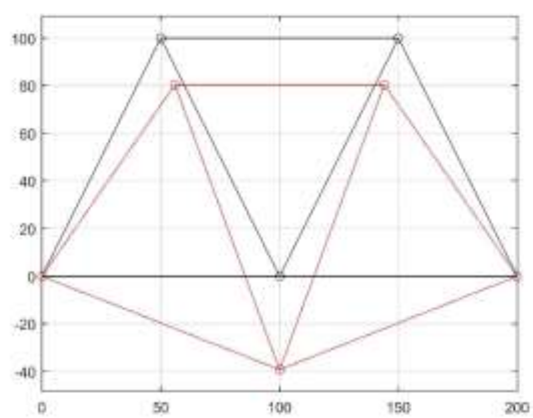
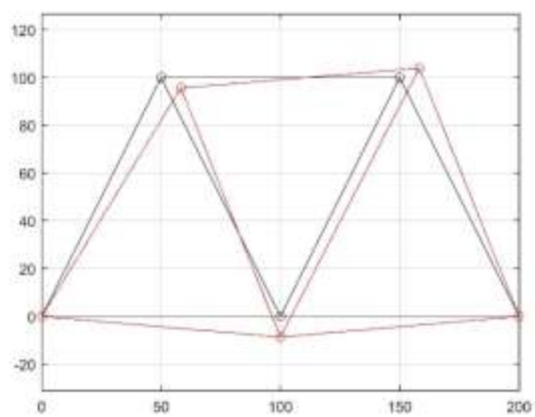
ME 260: Structural Optimization: Size, Shape, and Topology		
Assigned: Nov. 10, 2020	Homework 3	Due: Nov. 18, 2020

Problem 1 (15 points) [Download Matlab files for this problem.]

Barnett truss and Barnett compliant mechanism are shown below. The left and right vertices of the bottom of the truss are fixed. When we apply a downward force at the mid vertex of the bottom of the truss, all three moving vertices move down. Note that size optimization of the Barnett truss shown in the figure below can give a compliant mechanism as illustrated here if we adjust the areas of cross section to satisfy the desired displacement requirement. Now, the top right vertex moves up even though the applied force is downward at the mid vertex of the bottom of the truss. Many designs are possible to get this non-intuitive mechanism behavior. So, weight can be minimized or strain energy can be constrained to obtain a unique compliant mechanism.

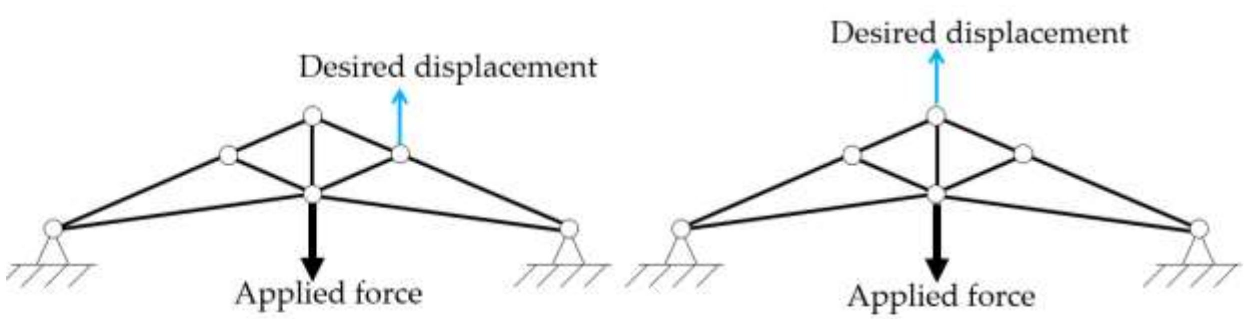


Barnett truss



Barnett compliant mechanism

Now, consider a roof-truss and make it a mechanism for two non-intuitive displacement behaviors shown here, using size optimization. Minimize the weight while meeting the displacement requirement. Assume steel as the material and circular cross section for all members. You may relocate the fixed vertices as necessary to get the desired behavior.



Non-intuitive behaviors using a roof truss upon size optimization of areas of cross section

Problem 2 (2 + 4 + 5 = 11 points)

Write Gateaux (first) variation of the following functionals.

1. $\int_0^l \left(y^2 + y'^2 - 2y\sin x\right) dx$

2. $\dfrac{\int_0^l \left(w''(x)\right)^2 dx}{\int_0^l \left(w(x)\right)^2 dx}$

3. $\frac{1}{2}\int_0^T \left[v_0 \sin\alpha(t) \left\{ x_0 + w_0 t + v_0 \int_0^t \cos\alpha(\tau) d\tau \right\} - \{ v_0 \cos\alpha(t) + w_0 \} \left\{ y_0 + v_0 \int_0^t \sin\alpha(\tau) d\tau \right\} \right] dt$

Problem 3 (3 + 4 + 4 + 4 = 14 points)

Write the Euler-Lagrange equations with boundary conditions for the following unconstrained minimizations of functionals.

$$J2 = \int_0^T \left(m\dot{x}\dot{y} - \frac{b}{2} x\dot{y} + \frac{b}{2}\dot{x}y - kxy \right) dt$$

$$\underset{f(x)}{Min}\, J = \left\{ \int_{x_1}^{x_2} \left\{ f^2(x)\ inv(f(x)) \right\} dx \right\}$$

$$\underset{u(x,y,t)}{Min}\ J = \frac{1}{2}\int_\Omega \left\{ \left(\frac{\partial u}{\partial t}\right)^2 - \left(c\frac{\partial u}{\partial x}\right)^2 - \left(c\frac{\partial u}{\partial y}\right)^2 \right\} d\Omega$$

$Data : c$

$$\underset{\phi(x,y,t),\, \psi(x,y,t)}{\text{Min}}\ J = \int_\Omega \left(\frac{\partial \phi}{\partial x}\frac{\partial \psi}{\partial x} + \frac{\partial \phi}{\partial y}\frac{\partial \psi}{\partial y} + \frac{c}{2}\frac{\partial \phi}{\partial t}\psi - \frac{c}{2}\frac{\partial \psi}{\partial t}\phi \right) d\Omega$$

Data $: c$

Problem 4 (10 points)

A beam is attached to a spring-restrained slider-crank linkage as shown in the figure here. Use the principle of minimum potential energy to write the governing differential equation and boundary conditions for the beam. Use I and E to denote the second moment of area of cross section of the beam and Young's modulus, respectively.