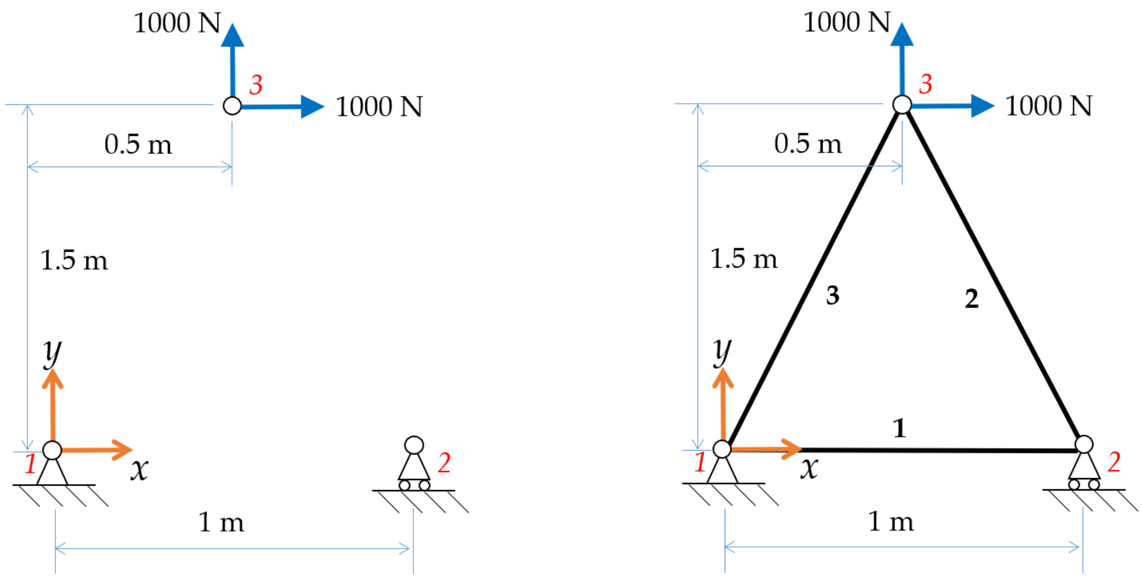
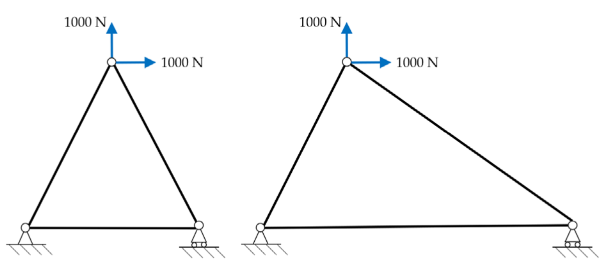
ME 260: Structural Optimization: Size, Shape, and Topology		
Assigned: Aug. 12, 2025	Homework 1: Solution	Due: Aug. 19, 2025

Problem 1 (50 points) "Understanding the importance of topology, shape, and size"

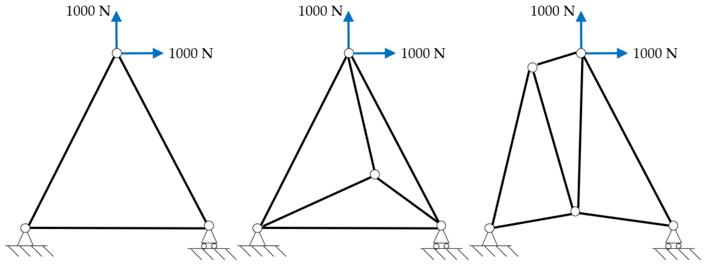
Consider the following planar truss problem.



On the left side, we see that three points are shown. Point 1 is fixed. Point 2 can move the x direction but it is fixed in the y direction. Point 3 has two external forces. On the right side, we see a three-bar truss. It has a particular topology (one hole) with a certain shape. If we move a point in a given topology, we get a differently shaped truss.



If we connect the three points in a different way by introducing one or more extra points, we get a different topology.



Now, we can change the shape of each topology to get differently shaped trusses for that topology. And then, when we vary the areas of cross-section of the truss members, we get different response of the truss for the applied external loads.

