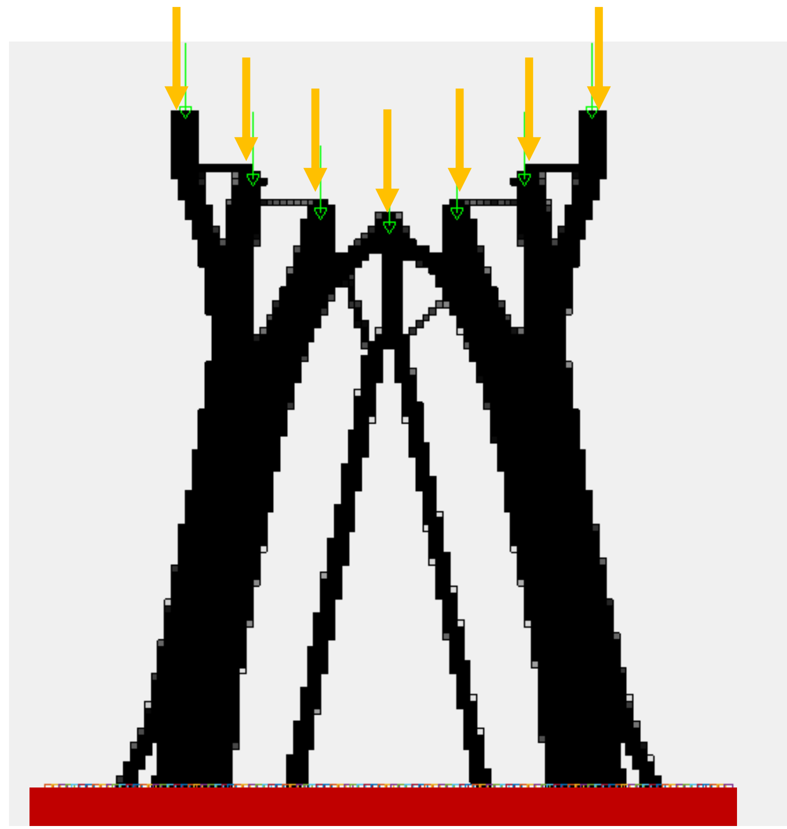
What about the shape of the connections?





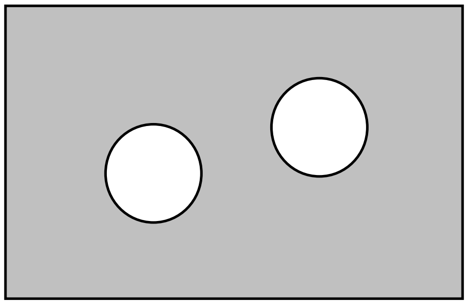
Here is one way to get all at once:
Size, Shape, and Topology





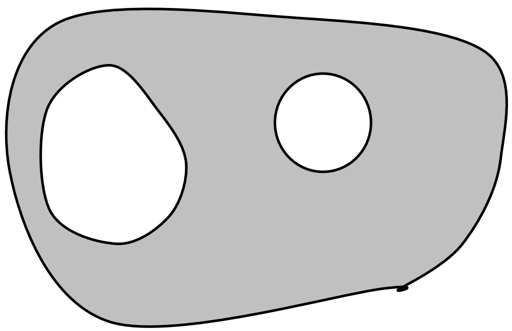
Stiffest structure for given volume of material.

Hierarchy in SO



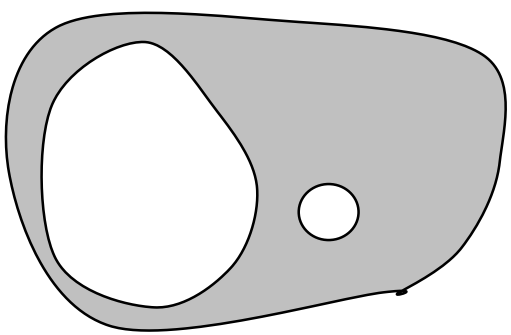
Topology: How many holes?

Topology optimization



Shape: What is the shape?

Shape optimization



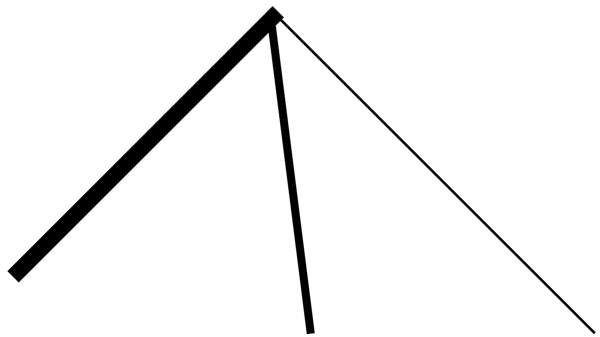
Size: What are the sizes?

