| ME 260: Structural Optimization: Size, Shape, and Topology | | |
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| Assigned: Oct. 17, 2023 | Homework 3 | Due: Oct. 26, 2023 |

**Problem 1** (3 + 5 + 7 = 15 points)

Write Gateaux (first) variation of the following functionals.

1. $\int_0^l \left(y^2 + y'^2 - 2y\sin x\right) dx$

2. $\dfrac{\int_0^l \left(w''(x)\right)^2 dx}{\int_0^l \left(w'(x)\right)^2 dx}$

3. $\frac{1}{2}\int_0^T \left[ v_0 \sin\alpha(t)\left\{x_0 + w_0 t + v_0 \int_0^t \cos\alpha(\tau)d\tau\right\} - \{v_0\cos\alpha(t) + w_0\}\left\{y_0 + v_0\int_0^t \sin\alpha(\tau)d\tau\right\}\right] dt$

**Problem 2** (5 + 8 + 5 + 7 = 25 points)

Write the Euler-Lagrange equations with boundary conditions for the following unconstrained minimizations of functionals.

$$J2 = \int_0^T \left( m\dot{x}\dot{y} - \frac{b}{2}x\dot{y} + \frac{b}{2}\dot{x}y - kxy \right) dt$$

$$\underset{f(x)}{Min}\, J = \left\{ \int_{x_1}^{x_2} \left\{ f^2(x)\ inv(f(x)) \right\} dx \right\}$$

$$\underset{u(x,y,t)}{Min}\ J = \frac{1}{2} \int_{\Omega} \left\{ \left( \frac{\partial u}{\partial t} \right)^2 - \left( c \frac{\partial u}{\partial x} \right)^2 - \left( c \frac{\partial u}{\partial y} \right)^2 \right\} d\Omega$$

$Data : c$

$$\underset{\phi(x,y,t),\, \psi(x,y,t)}{\text{Min}}\ J = \int_{\Omega} \left( \frac{\partial \phi}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial \psi}{\partial y} + \frac{c}{2} \frac{\partial \phi}{\partial t} \psi - \frac{c}{2} \frac{\partial \psi}{\partial t} \phi \right) d\Omega$$

Data : $c$

**Problem 3** (10 points)

A beam is attached to a spring-restrained slider-crank linkage as shown in the figure here. Use the principle of minimum potential energy to write the governing differential equation and boundary conditions for the beam. Use $I$ and $E$ to denote the second moment of area of cross section of the beam and Young's modulus, respectively.