| ME 260: Structural Optimization: Size, Shape, and Topology | | |
|---|---|---|
| Assigned: Oct. 22, 2026 | Homework 4 | Due: Oct. 29, 2026 |

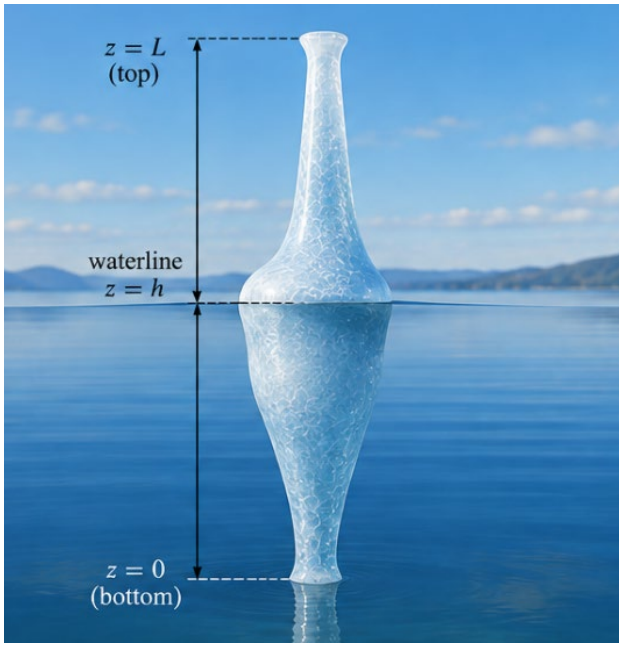
**Problem 1** (13 points) "Understanding Weierstrass-Erdmann corner conditions with a rendezvous curve, which is a variant of the Brachistochrone problem"

Consider points A and B with coordinates (0,0) and (1,0.5). Assuming gravity acting in the negative $y$ direction, find a curve joining points A and B such that two particle masses starting from rest at A and B and sliding along the curve meet each other in the least time. Pose this problem in the framework of calculus of variations with the unknown point where the two masses meet. Solve the problem and check if the curve is smooth (slope is continuous) and if its curvature is continuous at the point where the masses meet. Comment on anything interesting you observed in this problem.

*Bonus points if you write (or vibe-code) a Matlab script for simulating the motion of the particles.*

**Problem 2** (14 points) "Slender iceberg problem for in-depth appreciation of bar optimization"

A vertical, axisymmetric rod of length $L$ and variable circular cross-section $A(z)$ floats upright in calm water, as shown in the figure here. The picture was generated by ChatGPT for the purpose of illustration.




The ice has density $\rho_{ice}$ = 900 $kg/m^3$ and Young's modulus $E$ = 9 GPa; the water density $\rho_w$ = 1000 $kg/m^3$. The volume of the ice rod is $V^* = 4 \times 10^{-4}\, m^3$. Let $z = 0$ be at the bottom of the rod and $z = L$ = 5 $m$ be at the top. And $z = h$ is the distance to the waterline from the bottom of the rod. The rod is submerged in water from to $z = 0$ to $h$ , and is above the waterline from $z = h$ to $L$. The value of $h$ needs to be determined by balancing gravity and buoyancy based on $A(z)$, which also needs to be determined to minimize the axial strain energy of the rod under self-weight and buoyancy. Pose and solve this as a constrained optimization problem in the framework of calculus of variations with the objective of minimizing strain energy subject to the volume constraint ($\int_0^L A(z)dz - V^* = 0$) and floatation constraint (i.e., $\int_0^h A(z)dz - \frac{\rho_{ice}}{\rho_w} V^* = 0$).

*Bonus points if you also include constraints on hydrostatic stability and elastic buckling.*

**Problem 3** (13 points) "Analytical sensitivity using the adjoint method"

Consider the functional $\Omega = \frac{\int_0^L EI(x)\phi''^2 dx}{\int_0^L \rho A(x)\phi^2 dx}$, assume that $I(x) = \alpha A(x)$ and derive the sensitivity of $\Omega$ w.r.t. $A(x)$. Note that the governing equation for the state variable $\phi$ is given by $(EI\phi'')'' - \rho A\Omega\phi = 0$. Avoid computing the sensitivity of the state variable by using the adjoint method.

There will be bonus points for those who exceed instructor's expectations and those who write all steps clearly and show your thorough understanding of the concepts and analysis.