Size optimization

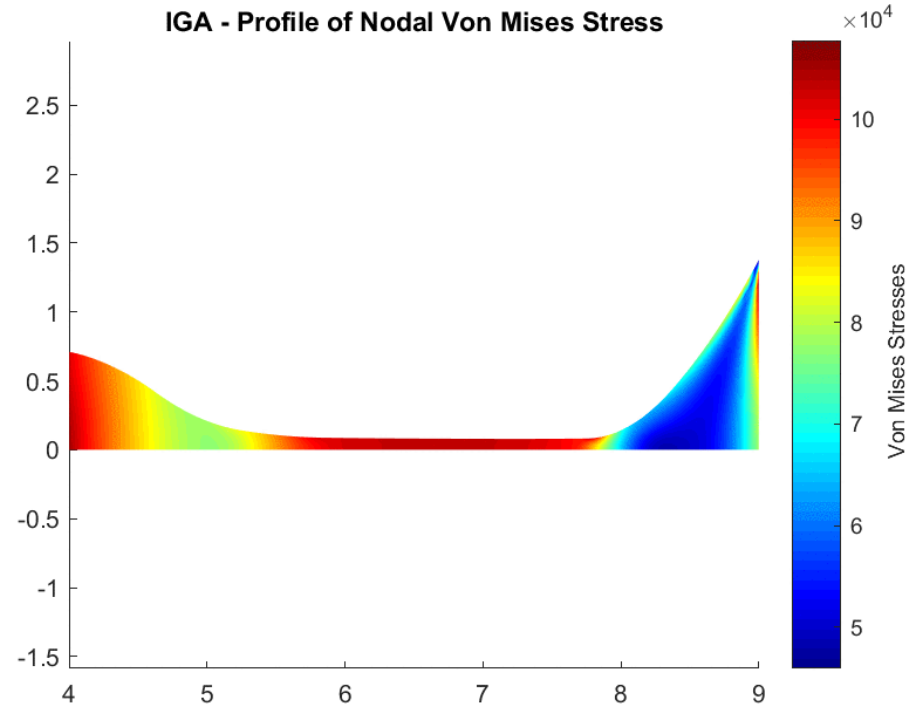
or

Parameteric optimization

Size and shape optimization



Size optimization:
Each segment has a different cross section.



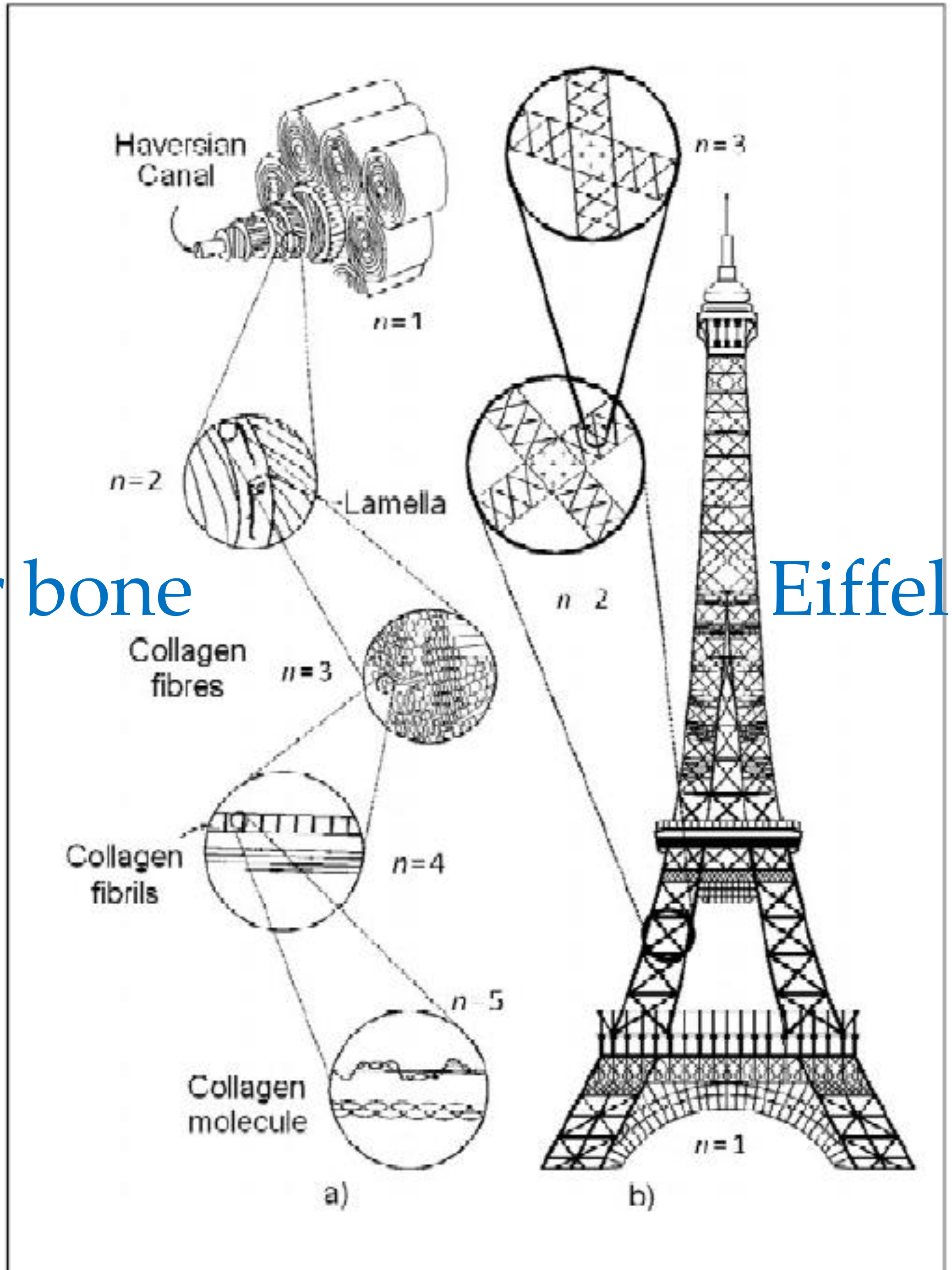
Shape optimization:
Shape of the boundary is changing.