- Choosing the right topology is topology optimization.
- Choosing the right shape is shape optimization.
- Choosing the areas of cross-sections is size optimization.

The truss shown on the right side, whose topology and shape are chosen, size optimization gives strain energy, SE = 0.4881 J for the given volume V^*= 4.1623E-4 m^3. The Matlab files from

https://mecheng.iisc.ac.in/suresh/me260/Matlab/Explore3BarTruss.zip

can be used to verify this. You can sue the same files to check how good your topology and shape are. There is a Matlab script for size optimization. You can modify it for shape optimization too.

What you need to do:

Choose a different topology and shape and do size optimization and get as low a value of SE as possible but without changing V^*= 4.1623E-4 m^3. You cannot move points 1 and 3 at all. Point 2 may be moved only in the x direction to the left or right. You can introduce as many points as you need or wish. If you want to keep it as a statically determinate truss, then ensure that $2v - b - 3 = 0$ (Maxwell condition) where v = the number of vertices and b = the number of bars (or truss members). If you disobey the Maxwell condition, then the truss is not statically determinate. And that is okay because our truss FEA (finite element analysis) code in Matlab will still work. You can design such a truss too but the volume of such a truss also should not exceed V^*= 4.1623E-4 m^3.

What you need to submit:

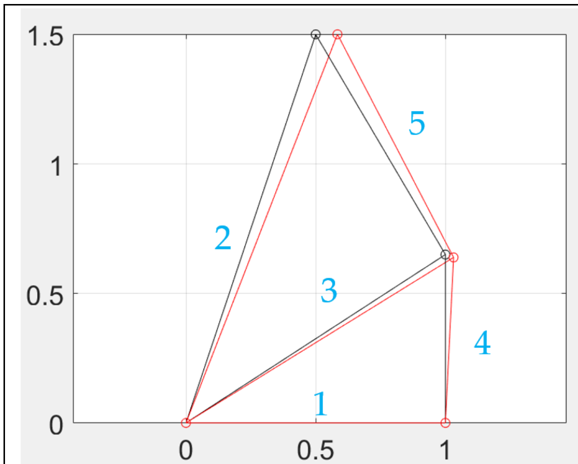
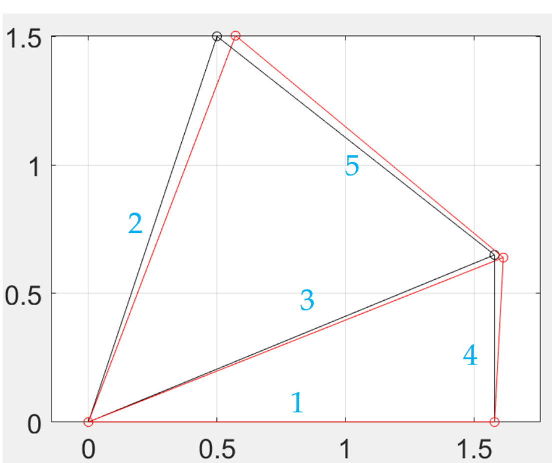
- Hand-written or typed description of how you arrived at your result. Show your work neatly.
- Four input files that contain the data of the truss you designed (node.dat, elem.dat, dispbc.dat, and forces.dat) printed neatly; also email them one zip file whose name is your name.zip.

- Write down the results that include areas of cross-section of all truss members, strain energy, and stresses in each member.
- A picture of your optimized truss with all vertices and bars marked.

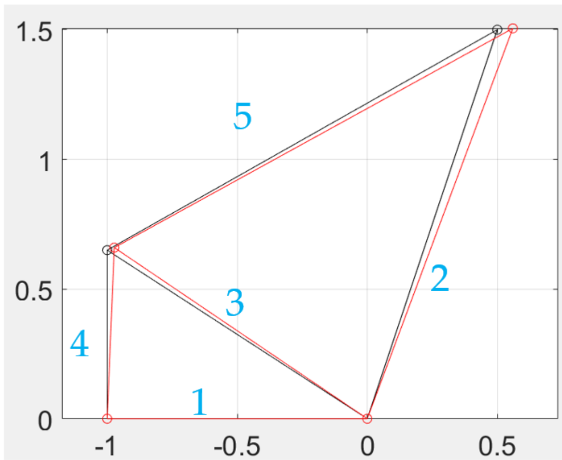
There will be bonus points for those who exceed my expectations. For instance, can you design a truss that has the same internal force (magnitude) in all its members? Or, do anything else you can think of for this data and surprise me, and get bonus points.

