Recitation 2a

Principle of minimum potential energy

ME260 Indian Institute of Science

Structural Optimization: Size, Shape, and Topology

G. K. Ananthasuresh

Professor, Mechanical Engineering, Indian Institute of Science, Bengaluru

suresh@iisc.ac.in

Principle of minimum potential energy

"A mechanical system has minimum potential energy in its stable static equilibrium position."

This is an alternative view to force balance.

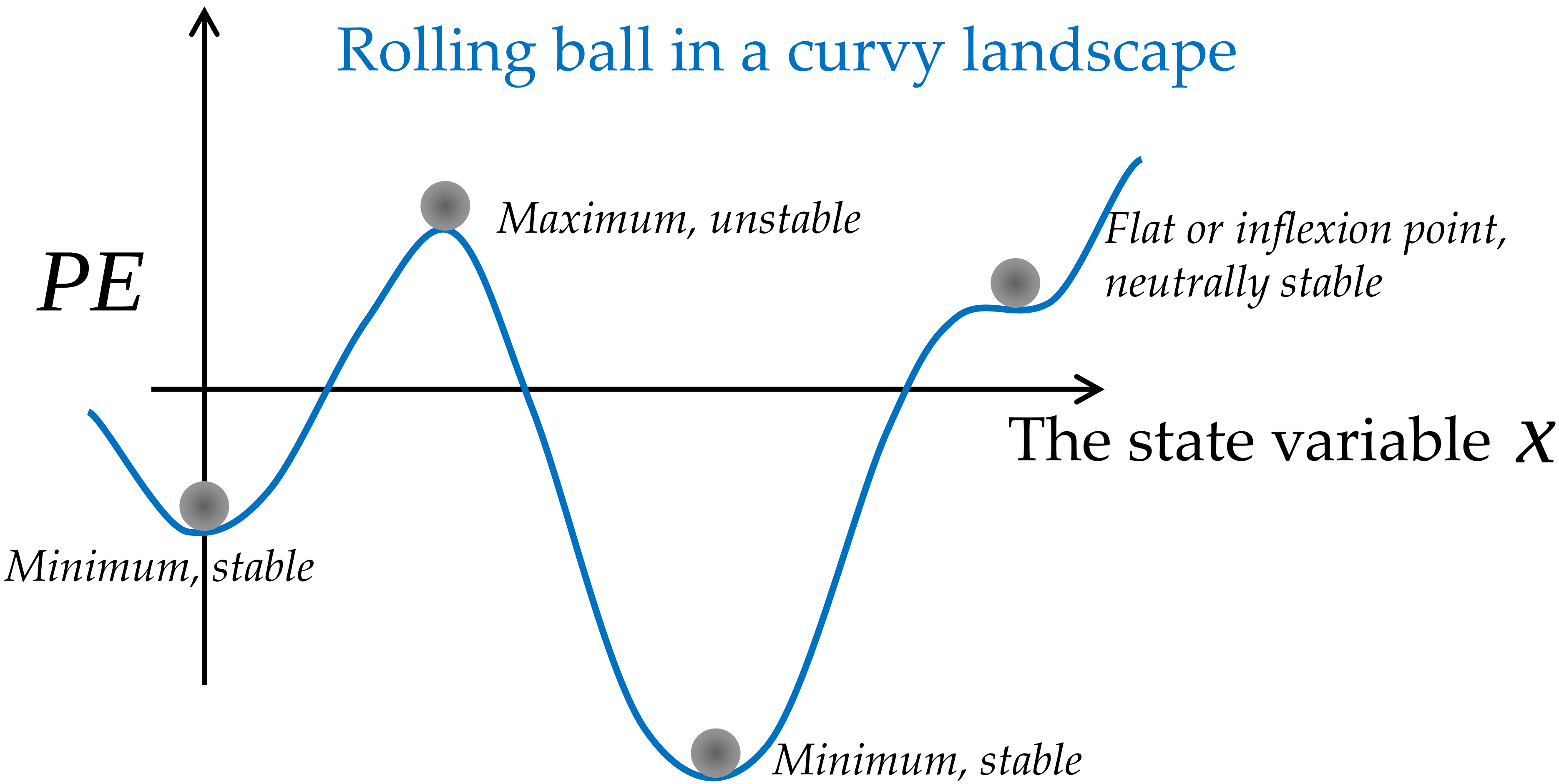
Potential energy = PE = SE + WP

SE = strain energy

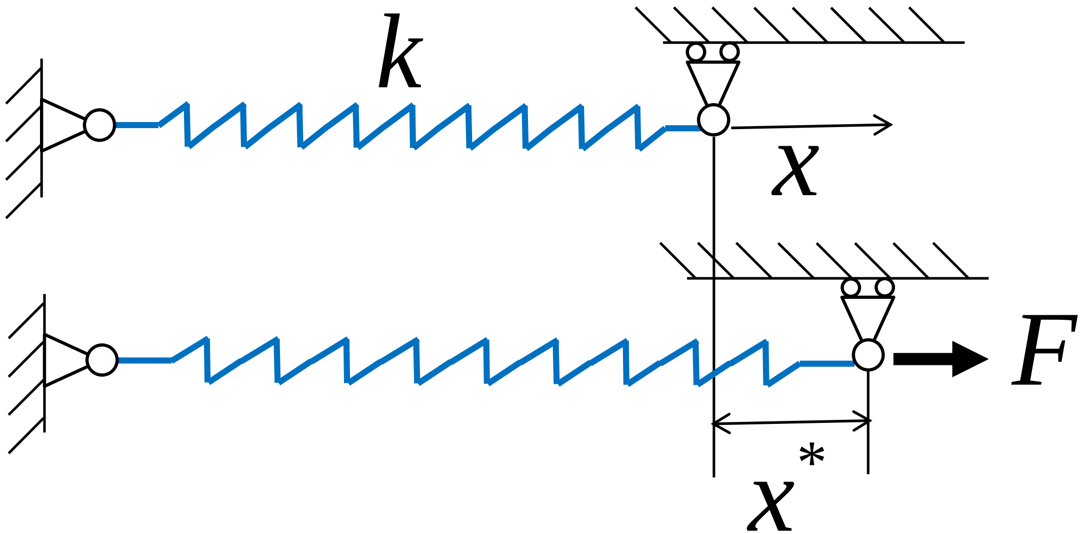
WP = negative of the work done by external forces

This principle involves only energy and work terms. So, it is completely scalar whereas force balance is a vectorial version.

Graphical portrayal of the principle of optimum potential energy



A simple example



We know from force balance:

$$F = kx^*$$

Strain energy = SE = $\frac{1}{2}kx^2$

Work potential = WP = $-Fx$

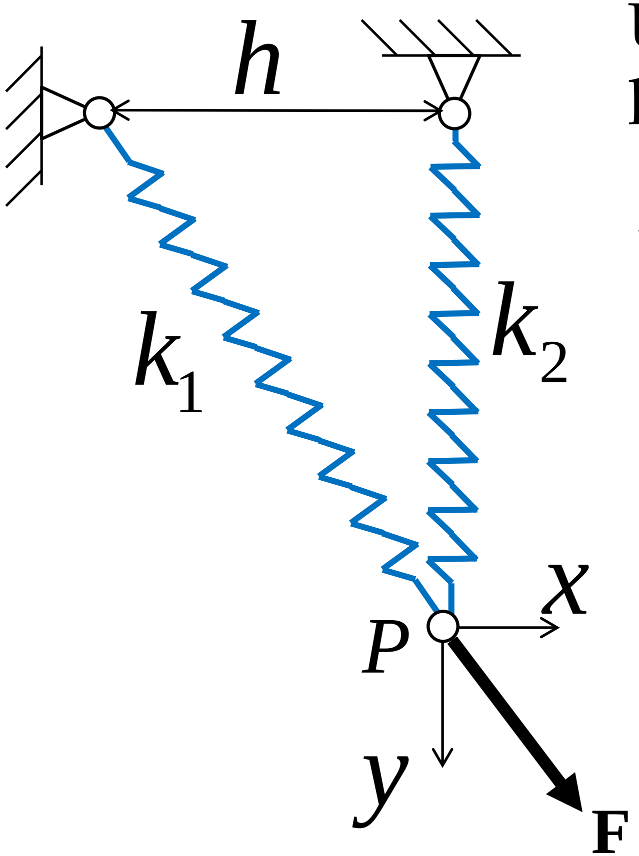
Potential energy = PE = SE + WP = $\frac{1}{2}kx^2 - Fx$

$$\underset{x}{\text{Min}}\ \ PE = \frac{1}{2}kx^2 - Fx$$

$$\frac{\partial PE}{\partial x} = kx - F = 0$$

So, $F = kx^*$

Another example