(Image created by Akshay Kumar, 2020

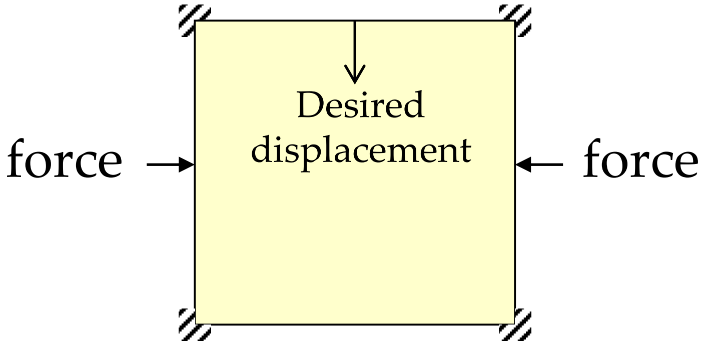
Hierarchy

Trabecular bone

Eiffel tower



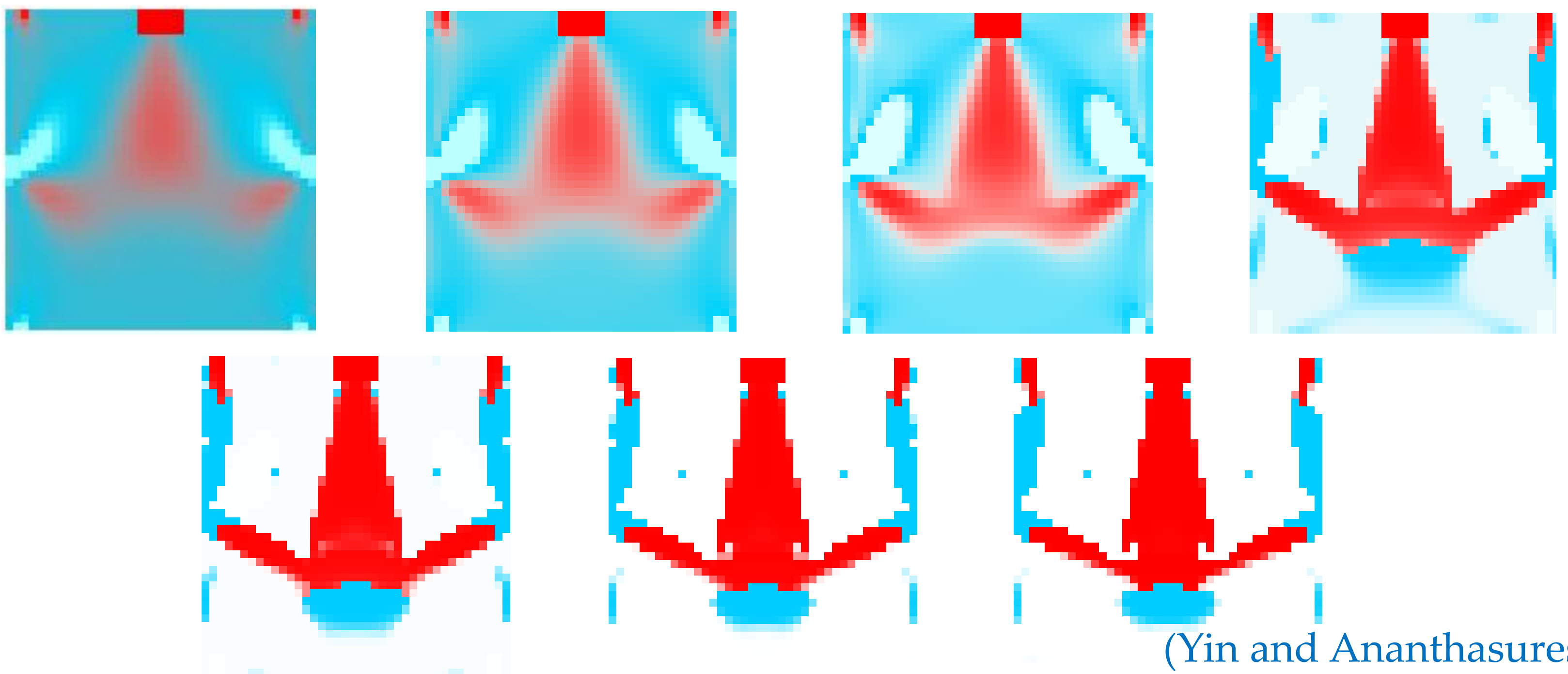
Two-material topologies



Single material design

