A Dual Algorithm to Design Statically Determinate Truss

Presented by

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The dual problem

Primal Problem

$$\underset{\mathbf{x}\in X}{\text{Min}}\ f(\mathbf{x})$$

Subject to

$$\boldsymbol{\mu}: \quad \mathbf{g}(\mathbf{x}) \le \mathbf{0}$$

$$\mathbf{g}(\mathbf{x}) = (g_1(\mathbf{x}),\ldots,g_m(\mathbf{x}))$$

Lagrangian

$$L(\mathbf{x},\boldsymbol{\mu}) = f(\mathbf{x}) + \sum_{i=1}^{m} \mu_i g_i(\mathbf{x})$$

Dual Problem

$$\underset{\boldsymbol{\mu}\ge 0}{\text{Max}}\ q(\boldsymbol{\mu})$$

$$q(\boldsymbol{\mu}) = \begin{cases} \underset{\mathbf{x}\in X}{\text{Min}}\ L(\mathbf{x},\boldsymbol{\mu}) & \forall\ \boldsymbol{\mu} \ge \mathbf{0}, \\ -\infty & \boldsymbol{\mu} < \mathbf{0} \end{cases}$$

$$f(\mathbf{x}^*) = \underset{\mathbf{x}\in X}{\text{Min}}\ L(\mathbf{x},\boldsymbol{\mu})\ ,\forall \mathbf{x} \in \mathbf{g}(\mathbf{x}) \le \mathbf{0}$$

$f^*\big|_{\mathbf{x}=\mathbf{x}^*}$ is the optimal value

Necessary Condition:

$$\nabla f(\mathbf{x}) + \boldsymbol{\mu}^T \nabla \mathbf{g}(\mathbf{x}) = 0,$$

Complementarity :

$$\boldsymbol{\mu}^T \mathbf{g}(\mathbf{x}) = 0 \ \ \&\ \ \boldsymbol{\mu} \ge \mathbf{0}$$

$$q(\boldsymbol{\mu}) = f(\mathbf{x}^*(\boldsymbol{\mu})) = \underset{\mathbf{x}}{\text{Min}}\, L(\mathbf{x},\boldsymbol{\mu})$$

$$q^* = q(\boldsymbol{\mu}^*) = \underset{\boldsymbol{\mu}\ge 0}{\text{Max}}\, L(\mathbf{x}^*,\boldsymbol{\mu})$$

$$\therefore L(\mathbf{x},\boldsymbol{\mu}^*) \le L(\mathbf{x}^*,\boldsymbol{\mu}^*) \le L(\mathbf{x}^*,\boldsymbol{\mu})\ \ \forall\ \mathbf{x} \in \mathbf{X}\ \&\ \boldsymbol{\mu} \ge \mathbf{0}$$

Primal function is not required to be convex but dual function is always concave if **x** is feasible.

Statically determinate stiff trusses

Minimize weight for given strain energy

Primal Problem

$$\underset{A_i}{\text{Min}}\ W = \sum_{i=1}^{n} \rho A_i l_i$$

Subject to

$$\Gamma: \quad \sum_{i=1}^{n} \frac{P_i^2 l_i}{2A_i E} - \text{SE}^* \le 0$$

Data: $\rho, l_{i=1,2....n}, P_{i=1,2....n}, E, \text{SE}^*$

Lagrangian

$$L(\mathbf{A},\Gamma) = \sum_{i=1}^{n} \rho A_i l_l + \Gamma\left(\frac{P_i^2 l_i}{2A_i E} - \text{SE}^*\right)$$

KKT Conditions:

Necessary Condition:

$$\frac{\partial L}{\partial A_i} = \rho l_i - \Gamma \frac{P_i^2 l_i}{2A_i^2 E} = 0$$

$$\Rightarrow A_i = \sqrt{\frac{\Gamma P_i^2}{2\rho E}}$$

Complementarity :

$$\Gamma\left(\frac{P_i^2 l_i}{2A_i E} - \text{SE}^*\right) = 0 \ \ \& \ \ \Gamma \ge 0$$

Dual Problem

$$\underset{\Gamma \ge 0}{\text{Max}}\ L(\Gamma) = \sum_{i=1}^{n} \rho A_i^* l_i + \Gamma\left(\frac{P_i^2 l_i}{2A_i^* E} - \text{SE}^*\right)$$

Where $A_i^* = \sqrt{\frac{\Gamma P_i^2}{2\rho E}}$

Strain energy minimization as the objective

Primal Problem

$$\underset{A_i}{\text{Min}}\ \text{SE} = \sum_{i=1}^{n} \frac{P_i^2 l_i}{2A_i E}$$

Subject to

$$\Gamma: \quad \sum_{i=1}^{n} \rho A_i l_i - W^* \leq 0$$

Data: $\rho, l_{i=1,2,\ldots,n}, P_{i=1,2,\ldots,n}, E, W^*$

Lagrangian

$$L(\mathbf{A},\Gamma) = \sum_{i=1}^{n} \frac{P_i^2 l_i}{2A_i E} + \Gamma\left(\rho A_i l_l - W^*\right)$$

KKT Conditions:

Necessary condition:

$$\frac{\partial L}{\partial A_i} = \frac{P_i^2 l_i}{2A_i^2 E} - \Gamma \rho l_i = 0$$

$$\Rightarrow A_i = \sqrt{\frac{P_i^2}{2\Gamma \rho E}}$$

Complementarity condition:

$$\Gamma\left(\rho A_i l_i - W^*\right) = 0 \ \ \& \ \ \Gamma \geq 0$$

Dual Problem

$$\underset{\Gamma \geq 0}{\text{Max}}\ L = \sum_{i=1}^{n} \frac{P_i^2 l_i}{2A_i^* E} + \Gamma\left(\rho A_i^* l_i - W^*\right)$$

Where $A_i^* = \sqrt{\frac{P_i^2}{2\Gamma \rho E}}$