Lecture 7b

Principle of virtual work and its implication in truss optimization

ME260 Indian Institute of Science

Structural Optimization: Size, Shape, and Topology

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Outline of the lecture

Principle of virtual work

Unit virtual (dummy) load method

What we will learn:

Interpreting and using the principle of virtual work

Principle of virtual work

"External virtual work is equal to the internal virtual work."

This is an alternative view to force balance and the principle of minimum potential energy.

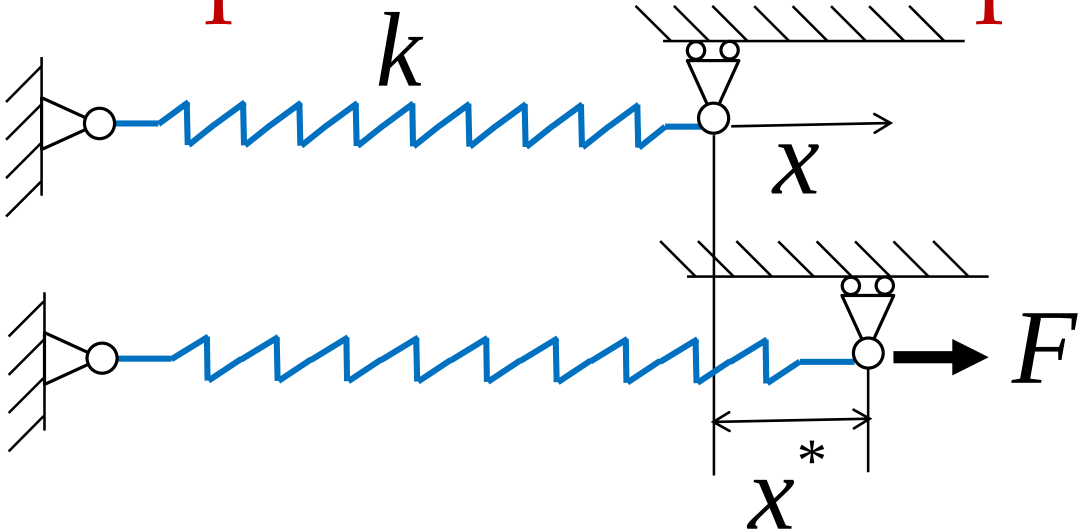
External virtual work = EVW = work done by the external real forces undergoing virtual displacements or work done by external virtual forces undergoing real displacements

Internal virtual work = IVW = work done by the real internal forces undergoing virtual displacements or work done by internal virtual forces undergoing real displacements

EVW = IVW is a powerful analytical tool.

It is a thought experiment either with virtual displacements or virtual forces.

A simple example used to understand the principle of minimum potential energy



We know from force balance:

$$F = kx^*$$

Strain energy = SE = $\frac{1}{2}kx^2$

Work potential = WP = $-Fx$

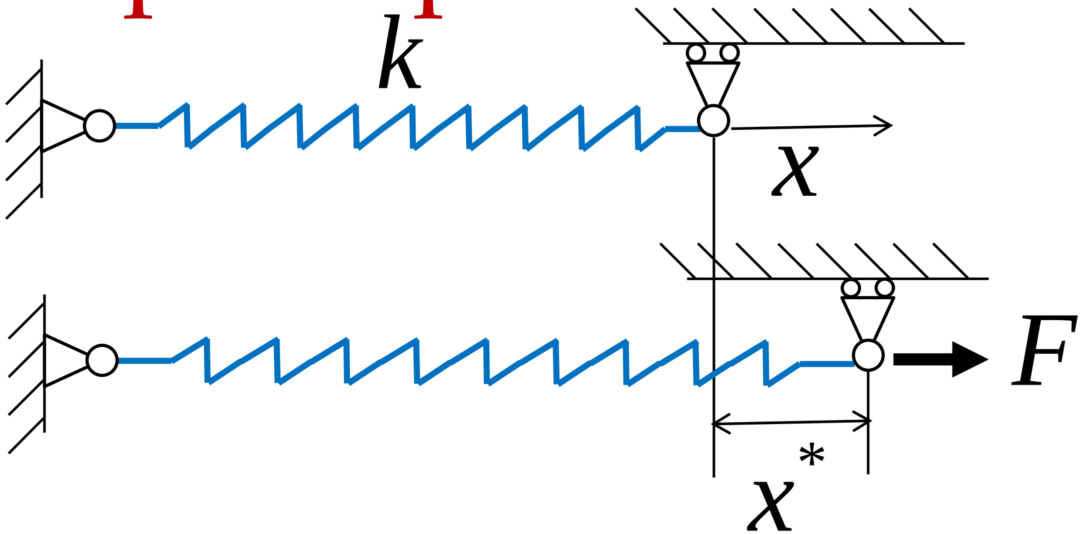
Potential energy = PE = SE + WP = $\frac{1}{2}kx^2 - Fx$