Stiff, flexible materials

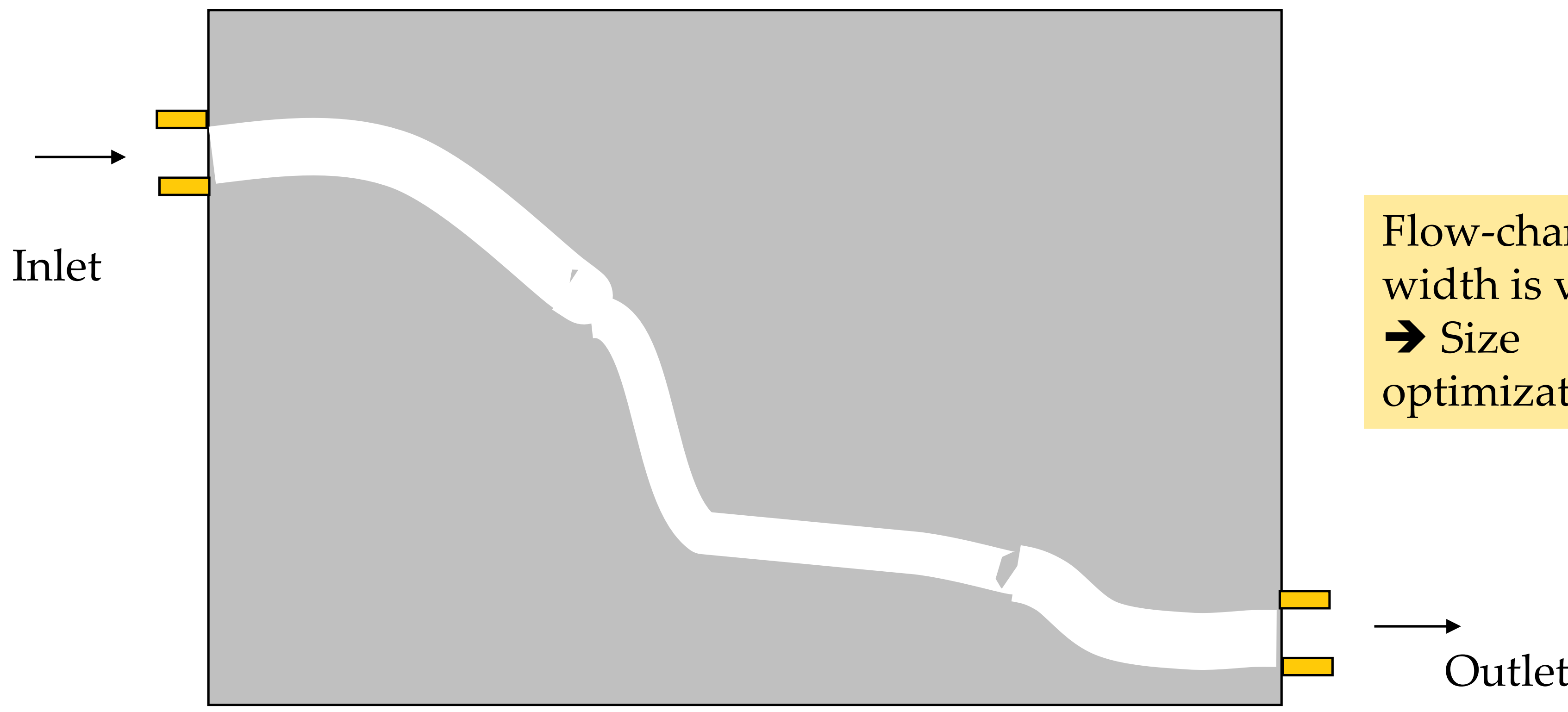
40x40 8-noded elements



(Yin and Ananthasuresh, 2002)

Flow-channel optimization

Need to vary solid/liquid states at every point to get the optimal solution.

