

Lecture 18a

More than a dozen problems in optimizing a bar

ME 260 at the Indian Institute of Science, Bengaluru

Structural Optimization: Size, Shape, and Topology

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Outline of the lecture

Some problems in optimizing the cross-section profile of a bar under axial loading.

What we will learn:

How to apply the concepts and ideas learned so far to solve problems in calculus of variations with particular examples of axially loaded bars.

Problem 1

$$\text{Min}_{A(x)} MC = \int_0^L p u dx$$

Subject to

$$\lambda(x): \quad (EAu')' + p = 0$$

$$\Lambda: \quad \int_0^L A dx - V^* \leq 0$$

Data : $L, p(x), E, V^*$

Minimize the mean compliance (a measure of stiffness) for given volume of material under given loading.

Problem 2

$$\text{Min}_{A(x)} SE = \int_0^L \frac{1}{2} EAu'^2 dx$$

Subject to

$$\lambda(x): \quad (EAu')' + p = 0$$

$$\Lambda: \quad \int_0^L A dx - V^* \leq 0$$

Data: $L, p(x), E, V^*$

Minimize the strain energy (a measure of stiffness) for given volume of material under given loading.

Problem 3

$$\underset{A(x)}{\text{Min}} V = \int_0^L A dx$$

Subject to

$$\lambda(x): \quad (EAu')' + p = 0$$

$$\Lambda: \quad \int_0^L \frac{1}{2} EAu'^2 dx - SE^* \leq 0$$

Data : $L, p(x), E, SE^*$

Minimize volume of material to be used with an upper bound constraint on strain energy under given loading.

Problem 4

$$\text{Min}_{A(x)} V = \int_0^L A dx$$

Subject to

$$\lambda(x): \quad (EAu')' + p = 0$$

$$\Lambda: \quad \int_0^L pu dx - MC^* \leq 0$$

Data: $L, p(x), E, MC^*$

Minimize volume of material to be used with an upper bound constraint on mean compliance under given loading.

Problem 5

$$\text{Min}_{A(x)} SE = \int_0^L \frac{P^2}{2AE} dx$$

Subject to

$$\Lambda: \int_0^L A dx - V^* \leq 0$$

Data : $L, P(x), E, V^*$

Minimize the strain energy, expressed in terms of internal forces in a bar, subject to the volume constraint.

Problem 6

$$\underset{A(x)}{\text{Max}} \underset{u(x)}{\text{Min}} PE = \int_0^L \left(\frac{1}{2} EAu'^2 - pu \right) dx$$

Subject to

$$\Lambda : \int_0^L A dx - V^* \leq 0$$

Data : $L, p(x), E, V^$*

A min-max formulation for the stiffest bar for given volume of material based on the Clayperon's theorem.

Problem 7

$$\text{Min}_{A(x)} MC = \int_0^L p u dx$$

Subject to

$$\Gamma: \int_0^L (EAu'v' - pv) dx = 0$$

$$\Lambda: \int_0^L A dx - V^* \leq 0$$

Data : $L, p(x), E, V^*$

Minimize the mean compliance for given volume of material with the governing equation in the weak form.

Problem 8

$$\text{Min}_{A(x)} MC = \int_0^L p u dx$$

Subject to

$$\lambda(x): \quad (EAu')' + p = 0$$

$$\Lambda: \quad \int_0^L A dx - V^* \leq 0$$

$$\mu_u(x): \quad A - A_u \leq 0$$

$$\mu_l(x): \quad A_l - A \leq 0$$

$$\text{Data: } L, p(x), E, V^*, A_l, A_u$$

Minimize the mean compliance for given volume and with upper and lower bounds on the area of cross-section.

Problem 9

$$\underset{A(x)}{\text{Min}} V = \int_0^L A dx$$

Subject to

$$\Lambda : \int_0^L \frac{PP_d}{AE} dx - \Delta^* = 0$$

$$\text{Data} : L, P(x), P_d(x), E, \Delta^*$$

Minimize the volume of a statically determinate bar with a deflection constraint at a point somewhere on its axis. Internal force, P , due to some applied load, and that due to a unit virtual load at the point of interest are given.

Problem 10

$$\text{Min}_{A(x)} V = \int_0^L A dx$$

Subject to

$$\lambda(x): \quad (EAu')' + p = 0$$

$$\lambda_d(x): \quad (EAu'_d)' + p_d = 0$$

$$\Lambda: \quad \int_0^L EAu'u'_d dx - \Delta^* = 0$$

$$\text{Data: } L, p(x), p_d(x), E, \Delta^*$$

Minimize the volume of a bar (indeterminate or determinate) for a deflection constraint at a point.

Problem 11

$$\text{Min}_{A(x)} V = \int_0^L A dx$$

Subject to

$$\lambda(x): \quad (EAu')' + p = 0$$

$$\lambda_d(x): \quad (EAu'_d)' + p_d = 0$$

$$\Lambda: \quad \int_0^L EAu'u'_d dx - \Delta^* = 0$$

$$\Gamma: \quad \int_0^L \frac{1}{2} EAu'^2 dx - SE^* = 0$$

$$\text{Data} : L, p(x), p_d(x), E, \Delta^*, SE^*$$

Minimize the volume of a bar (indeterminate or determinate) for a deflection constraint at a point and strain energy constraint.

Problem 12

$$\text{Min}_{A(x)} MC = \int_0^L p u dx$$

Subject to

$$\lambda(x): \quad (EAu')' + p = 0$$

$$\Lambda: \quad \int_0^L A dx - V^* \leq 0$$

$$\mu_u(x): \quad u - u_u \leq 0$$

$$\mu_l(x): \quad u_l - u \leq 0$$

$$\text{Data: } L, p(x), E, V^*, u_l, u_u$$

Minimize the mean compliance of a bar for given volume and upper and lower bounds on the displacement.

Problem 13

$$\text{Min}_{A(x)} MC = \int_0^L p u dx$$

Subject to

$$\lambda(x): \quad (EAu')' + p = 0$$

$$\Lambda: \quad \int_0^L A dx - V^* \leq 0$$

$$\mu_t(x): \quad Eu' - \sigma_t \leq 0$$

$$\mu_c(x): \quad \sigma_c - Eu' \leq 0$$

$$\text{Data: } L, p(x), E, V^*, \sigma_t, \sigma_c$$

Minimize the mean compliance of a bar for given volume with stress constraints.

Problem 14

$$\underset{p(x)}{\text{Max}} \quad MC = \int_0^L p u \, dx$$

Subject to

$$\lambda(x): \quad (EAu')' + p = 0$$

$$\Lambda: \quad \int_0^L p \, dx - W^* \leq 0$$

Data : $L, A(x), E, W^*$

Find the world load distribution for a bar of given geometry. Notice the total load is specified.

Problem 15

$$\text{Min}_{A(x)} \text{MSC} = \int_0^L p u^2 dx$$

Subject to

$$\Gamma: \int_0^L (EAu'v' - pv) dx = 0$$

$$\Lambda: \int_0^L A dx - V^* \leq 0$$

Data : $L, p(x), E, V^*$

Minimize a general objective function for given volume.

The end note

Practice problems in calculus of variations

Axially deforming bar is the simplest structural optimization problem.

Mean compliance and strain energy are measures of stiffness.

Volume of material used is a cost-measure.

Objective function and functional constraint can be interchanged without affecting the nature of the solution.

Equilibrium equation can be posed in strong or weak form without changing the nature of the solution.

Constraints can also be imposed on profile of the area of cross-section.

Constraints on displacements and strains (stresses) can be imposed.

Thanks