Problem 1 (15 points)

Find the sensitivity of the objective function of the following electro-thermal-compliant mechanism problem. Outline the procedure to solve it using COMSOL. In Programming assignment 2, you need to verify it with the finite difference method.

$$\underset{0\le\rho\le 1}{Min}\, MSE = -t\int_{\Omega} \boldsymbol{\varepsilon}_{\mathbf{u}}^T \mathbf{D} \boldsymbol{\varepsilon}_{\mathbf{v}}\, d\Omega$$

Subject to

$$\Gamma_u : t\int_{\Omega} \left(\boldsymbol{\varepsilon}_{\mathbf{u}}^T \mathbf{D} \boldsymbol{\varepsilon}_{\mathbf{u}_v} - \boldsymbol{\varepsilon}_{th}^T \mathbf{D} \boldsymbol{\varepsilon}_{\mathbf{u}_v} \right) d\Omega \; = 0$$

$$\Gamma_v : t\int_{\Omega} \left(\boldsymbol{\varepsilon}_{\mathbf{v}}^T \mathbf{D} \boldsymbol{\varepsilon}_{\mathbf{v}_v} \right) d\Omega \; = 0 \text{ with unit load at the output DoF}$$

$$\Gamma_\phi : t\int_{\Omega} \left(\nabla \phi^T k_{th} \nabla \phi_v - \nabla V^T k_e \nabla V \phi_v \right) d\Omega \; = 0$$

$$\Gamma_V : t\int_{\Omega} \left(\nabla V^T k_e \nabla V_v \right) d\Omega \; = 0$$

$$\Lambda : t\int_{\Omega} \rho \, d\Omega \; - \left|\Omega^*\right| \le 0$$

Data: t, $\mathbf{D} = \mathbf{D}_0 \rho^3$, $\boldsymbol{\varepsilon}_{\mathbf{u}} = \dfrac{\nabla \mathbf{u} + \nabla \mathbf{u}^T}{2}$, $\boldsymbol{\varepsilon}_{\mathbf{u}_v} = \dfrac{\nabla \mathbf{u}_{\mathbf{v}} + \nabla {\mathbf{u}_{\mathbf{v}}}^T}{2}$, $\boldsymbol{\varepsilon}_{\mathbf{v}} = \dfrac{\nabla \mathbf{v} + \nabla \mathbf{v}^T}{2}$, $\boldsymbol{\varepsilon}_{\mathbf{v}_v} = \dfrac{\nabla \mathbf{v}_{\mathbf{v}} + \nabla {\mathbf{v}_{\mathbf{v}}}^T}{2}$,

$\boldsymbol{\varepsilon}_{th} = \alpha\left(\phi - \phi_0\right)\left\{1 \quad 1 \quad 0\right\}^T$, ϕ_0, α, $k_{th} = k_{th0}\,\rho^3$, $k_e = k_{e0}\,\rho^3$, $\left|\Omega^*\right|$.

Problem 2 (15 points)

Derive the shape sensitivity of mean compliance of a beam where the length of the domain is the design variable. That is, derive an expression for $\dfrac{d}{dl}\left(\int_0^l q w\, dx\right)$ at $l = L$ where the state variable $w(x)$ is governed by $EIw^{iv} = q$ with the boundary conditions, $w_{x=0} = 0$ and $w'\big|_{x=0} = 0$. Verify your answer with the beam finite element code in Matlab.