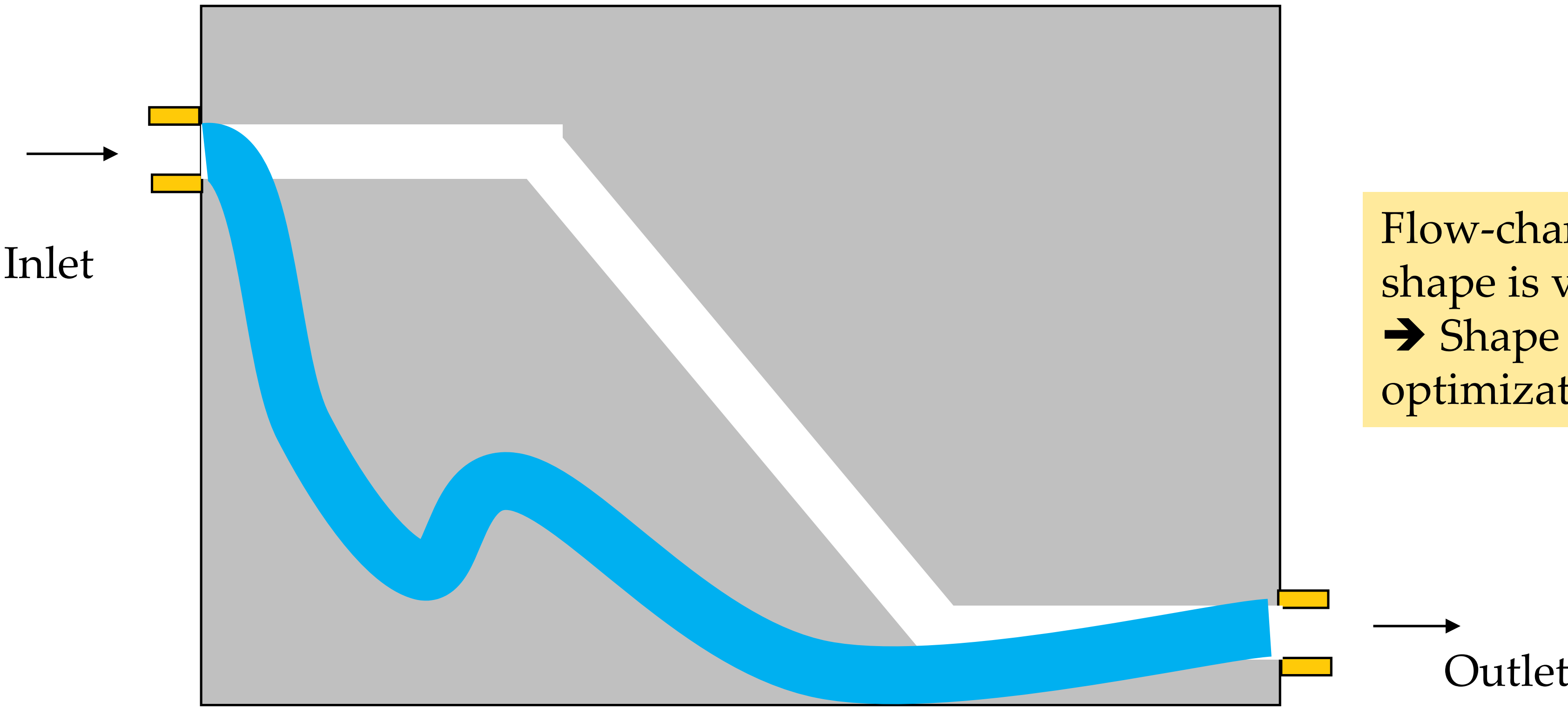


Flow-channel width is variable ➔ Size optimization

Maute et al. (2005)

Flow-channel optimization

Need to vary solid/liquid states at every point to get the optimal solution.

