ME 260: Structural Optimization: Size, Shape and Topology

Assigned on: Aug. 20, 2021 **Quiz1** **Due on:** Aug. 24, 2001

Question 1 (6 points)

For the function

$f(x_1, x_2) = 9(x_1^2 - x_2^2)^2 + (x_1 - 1)^2 + (x_2 - 1)^2,$ $\mathbf{x}^0 = [0,\ 0]^\top$

1. evaluate the gradient vector and Hessian matrix of $f(x_1, x_2)$ at $\mathbf{x}^0$,
2. develop linear and quadratic models of $f(x_1, x_2)$ at $\mathbf{x}^0$,
3. superpose the contours of the original function and its quadratic model and plot the gradient vector at $\mathbf{x}^0$ (Use MATLAB),
4. plot the contours of the original function, its linear and quadratic models at $\mathbf{x}^0$ (Use MATLAB).

Question 2 (8 points)

[PART 1]

For the problem

$$\begin{aligned} &\text{minimize} && f(\mathbf{x}) = 0.01x_1^2 + x_2^2 \\ &\text{subject to} && g_1(\mathbf{x}) = 25 - x_1x_2 \le 0, \qquad g_2(\mathbf{x}) = 2 - x_1 \le 0; \end{aligned}$$

1. write the KKT conditions,
2. obtain the solution using the KKT conditions,
3. sketch the domain with the solution and verify the second order sufficient condition for optimality,
4. Find the new optimal value of the function with $g_1(\mathbf{x}) = 25.1 - x_1x_2 \le 0,\ g_2(\mathbf{x}) = 1.9 - x_1 \le\ 0$ using the Lagrange multipliers and then verifying it by resolving the optimization problem..

[PART 2]

For the problem

$$\text{minimize} \quad f(\mathbf{x}) = (x_1 - 3)^2 + (x_2 - 3)^2, \quad \text{subject to} \quad 2x_1 + x_2 \le 2$$

develop the dual function, maximize it and find the corresponding point in $\mathbf{x}-$space.

Question 3 (10 points)

Consider a two-bar problem in Fig. 1. Say, $l_i|_{i=1,2}$, A_i and E_i represent the length, cross-sectional area and Young's modulus of the bar i respectively.

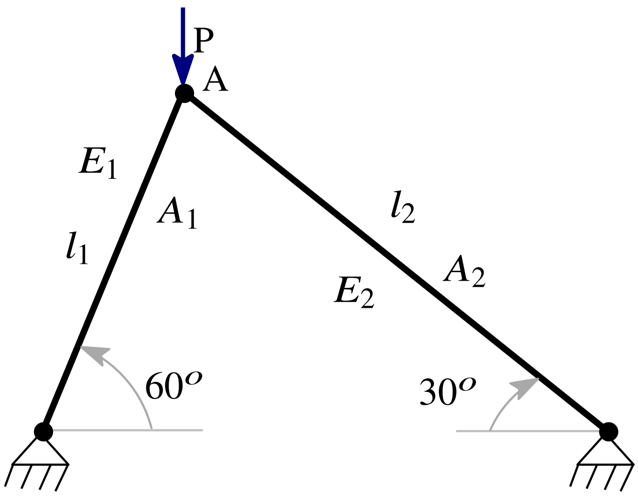

Figure 1: Two-bar problem

1. Formulate the minimization optimization problem. (i) for the objective function, take vertical displacement of the point A. (*Hint*: Equate twice the strain energy of the system to the work done by the force P). (ii) for constraints, use (a) upper limit for the stress in each bar as s_y (b) upper limit on the total volume of the system as V^*.

2. Solve the minimization problem and find the optimum cross-sectional areas A_i. Use the numerical data given as $E_1 = E2 = 210\,\text{GPa}$, $l_1 = 1\,\text{m}$, $l_2 = 1.73\,\text{m}$, P= $10\,\text{kN}$ and $s_y = 250\,\text{MPa}$.

Question 4 (16 points)

(a) Assume $y_i = \frac{1}{A_i}$, and solve the problem formulated in Question 3.

(b) Rewrite the problem formulation of Question 3 with additional buckling constraints arising from each bar and find the optimum cross sectional areas A_i. Assume I_i (second moment of area of bar i) $= \alpha A_i^2$, where α is a known constant.