ME 260: Structural Optimization: Size, Shape, and Topology		
Assigned: Aug. 28, 2026	Homework 2	Due: Sep. 8, 2026

Problem 1 (10 points) "Understanding the duality of the Lagrangian"

Consider the primal problem and solve it by writing KKT conditions.

$$\min_{x_1,x_2} f = x_1 + x_2^2$$

Subject to

$$g_1 = 1 - x_1^2 \leq 0$$

$$g_2 = x_1^2 - 4 \leq 0$$

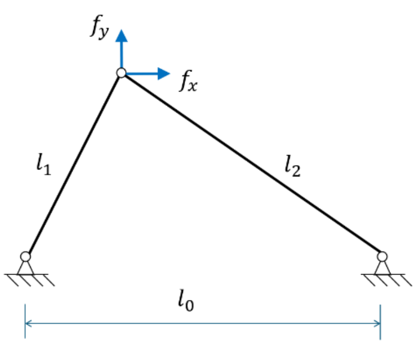
(i) Obtain one or more solutions. Clearly write the optimal values of $x_1^*, x_2^*, \mu_1^*, \mu_2^*$, and $L^* = p^*$ that you obtain.

(ii) Write the dual problem and obtain its solution for the solution(s) that you obtained for the primal problem. Clearly write the optimal values of $\mu_1^*, \mu_2^*, x_1^*, x_2^*$, and $L^* = d^*$ that you obtain.

(iii) Is there strong or weak duality in this problem?

Problem 2 (10 points) "A simple statically indeterminate light-stiff truss to understand optimality principle of uniform strain energy density in the presence of lower and upper bounds on areas of cross-sections"

Optimize the two-bar light-stiff truss with statically indeterminate boundary conditions. The objective is to minimize the strain energy subject to the volume constraints.



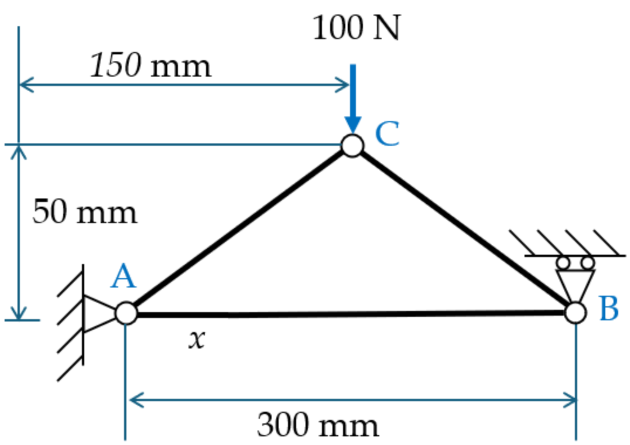
(i) Write down the KKT necessary conditions and obtain the solution symbolically. Use the symbols E for Young's modulus; a_1 and a_2 for areas of cross-sections of the two members, respectively; and V^* for the upper bound on the volume.

(ii) Next consider $l_0 = 1\,m; l_1 = 0.5\,m; l_2 = 0.8\,m; E = 70\,GPa; V^* = 0.001\,m^3$.

Use appropriate numerical values for symbols whose numerical values are not specified to see if uniform strain energy density is satisfied by the optimal solution or not for all five cases listed below. Clearly list the numerical values for each case.

a. Both areas of cross-section are within bounds.
b. a_1 or a_2 has reached the lower bound.
c. a_1 has reached the upper bound.
d. a_2 has reached the upper bound.
e. Both a_1 and a_2 have reached the upper bound.

Problem 3 (10 points) "A more comprehensive structural optimization problem of a light truss with stiffness, strength, and stability considerations."



Consider a statically determinate truss shown here. The coordinates of its six vertices are given in mm. Our objective is to design this truss for stiffness, strength, and stability and to make it as light as possible. The following optimization problem can be posed. Stiffness is imposed with g_1; strength by three constraints with g_2 to g_4; and stability by three constraints with g_5 to g_7.

$$\min_{a_i} V = \sum_{i=1}^{9} l_i a_i$$

Subject to

$$g_1 = v_C - 3 \leq 0$$

$$g_2 = \sigma_1 - S_y \leq 0$$

$$g_3 = \sigma_2 - S_y \leq 0$$

$$g_4 = \sigma_3 - S_y \leq 0$$

$$g_5 = p_1 - P_{cr1} \leq 0$$

$$g_6 = p_2 - P_{cr2} \leq 0$$

$$g_7 = p_3 - P_{cr3} \leq 0$$

Data: $l_{\{i=1,2,3\}}, S_y = 0.3\ GPa, P_{cri} = \frac{\pi^2 E I_i}{l_i^2}, E = 2\ GPa; I_i = 3a_i^2$

Note that v_C is the vertical displacement of vertex C; σ_i is the axial stress and p_i is the axial force in the i^{th} member.

By showing all steps, find the optimal areas of cross-section. Verify that your solution satisfies all constraints. Indicate which constraints are active.

Comment on when stiffness or strength or stability dominate the optimal structure. What decides this?

Which constraint (stiffness, strength, stability) is easier to deal with and why?

Problem 4 (10 points) "Understanding the role of Lagrange multipliers as sensitivity coefficients."

Consider the minimization problem.

$$\min_{x,y} f = \frac{1}{x} + \frac{1}{y}$$

Subject to

$$g_1 = 4x - 2y - 4 \le 0$$

$$g_2 = 2x + y - 6 \le 0$$

(i) Solve this problem by showing all the steps.

(ii) In the active constraint, if the value of the constraint (i.e., 4 or 6) is increased by 1%, estimate the new minimized objective function value without re-solving the problem.

(iii) Verify the accuracy of your estimate by solving the problem with the changed constraint.

There will be bonus points for those who exceed my expectations and those who write all steps clearly and show your thorough understanding of the concepts and analysis. If you use fmincon function in Matalb and verify your answers, that will also be rewarded with bonus points.