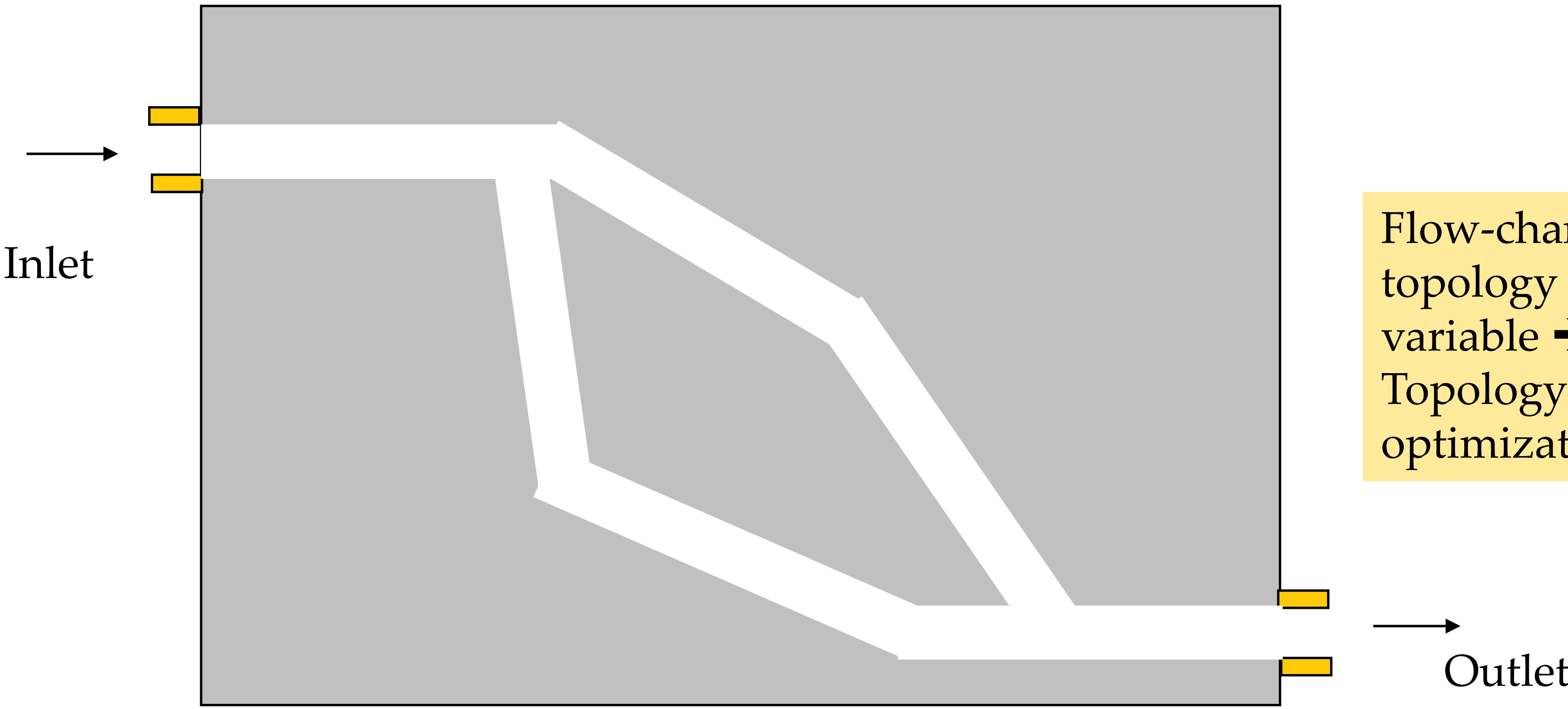


Flow-channel shape is variable
➔ Shape optimization

Maute et al. (2005)

Flow-channel optimization

Lot of possibilities; which is the optimal one?