Solution

1. Prompted by 2D topology solution given by YinSyn, we have the following optimal topology. Is we fix vertex 2 at (1,0), we get SE = 0.4214 J. Optimal areas of cross-section of all five members are shown in the table. If we move vertex 2 to (1.58,0), we get SE = 0.3743. This is the improvement due to shape design.

	$\boldsymbol{A}^*$ = [0.0000 0.1526 0.0426 0.0840 0.0705] × 10^{-3} m^2. SE = 0.4214 J σ = [0 11.9052 11.9067 -11.9056 -11.9058] MPa
	$\boldsymbol{A}^*$ = [0.0000 0.1330 0.0509 0.0564 0.0599] × 10^{-3} m^2. SE = 0.3743 J σ = [0 11.2200 11.2210 -11.2209 -11.2208] MPa

If we move vertex 2 to the negative side, there is further improvement: SE = 0.3230 J, as be seen next.



$\boldsymbol{A}^*$ = [0.0001 0.5403 0.9405 0.9594 0.9064] × 10^{-3} m^2.
SE = 0.3230 J
σ = [0 10.4237 -10.4231 10.4231 10.4231] MPa

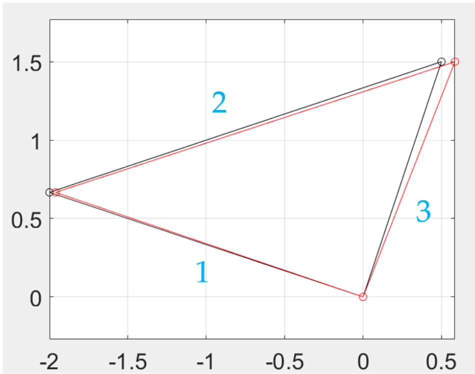
So, if we vary the x coordinate of vertex 3 and find an optimum value, we can find the best shape of the truss.

2. If we want to find a truss that has **equal force** in all three members, we need to move vertex 2 in both x and y directions. If we move vertex 2 to (-2,0.6667), we get equal forces in all three members. The equal force in this case is 790.57 N. After size optimization, we get the same stress (as we should) with the same area of cross-section, which is desirable from the manufacturing viewpoint. But note that the strain energy is much more than what is possible with unequal forces. Thus, there is a tradeoff between manufacturability and performance.

   If you wonder how this design was found, you can do shape design with two variables, which are the x and y coordinates of vertex 2, and impose constraints to have equal force in all three members. That will be two constraints in two equations. Try this interesting exercise.

   Actually, this solution was found using graphics statics. We will discuss that in passing sometime later.

<table>
<tr>
<td>



</td>
<td>

$\boldsymbol{A}^{*}$ = [0.0658 0.0658 0.0658] × 10^{-3} m^2.

SE = 0.4290 J

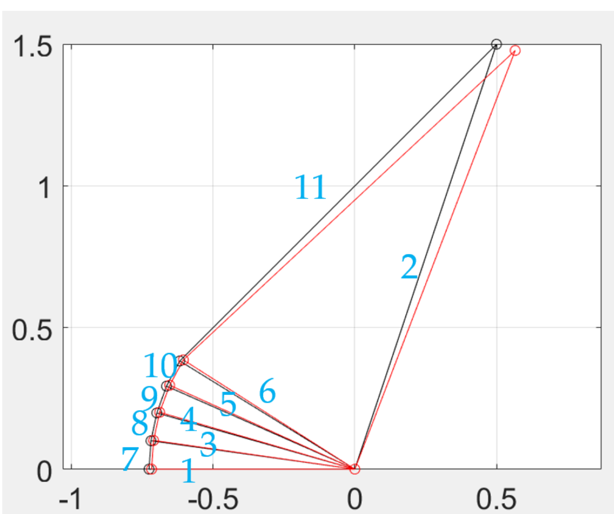
σ = [-12.0126 12.0126 12.0126] MPa

</td>
</tr>
</table>

3. Solutions submitted by the students have different topologies and shapes. All have optimized the areas of cross-section. This exercise shows that topology is the most important aspect of an optimal structure. But shape and size also matter. Only with the optimal topology, the best structure is obtained.