Undeformed (i.e., when there is no force) lengths of the two springs are: l_{10}, l_{20}

x and y are displacements of Point P.

$$SE = \frac{1}{2}k_1\left\{\sqrt{(h+x)^2+(l_{20}+y)^2}-l_{10}\right\}^2 + \frac{1}{2}k_2\left\{\sqrt{(x)^2+(l_{20}+y)^2}-l_{20}\right\}^2$$

$$WP = -\mathbf{F}\cdot\left(x\mathbf{i}+y\mathbf{j}\right)$$

$$PE = SE + WP$$

$$\mathbf{F} = F\left(\frac{h}{\sqrt{l_{10}^2+l_{20}^2}}\hat{\mathbf{i}}+\frac{l_{20}}{\sqrt{l_{10}^2+l_{20}^2}}\hat{\mathbf{j}}\right)$$

Stable equilibrium conditions

$$PE = SE + WP = \frac{1}{2}k_1\left\{\sqrt{(h+x)^2+(l_{20}+y)^2}-l_{10}\right\}^2 + \frac{1}{2}k_2\left\{\sqrt{(x)^2+(l_{20}+y)^2}-l_{20}\right\}^2 - \mathbf{F}\cdot\left(x\mathbf{i}+y\mathbf{j}\right)$$

Involves a lot of algebra to solve but it is straightforward to do once we write SE and WP.

$$\frac{\partial PE}{\partial x}=0$$

$$\frac{\partial PE}{\partial y}=0$$

Equilibrium equations (necessary conditions for a minimum and a maximum)

$$\begin{bmatrix} \partial^2 PE / \partial x^2 & \partial^2 PE / \partial x \partial y \\ \partial^2 PE / \partial x \partial y & \partial^2 PE / \partial y^2 \end{bmatrix} = \mathbf{H}$$

Hessian should be positive definite is the sufficient condition.

There are many solutions to this problem.