ME 260: Structural Optimization: Size, Shape, and Topology		
Assigned: Aug. 29, 2023	Homework 1	Due: Sep. 7, 2023

Problem 1 (10 points)

$$\underset{n,k_{co},k_{ci}}{Max}\ \phi = \frac{n k_{co} k_w}{k_{ci}(k_{co}+k_w)+n^2 k_{co} k_w}$$

Subject to

$$\lambda: \qquad \frac{k_{ci}k_{co}}{k_{ci}+n^2 k_{co}} - s^* = 0$$

$$Data: s^* = 8, k_w = 80$$

(a) Firstly, by solving the problem by hand, decide if all three optimization variables (n, k_{ci}, k_{co}) are really needed. Argue why k_{ci} need not be considered in optimization.

(b) Solve the above optimization problem by using *fmincon* routine in Matlab, and then graphically by plotting the contours in Matlab. Plot $\phi(n, k_{co})$ as a surface and see if your answer is indeed a local minimum subject to the constraint. Find also the values of the the Lagrange multiplier, λ.

(c) If s^* is decreased by 1%, compute the change in the optimized value of ϕ without re-solving the problem.

Problem 2 (20 points)

Constructing special numerical examples to make a subtle point is fun. Please do this for one of the following subtle points in the KKT theory for constrained minimization. Use Chat GPT to get your answers and verify them. Do not forget to enclose the script of your interaction with Chat GPT. There is a lot all of us learn in explaining something to an AI agent.

(i) If the regularity condition is not satisfied by a KKT point, then such a KKT point may not be a constrained minimum.

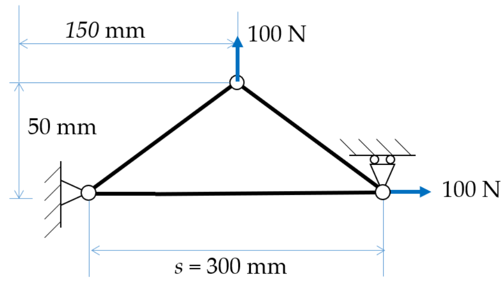
(ii) The Hessian of the Lagrangian need not be positive definite for the sufficiency condition of a KKT point to be a minimum as long as the Bordered Hessian satisfies the conditions set forth for it.

In the numerical example you come up with it, the number of variables, numbers of equality and inequality constraints can be whatever you can muster.

Problem 3 (20 points)

(a) Solve the following three-bar truss problem to find the areas of cross-sections to minimize the strain energy subject to a volume constraint. Use E = Young's modulus =

210 GPa and V^* = upper bound on volume = 30,000 mm^3. Do it by hand and by using *fmincon*.



(b) Now, pose and solve the same problem as a shape optimization problem in one variable wherein the location of the moving pivot is variable from the fixed pivot. That is, 300 mm (call it s) can be varied and its optimum value needs to be found. Plot strain energy and volume as functions of s and verify that the optimum you found is indeed so.