ME 260: Structural Optimization: Size, Shape, and Topology		
Assigned: Oct. 11, 2024	Homework 3	Due: Oct. 22, 2024

Problem 1 (16 points) "A gaming way to appreciate the key concepts of the course"

We have discussed many mathematical, engineering, and intuitive concepts in this course. There are obvious and subtle connections among them. So, let us see it from the perspective of New York Times game "Connections". See https://www.nytimes.com/games/connections. Play it couple of days to understand how it works. And then, come up with your own set of four groups of four words related to the content of the course. Use your "Connections" game on one of your classmates and write about how well they played and where they went wrong (or you went wrong in formulating the game). But do try to mislead and trick to make the game interesting and debatable. Here is a simple example:

Gâteaux	Non-negative	Euler	Fréchet
Triangular inequality	Lagrange	Lebesgue	Variational derivative
Raymond	Discretization	Banach	Sobolev
Zero	Hilbert	Symmetry	Partial derivative

Answer

Function spaces Banach, Hilbert, Lebesgue, Sobolev
Properties of a metric (or a norm) Non-negative, zero, symmetry, triangle inequality
Terms underlying alternate derivation of Euler-Lagrange Equations Discretization, Euler, Partial derivative, Variational derivative
Mathematicians who made variational calculus rigorous Fréchet, Gâteaux, Lagrange, Raymond

Problem 2 (20 points) "Understanding how constraints are handled in calculus of variations from an analytical viewpoint"

Use Lagrange's variation concept and alongside any of the concepts of Gateaux variation, Frechet differential, Frechet derivative (but not the variational derivative), to form the basis for constrained optimization in the framework of calculus of variations. In other words, the derivation of the Lagrangian and the Lagrange multiplier should be different from the we did in the class using the variational derivative. In your derivation, the "sensitivity" meaning of the Lagrange multiplier should be evident too. Use functional (global) or function (local) constraints.

Extra 5 points if you do the derivation for both functional and function constraints.

Problem 3 (24 points) "Appreciating Hamilton's principle for writing the equations of motion of elastically deformable bodies"

Consider $\omega = \{u \quad v\}^T$ as the displacement vector of a planar elastically deformable body where $u(x, y, t)$ and $v(x, y, t)$ are functions of spatial and temporal variables. Now, write Hamilton's principle of extremizing the action integral. And then write the equations of motion and interpret the "boundary" conditions to reconcile with our understanding of applying D'Lambert's principle and Newton's second law.