ME 260: Structural Optimization: Size, Shape, and Topology		
Assigned: Sep. 20, 2022	Homework 3	Due: Sep. 27, 2022

Problem 1 (10 points)

Obtain the Gateaux variation of the following functionals.

$$J1 = \int_a^b \frac{\sqrt{1+y'^2}}{y} dx$$

$$J2 = \int_0^T \left(m\dot{x}\dot{y} - \frac{b}{2}x\dot{y} + \frac{b}{2}\dot{x}y - kxy \right) dt$$

$$J3 = \frac{\int_0^L p y''^2 dx}{\int_0^L q y'^2 dx}$$

where m, b, k, p, q are known constants

$$J4 = \int_{x_1}^{x_2} \left\{ y^2(x)\ inv(y(x)) \right\} dx$$

Problem 2 (10 points)

Write the Euler-Lagrange equations along with the boundary (or end) conditions for the following problems.

(a) $\underset{w(x,y)}{Min}\ J = \iint_\Omega \left[D\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2D(1-\nu)\left\{ \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right\} - q(x,y)w \right] dxdy$ where D, ν, and q are known. The domain Ω is a closed area in two dimensions.

(b) $\underset{u(t)}{Min}\ J = \frac{1}{2}\int_0^1 \left\{ x(t)^2 + u(t)^2 \right\} dt$ where $\frac{dx}{dt} = u(t) - x(t)$ for $0 \le t \le 1$ with $x(0) = c$.