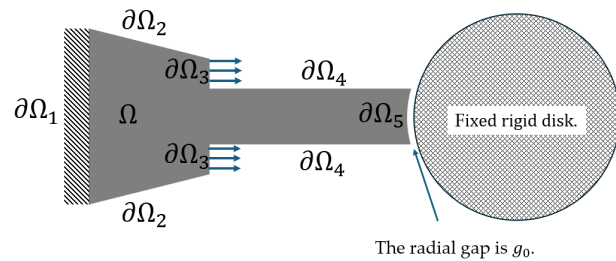


Problem 1 (10 points) “Understanding Euler-Lagrange equations when there are multiple spatial and temporal independent variables”

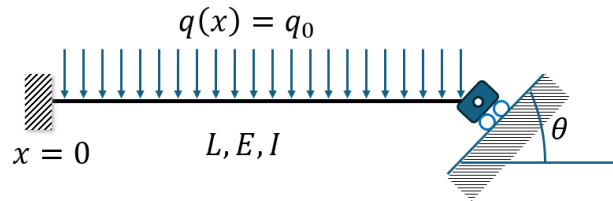
Consider a vibrating planar elastic body shown in the figure here. Using Hamilton’s principle of extremizing the action integral, derive the equation of motion and all boundary conditions. The domain (Ω) and five segments of the boundaries ($\partial\Omega_1$ to $\partial\Omega_5$) are marked. Do not guess the boundary



conditions; derive them systematically using calculus of variations. Note that the blue arrows indicate distributed force on the boundary. Denote that force with vector $\mathbf{f}(t)$. Use E for Young’s modulus, ν for Poisson’s ratio, ρ for mass density, and other standard symbols for other quantities.

Problem 2 (10 points) “Understanding variable end conditions”

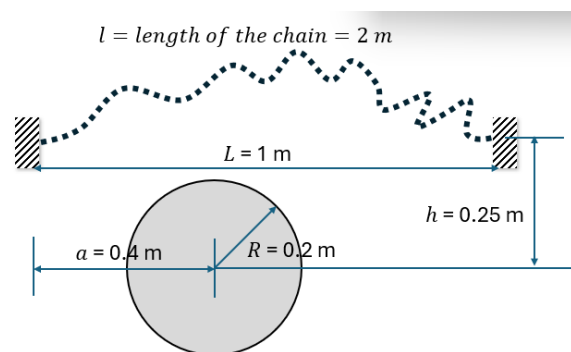
Consider the beam shown in the figure here. Its left end is clamped, and the right end is guided along an inclined line. So, the right end is not known for $w(x)$, which denotes the transverse displacement of the beam.



Using calculus of variations, derive the boundary condition for the right end of the beam. And interpret it physically. Verify your physical interpretation with a numerical example to be sure of your interpretation. Also verify for familiar cases by using $\theta = 0$ or $\pi/2$.

Problem 3 (20 points) “Understanding constrained optimization problem with function and functional constraints”

Consider a chain of length 2 m with a uniform linear mass density of 1 kg/m. It is fixed at both ends as shown in the figure here. There is a fixed rigid circular disk obstructing the fall of the chain if the gravity is suddenly turned on. Which shape does this chain take? Use minimum potential energy



principle to formulate the problem and solve it numerically. Plot the solution. Is there only one solution to this problem?

There will be bonus points for those who exceed my expectations and those who write all steps clearly and show your thorough understanding of the concepts and analysis.