<table>
<tr>
<td>

Raghavendra Katragadda



</td>
<td>

$\boldsymbol{A}^{*}$ = [0.0110 0.0000 0.0222 0.0222 0.0222 0.0488 0.1604 0.1604 0.1604 0.1604 0.1644] × 10^{-3} m^2.

SE = 0.2201 J (the best we have)

σ = [-8.6087 0.0000 -8.6058 -8.6059 -8.6059 -8.6042 8.6035 8.6035 8.6034 8.6034 8.6034] MPa

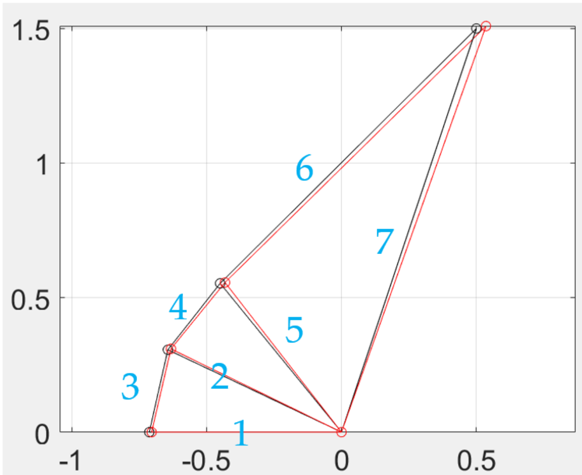
The stress of 8.61 MPa is also the best we have.

The algorithm does not want element 2 as it has nearly zero internal force. So, it pushes it to the lower bound (here, 1E-8 m^2).

</td>
</tr>
</table>

<table>
<tr>
<td>

Harshavardhan L.



</td>
<td>

$\boldsymbol{A}^* = [0.0365 \quad 0.0732 \quad 0.1662 \quad 0.1661 \quad 0.0202 \quad 0.1629 \quad 0.0005] \times 10^{-3}$ m^2.

SE = 0.2227 J

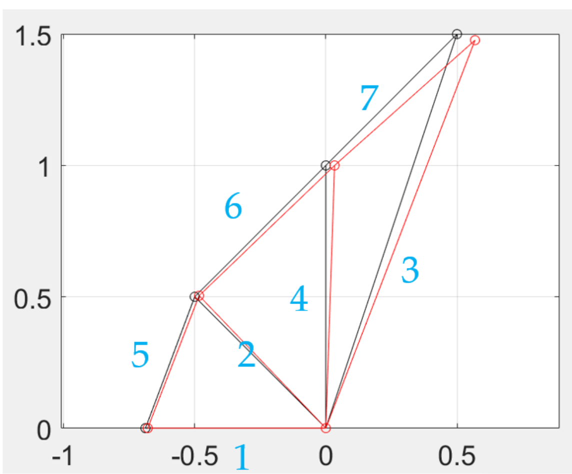
$\sigma = [-8.6563 \quad -8.6555 \quad 8.6553 \quad 8.6565 \quad -8.6555 \quad 8.6554 \quad 8.7138]$ MPa

It may look like element 7 is not needed as it has nearly zero force, but removing it will give an error in FEA.

</td>
</tr>
</table>

<table>
<tr>
<td>

Subhom Das



</td>
<td>

$\boldsymbol{A}^* = [0.0622 \quad 0.0730 \quad 0.0000 \quad 0.0000 \quad 0.1766 \quad 0.1609 \quad 0.1609] \times 10^{-3}$ m^2.

SE = 0.2298 J

$\sigma = [-8.7913 \quad -8.7912 \quad 0.0000 \quad 0 \quad 8.7907 \quad 8.7907 \quad 8.7907]$ MPa

Elements 3 and 4 are redundant as they have nearly zero force but if we remove them, FEA would have a problem as elements 6 and 7 will become loose (i.e., dangling) elements.

</td>
</tr>
</table>

4. Is there a correlation between strain energy and maximum stress in the optimized structures?

In this case, as stiffness increases the maximum stress (equal stress, really) decreases. Thus, stiffest truss is also the strongest truss. But it does not have to be this way in all cases.