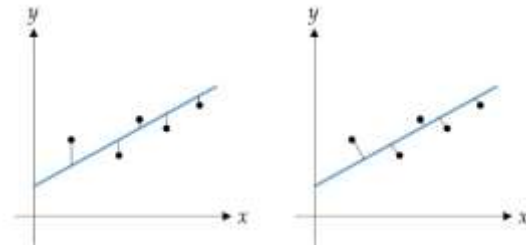


Problem 1 (10 points)

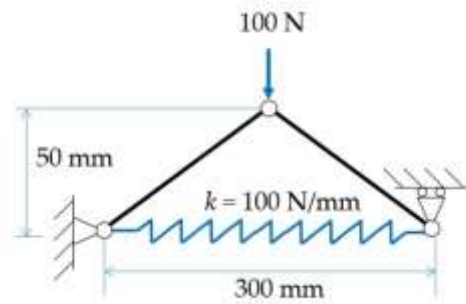
Given a set of points in 2D, fit the best line to the data in two different ways: (i) the usual least squares line fit, and (ii) another least squares fit that is invariant to the coordinate-system. The figures on the right illustrate the idea.



Data set: (1,3), (2.5, 2.5), (3,4), (3.5,3.5), (4.5, 4.2)

Problem 2 (10 points)

Find x and y displacements of the apex of a spring-constrained truss where a load of 100 N is applied, using the principle of minimization of potential energy.

**Extra credit of 10 points**

What happens if you keep increasing the load? Plot load vs. vertical displacement of the apex.

Problem 3 (10 points)

$$\text{Min}_{x,y} \quad f = \frac{1}{x} + \frac{1}{y}$$

Subject to

$$\mu_1: \quad g_1 = 4x - 2y - 3 \leq 0$$

$$\mu_2: \quad g_2 = 2x + y - 5 \leq 0$$

- Solve the above optimization problem (i) by hand, and (ii) using *fmincon* routine in Matlab, and (iii) graphically by plotting the contours in Matlab. Plot $f(x, y)$ as a surface and see if your answer is indeed a local minimum subject to the constraints. Find also the values of the also the Lagrange multipliers μ_1 and μ_2 .
- If 5 in g_2 is changed to 5.01, compute the change in the optimized value of f without re-solving the problem.

Problem 4 (10 points)

Formulated and solve one of the two optimization problems verbally stated here.

Extra credit of 10 points if we solve both.

- (a) Given three points in a plane, find a fourth point such that the sum of its distances from the given three points is a minimum.
- (b) Given a triangle, circumscribe the largest possible equilateral triangle about it.

The data for both problems are: $(0,0)$, $(10,0)$, $(0,6)$, which are coordinates of three points that form a triangle.

Problem 5 (10 points)

In Lecture 1 and Recitation 1, we discussed at length that there must be a conflict in order to have meaningful structural optimization problems. Let us revisit that point. Can you pose an unconstrained minimization problem that is in the realm of structural optimization? Show your attempts and write about your thought process that enabled to arrive at your answer.