Lecture 19b

Size optimization of beams for strength

ME 260 at the Indian Institute of Science, Bengaluru

Structural Optimization: Size, Shape, and Topology

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Outline of the lecture

Posing and solving the topology optimization of 2D frames in which design variables are cross-section dimensions of beam elements.

Considering stiffness and flexibility together.

What we will learn:

How to apply the six steps to identify the optimality criterion and use it in the numerical method.

How to ensure that we get a realistic solution when non-intuitive displacement is desired for given applied force.

Problem S1

Minimize the volume of material of a beam subject to strength constraints.

$$\underset{A(x)}{Min}\ V = \int_0^L A\,dx$$

$$Subject\ to$$

$$\mu_t(x): \qquad E\left(\frac{d}{2}\right)w'' - S_t \le 0$$

$$\mu_c(x): \qquad S_c - E\left(\frac{d}{2}\right)w'' \le 0$$

$$\lambda(x): \quad \left(E\alpha A w''\right)'' - q = 0$$

$$Data: L, q(x), \alpha = {t^2}\big/{12}, E, S_t, S_c$$

Problem S2

Minimize the maximum stress subject to a volume constraint.

$$\underset{A(x)}{Min}\ \underset{x}{Max}\ \sigma = E\frac{d}{2}w''$$

Subject to

$$\Lambda: \quad \int_0^L A\,dx - V^* \leq 0$$

$$\lambda(x): \quad \left(E\alpha A w''\right)'' - q = 0$$

$$Data: L, q(x), \alpha = t^2/12, E, V^*$$

Do you see a problem here?

How do you take variation of a functional that is maximum of a function?

Maximum of a function over the spatial domain is indeed a functional. But how do you take the variation?

We use a trick here. See next…

Problem S2

Minimize the maximum stress subject to a volume constraint.

Two equivalent formulations
(The latter is the trick!)

$$\underset{A(x)}{Min}\ \underset{x}{Max}\ \sigma = E\frac{d}{2}w''$$

Subject to

$$\Lambda: \quad \int_0^L A\,dx - V^* \le 0$$

$$\lambda(x): \quad \left(E\alpha A w''\right)'' - q = 0$$

$$Data: L, q(x), \alpha = t^2/12, E, V^*$$

$$\Leftrightarrow$$

$$\underset{A(x),\beta}{Min}\ \beta$$

Subject to

$$\mu(x): \ E\frac{d}{2}w'' - \beta \le 0$$

$$\Lambda: \quad \int_0^L A\,dx - V^* \le 0$$

$$\lambda(x): \quad \left(E\alpha A w''\right)'' - q = 0$$

$$Data: L, q(x), \alpha = t^2/12, E, V^*$$

The end note

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- Observe how we used the beta-formulation for handling mix-max problems
- We follow six steps to solve the discretized (or finite-variable optimization) problem.
- Identify the optimality criterion.
- Interpret the optimality criterion.
- Iterative numerical solution, when it is needed, remains the same.

Thanks