Problem 1 (10 points)
$\operatorname{Min}_{x, y} f=\frac{1}{x}+\frac{1}{y}$
Subject to

$$
\begin{array}{ll}
\mu_{1}: & g_{1}=4 x-2 y-4 \leq 0 \\
\mu_{2}: & g_{2}=2 x+y-6 \leq 0
\end{array}
$$

(a) Solve the above optimization problem (i) by hand, and (ii) using fmincon routine in Matlab, and (iii) graphically by plotting the contours in Matlab. Plot $f(x, y)$ as a surface and see if your answer is indeed a local minimum subject to the constraints. Find also the values of the also the Lagrange multipliers $\mu_{1}$ and $\mu_{2}$.
(b) In the active constraint, change the value of the constant (either 4 or 6 ) by $1 \%$, and compute the change in the optimized value of $f$ without re-solving the problem.

## Problem 2 (20 points)

(a) Solve the following three-bar truss problem to find the areas of cross-sections to minimize the strain energy subject to a volume constraint. Use $E=$ Young's modulus = 210 GPa and $V^{*}=$ upper bound on volume $=30,000 \mathrm{~mm}^{3}$. Do it by hand and by using fmincon.

(b) Now, pose and solve the same problem as a shape optimization problem in one variable wherein the location of the moving pivot is variable from the fixed pivot. That is, 300 mm (call it $s$ ) can be varied and its optimum value needs to be found. Plot strain energy and volume as functions of $s$ and verify that the optimum you found is indeed so.