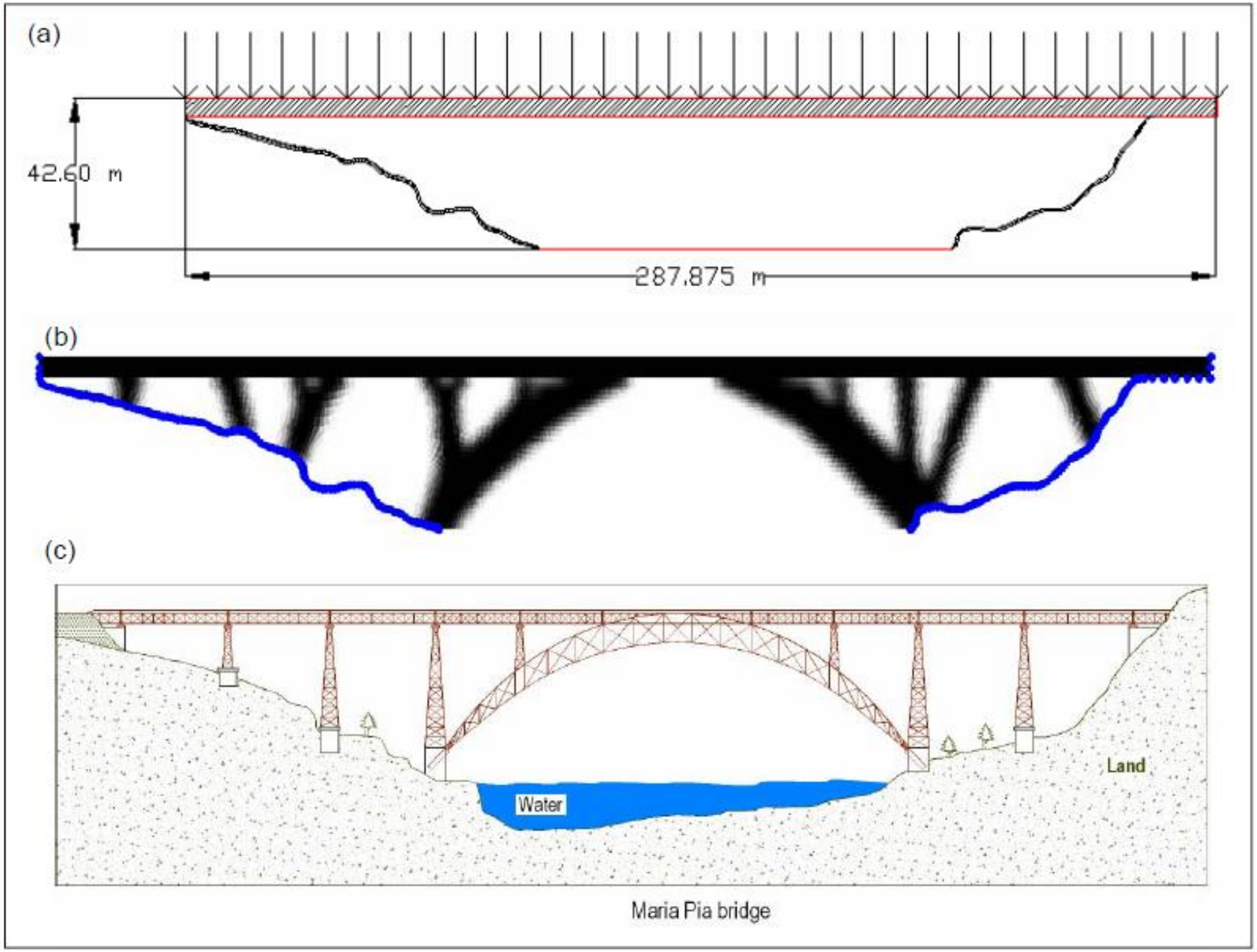


Flow-channel topology is variable ➔ Topology optimization

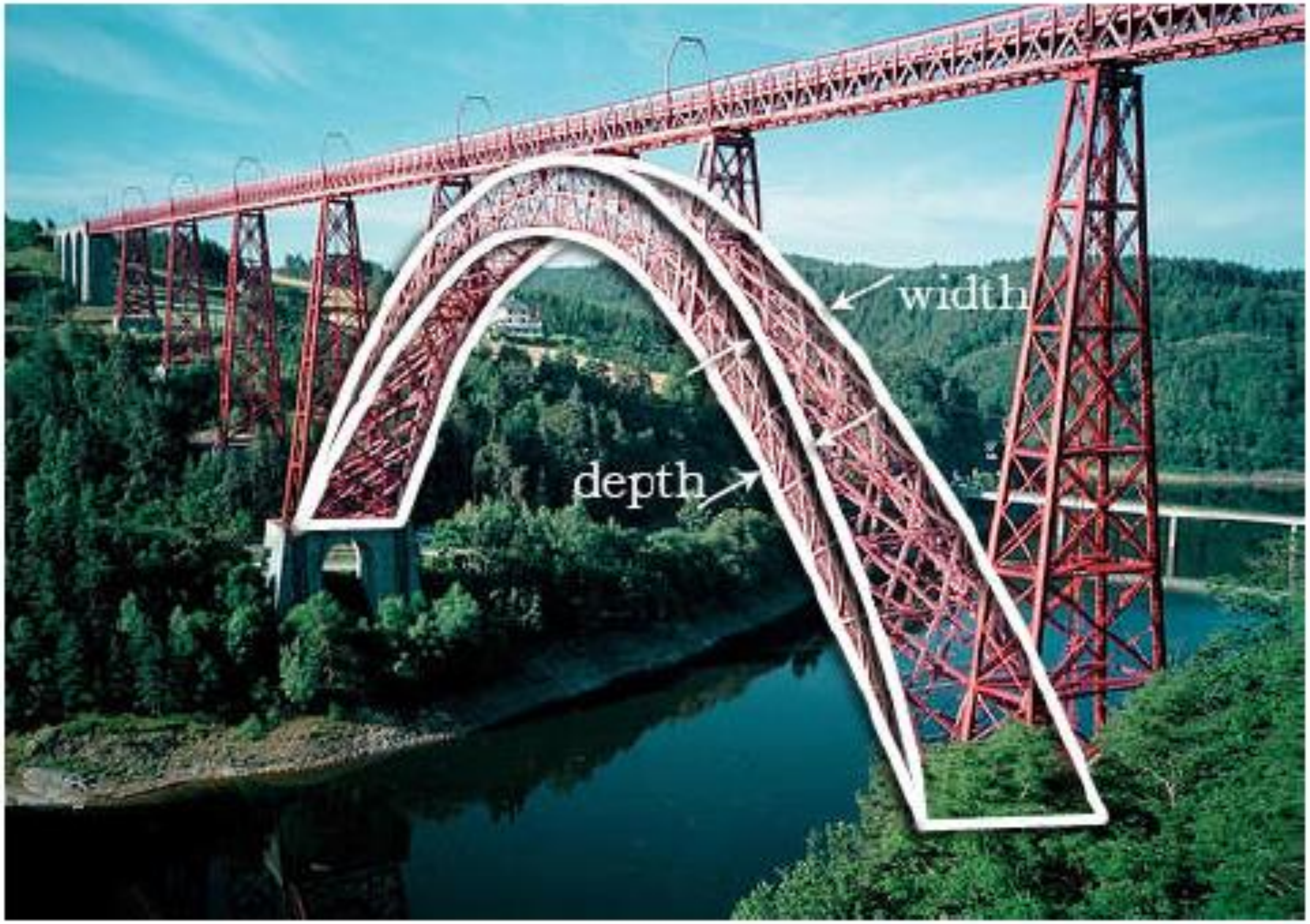
Maute et al. (2005)

