ME 260: Structural Optimization: Size, Shape, and Topology

Problem 1 (20 points)
Solve the following problem analytically to derive the design and adjoint equations and then modify the beam optimization code for solving this remodeling problem.

A beam of constant square cross section of side $s$, length $L$, and Young's modulus $E$ under distributed linearly-tapering transverse load $q(x)=q_{1} x$ (where $x=0 \cdots L$ ) needs to be remodeled by adding or subtracting material symmetrically along the breadth direction to make the cross section rectangular with constant depth of $s$. The objective is to make the beam as stiff as possible using additional material $V^{*}$. The condition is that the cross section can be modified to rectangular section over a continuous span of length $l$ that is smaller than the beam length $L$. See the figure for further clarity. Pose this as an optimization problem. Write the design and adjoint equations along with the boundary conditions. You need not solve the equation analytically to the extent possible by simplifying the design, adjoint, and other equations.


Side view


Top view

## Problem 2 (20 points)

Verify the analytical sensitivity expression for the output displacement of an electro-thermal-elastic problem using the finite-difference method in COMSOL. Note that you need to ensure that the finite-difference derivative is correct by doing a parametric variation of the perturbation.

