ME 260: Structural Optimization: Size, Shape, and Topology		
Assigned: Aug. 22, 2024	Homework 1	Due: Aug. 29, 2024

Problem 1 (20 points) "Understanding the meaning of Lagrange Multipliers"

$$\min_{x_1, x_2} f = 0.01 x_1^2 + x_2^2$$

Subject to

$$\mu_1 : \quad g_1 = 25 - x_1 x_2 \le 0$$

$$\mu_2 : \quad g_2 = 2 - x_1 \le 0$$

a. Write the KKT conditions and solve the above problem.
b. Estimate the new optimal value of the objective function if 25 in the first constraint is changed to 25.05 and 2 in the second constraint is changed to 1.95, using the Lagrange multipliers. And then, resolve the problem with 25.05 and 1.95 and see how close your estimate is.
c. 10 extra points for plotting the contours of the objective function and superposing the constraint on it along with the optimum point. Verify the geometric meaning of the first of the KKT conditions by plotting the gradients of the objective function and the active constraint(s).

Problem 2 (10 points) "Understanding the duality of the Lagrangian"

$$\min_{x_1, x_2} f = (x_1 - 3)^2 + (x_2 - 3)^2$$

Subject to

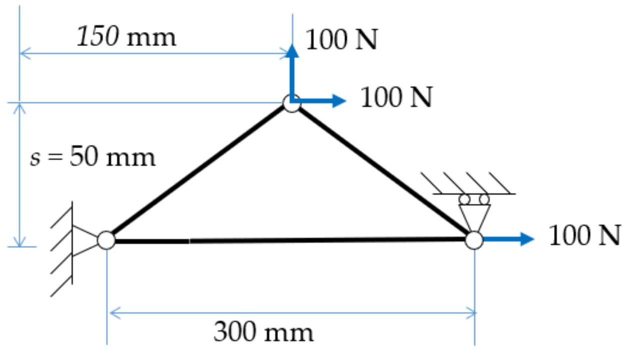
$$\mu : \quad g = 2x_1 + x_2 - 2 \le 0$$

Write the Lagrangian in terms of only μ and solve it to find the solution, μ^*, x_1^*, x_2^*. Be sure to check the second order sufficiency condition.

10 extra points for plotting the minimum of the Lagragian w.r.t. the primal variables and maximum w.r.t. the dual variable.

Problem 3 (30 points) "Initiation into structural optimization of trusses"

(a) Solve the following three-bar truss problem to find the areas of cross-sections to minimize the strain energy subject to a volume constraint. Use E = Young's modulus = 210 GPa and V^* = upper bound on volume = 30,000 mm^3. Do it by hand and by using *fmincon*.



(b) Now, pose and solve the same problem as a shape optimization problem in one variable wherein the height of the truss is variable. That is, 50 mm (call it s) can be varied and its optimum value needs to be found. Plot the strain energy and volume as functions of s and verify that the optimum you found is indeed so.