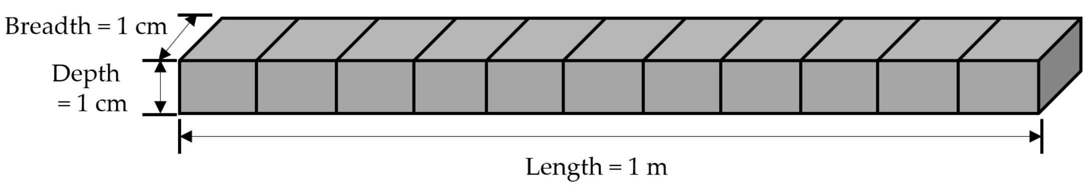
ME 260: Structural Optimization: Size, Shape, and Topology		
Assigned: Oct. 1, 2024	Homework 2	Due: Oct. 8, 2024

Problem 1 (10 points) "Deeper appreciation of Euler's derivation of Euler-Lagrange equation"

a. (5 points) Consider $J = \int_{x_1}^{x_2} F\left(y, y', y''\right) dx$ and use the discretized finite-variable optimization approach (and then taking the limit) to derive the Euler-Lagrange equation in the manner Euler apparently did.
Extra 5 points for deriving the boundary conditions using this same method.

b. (3 points) Consider a fixed-fixed beam with a distributed transverse load of 100 N/m in the depth direction. Think of its discretized model with 11 subunits. If you were to decrease the breadth of the beam by 10% in only one subunit and increase the strain energy by the least amount, which subunit would you choose? Justify your answer.



A fixed-fixed beam with load in the depth direction; $E = 210$ GPa

c. (2 points) And would your answer change if there was a concentrated load of 100 N at the middle of the third subunit from the left end, instead of the distributed load? Justify your answer here too.
Extra 5 points if you draw a graph of the predicted increase in strain energy against the reduced breadth of each subunit for cases (b) and (c).

Problem 2 (10 points) "Routine practice of writing Euler-Lagrange equations and solving them"

a. (5 points) Solve the following problem to obtain $y(x)$. Draw it too.

$$\underset{y(x)}{\text{Extremize}} \; J = \int_1^2 \left(x^2 y'^2 + y\right) dx \text{ with } y(1) = y(2) = 1$$

Consider an arbitrary variation $h(x)$ (which satisfies the Dirichlet boundary conditions) and verify if the solution you obtained is a minimum or a maximum. Draw $F = x^2\left(y' + h'\right)^2 + \left(y + h\right)$ vs. x and superimpose it with $F = x^2 y'^2 + y$ to appreciate the change in J after the arbitrary variation.

b. (5 points) Consider

$$\underset{x(t)}{Min}\ J_1 = \int_0^1 \left(\frac{x'^2}{2} + x \right) dt \ \text{ with } \ x(0) = x(1) = 0$$

and

$$\underset{x(t)}{Min}\ J_2 = \int_0^1 \left(\frac{x'^2}{2} + x \right) dt \ \text{ with } \ x(0) = 0$$

Whose minimum value is larger, J_1 or J_2 , and why? Solve and discuss.

Problem 3 (30 points) "Appreciation of various forms of functionals and how to take their first variations"

Write the first or Gâteaux variation for the following cases.

a. $\underset{y(x)}{\text{Min}}\ J = \int_{x_1}^{x_2} \left(\sin\left(y'^2\right) + y^3 y'' \right) dx$

b. $\underset{y(x)}{\text{Min}}\ J = \dfrac{\int_{x_1}^{x_2} y''^2 dx}{\int_{x_1}^{x_2} y'^2 dx}$

c. $\underset{y(x)}{\text{Min}}\ J = \left(\int_{x_1}^{x_2} y''^2 dx \right) \left(\int_{x_1}^{x_2} y^2 dx \right)$

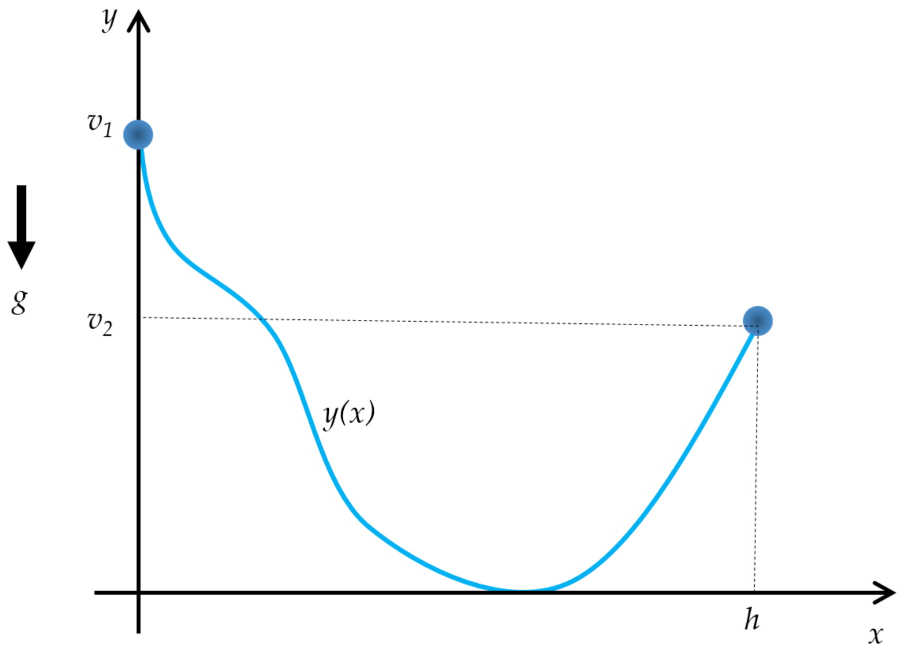
d. $\underset{y(x)}{\text{Min}}\ J = \int_{x_1}^{x_2} \left[y^2 \left\{ \int_0^x \sin\left(y(z)\right) dz \right\} \right] dx$

e. $\underset{y(x)}{\text{Min}}\ J = \int_{x_1}^{x_2} \left(y^2 inv(y) \right) dx$

f. $\underset{y(x)}{\text{Min}}\ J = \max_x (y)$

Extra problem (10 extra points) "Understand more about the brachistochrone curve—that it is a tautochrone too."

Consider two points as shown in the figure. Which curve between the two points will give the least time for the balls to meet if they slide without friction along the curve, one starting at A and another at B, both at rest when they are released to fall under gravity?



Simulate it by writing a Matlab (or any other) code to show that your answer is correct. Where do they meet? Do not forget to show the solution graphically; animation will be even better. Take $v_1 = 1$ m, $v_2 = 0.75$ m, and $h = 2$ m.