$$\underset{x}{\text{Min}}\ PE = \frac{1}{2}kx^2 - Fx$$

$$\frac{\partial PE}{\partial x} = kx - F = 0$$

So, $F = kx^*$

The same simple example to illustrate the principle of virtual work



We know from force balance:

$$F = kx^*$$

$$\Rightarrow x^* = F / k$$

$$\underset{x}{\text{Min}}\ PE = \frac{1}{2}kx^2 - Fx$$

$$\frac{\partial PE}{\partial x} = kx - F = 0$$

So, $F = kx^*$

Imagine virtual displacement δx

$$\left.\begin{aligned} EVW &= F\,\delta x \\ IVW &= kx^*\,\delta x \end{aligned}\right\} EVW = IVW \Rightarrow F = kx^*$$

Imagine virtual force δF

$$\left.\begin{aligned} EVW &= \delta F\ x^* \\ IVW &= k\delta x\ x^* \end{aligned}\right\} \begin{aligned} &EVW = IVW \\ &\Rightarrow \delta F\left(F / k\right) = k\left(\delta F / k\right)x^* \Rightarrow F / k = x^* \end{aligned}$$

Let us apply the principle of virtual work to trusses

$$\underset{n\times n}{\mathbf{K}}\underset{n\times 1}{\mathbf{u}}=\underset{n\times 1}{\mathbf{p}}$$

Imagine virtual displacements $\delta\mathbf{u}$

$$EVW=\mathbf{p}^T\delta\mathbf{u}$$

$$IVW=(\mathbf{K}\mathbf{u})^T\delta\mathbf{u}=\mathbf{u}^T\mathbf{K}\delta\mathbf{u}$$ because **K** is symmetric.

$$EVW=IVW\Rightarrow\mathbf{p}^T\delta\mathbf{u}=\mathbf{u}^T\mathbf{K}\delta\mathbf{u}$$

$$\Rightarrow\mathbf{p}^T=\mathbf{u}^T\mathbf{K}$$

$$\Rightarrow\mathbf{K}\mathbf{u}=\mathbf{p}$$

Unit virtual (dummy) load method

$$\mathbf{Ku} = \mathbf{p}$$

Imagine virtual force $\delta\mathbf{p}^T = \begin{Bmatrix} \overset{1}{0} & 0 & \overset{i}{1} & \cdots & \cdots & 0 & \overset{n}{0} \end{Bmatrix}$

$$EVW = \delta\mathbf{p}^T\ \mathbf{u}^*$$

$$IVW = \left(\mathbf{K}\delta\mathbf{u}\right)^T\ \mathbf{u}^* = \delta\mathbf{u}^T\mathbf{K}\mathbf{u}^*$$

$$EVW = IVW \Rightarrow \delta\mathbf{p}^T\ \mathbf{u}^* = \delta\mathbf{u}^T\mathbf{K}\mathbf{u}^*$$

$$\Rightarrow u_i = \left(\delta\mathbf{u}^T\mathbf{K}\right)\mathbf{u}^*$$

$\mathbf{u}^*$ = displacements due to applied real loads

$\delta\mathbf{u}$ = displacements due to the unit virtual load

The end note

Truss optimization

- Principle of virtual work
- Applying the principle of virtual work to a simple spring
- **Think about the implication of the principle of virtual work for trusses**
- The second alternative to force balance.
  The first alternative is the principle of minimum potential energy.

Thanks