Optimization hinders evolution.

Topology in Eiffel's bridge



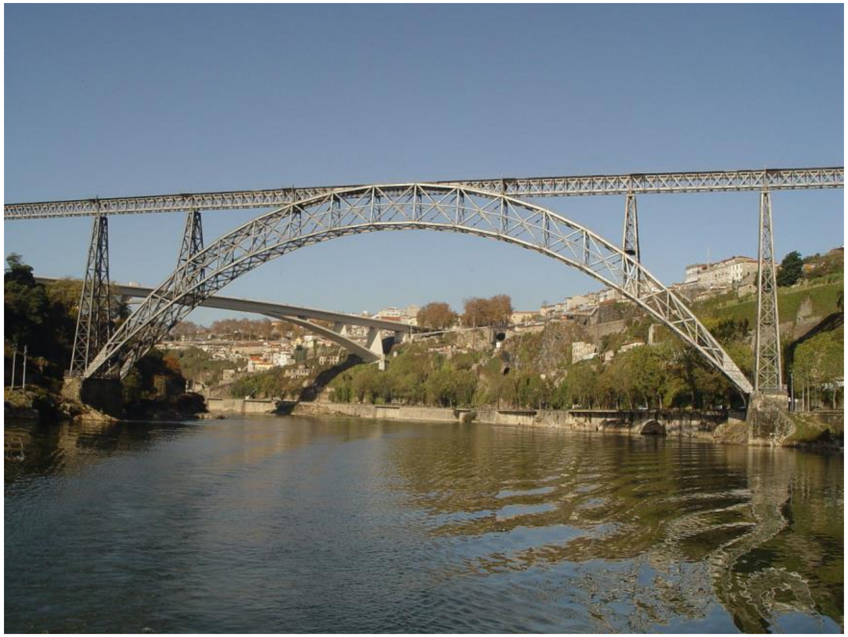
Maria-Pia bridge by Eiffel



Chenab river bridge under construction

Are these similar?
Are they exactly alike?

Chenab river bridge, India



Maria-Pia bridge, Portugal

Hierarchical design in our own country…

For more information…

Sundaram, M. and Ananthasuresh, G.K., “Gustave Eiffel and his Optimal Structures”, *Resonance: Science Education Journal*, Vol. 14, No. 8, Sep. 2009, pp. 849-865.

The end note

Structural Optimization

Topology

Topology is the highest level in structural hierarchy.
Topology decides connectivity and the number of holes in a structure.
Identifying the variables that decide the topology is the main challenge.
Topology should be decided first.

Shape

Shape of the segments in a structure.
Shape of a feature in a structure.
Shape of a hole in a structure.
Identifying the variables that decide the shape of a segment, feature, or a hole is easier compared to topology.
Shape optimization follows topology optimization.

Size or **parameter**

Size optimization is the easiest and is the last step.
Parameter (radius of a circular hole) is also a size variable.

All three- topology, shape, and size can be at once too.

Thanks