

ME 260: Structural Optimization: Size, Shape, and Topology		
Assigned: Sep. 15, 2026	Mini Project	Due: Oct. 1, 2026

Mini Project (10 points) “Learning coding for structural optimization”

In Homework 1, you understood the importance of topology, shape, and size in the design of light-stiff trusses for given boundary conditions. You also understood the theory for the optimality conditions and the optimality criteria method to obtain optimal solutions. But if boundary conditions are not fully prescribed? You saw that in Homework 1 wherein we did not know the location of the roller support. In Homework 2, you considered objectives and constraints other than volume and stiffness. In this Mini Project, let us take a leap by exploring general structural problems of large sizes. Coding is straightforward today because all the tools are at your disposal. Using those tools properly and efficiently is what we want you to learn in this Mini Project.

Problem Statement

Write a general-purpose truss optimization code that can handle at least two of the following in Category 1 and at least one in Category 2. You may consider planar (i.e., 2D) or spatial (i.e., 3D) trusses.

Category 1

- Multiple force cases with fully prescribed boundary conditions over a fixed ground structure.
- Single force case with variable boundary conditions over a fixed ground structure.
- Single force case with fully prescribed boundary conditions over a variable ground structure.
- Single force case with fully prescribed boundary conditions using lattice units.

Category 2

- Minimize strain energy for given volume.
- Minimize volume with stress constraints.
- Maximize fundamental natural frequency for given volume.
- Maximize displacement of a degree of freedom with a constraint on strain energy for given volume.

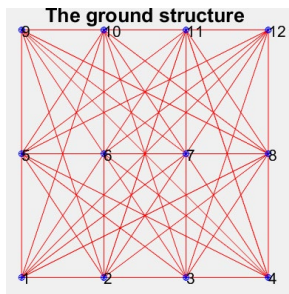
This leads to a total number of $6 \times 4 = 24$ cases. You will be assigned one of these cases.

All the cases will be discussed in the class to get you good insights into all the cases.

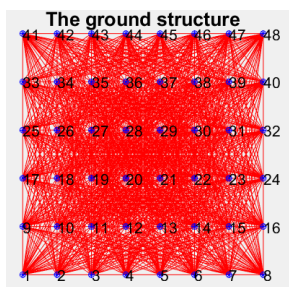
There will be bonus points for those who exceed my expectations and those who write all steps clearly and show your thorough understanding of the concepts and analysis.

What is a ground structure?

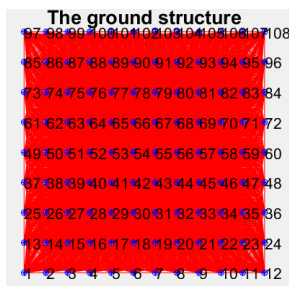
The ground structure of a truss is a set of connected truss elements over a given grid of points wherein all grid points are connected to one another, as shown next.



This 4×3 grid has 49 truss elements.

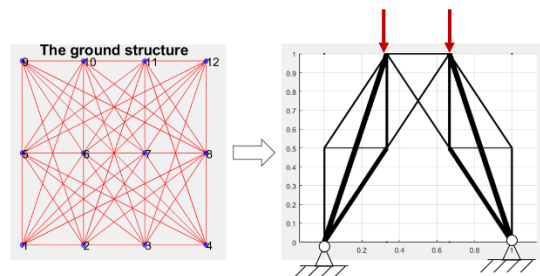


This 6×8 grid has 730 elements.

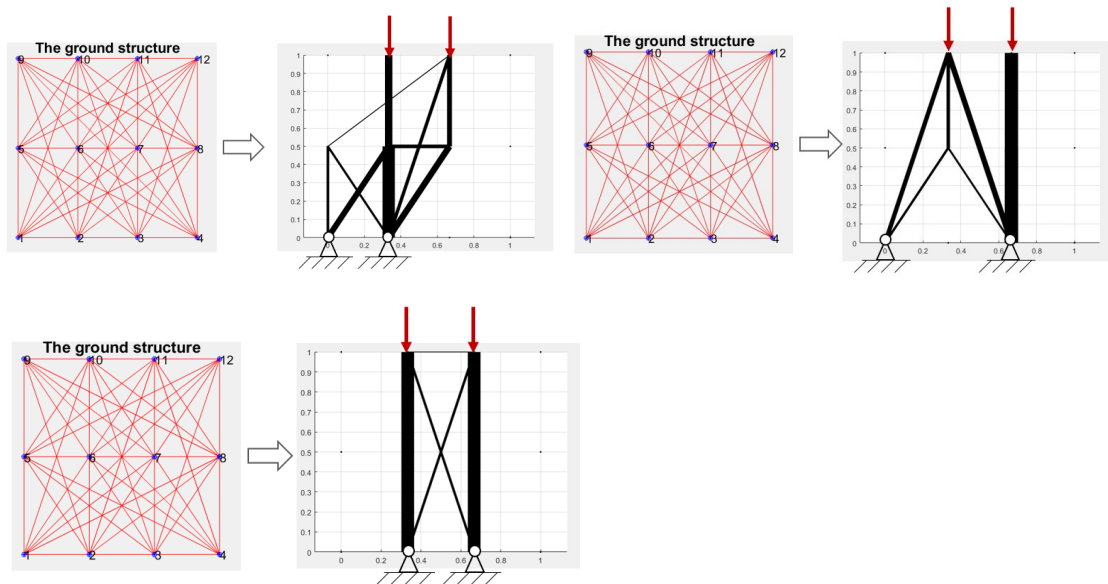


This 12×9 grid has 3,607 elements. It is so dense that you hardly see element lines. If we use such a dense ground structure, we get a topology that is closer to optimal design. But the price we pay is a large optimization problem with a lot of optimization variables. Today, with high-performance computing, we can handle millions of variables.

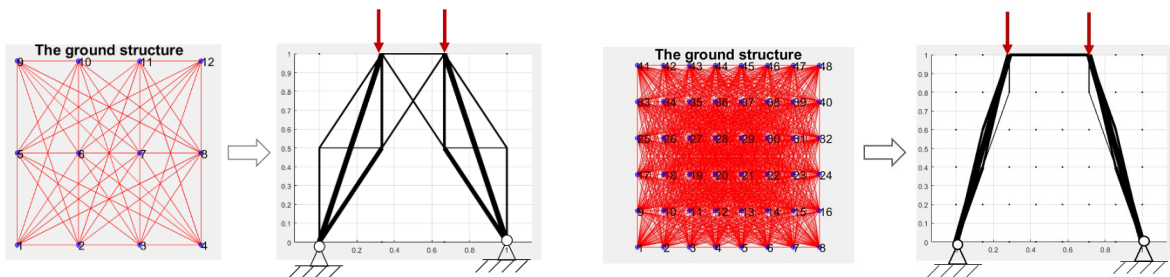
We use a ground structure to decide which truss elements should remain in the optimal truss for given load and boundary conditions. We do this by making the area of cross-section of each truss element in the ground structure an optimization variable. We also impose lower (nearly zero) and upper bound on the areas of cross-section. The optimization algorithm will determine the optimal values of cross-section areas. This is size optimization. If the cross-section area of a truss element reaches the lower bound, that element is considered absent in the optimal design. Therefore, some elements disappear and thus define the topology. This is topology optimization. Here is an example. Only some elements remained and define an optimal topology.



See more examples to see what an optimization algorithm does for different boundary conditions. You should write such a general-purpose code so that an optimal topology can be obtained for any given load and boundary conditions.



And, for the same load and boundary conditions, a denser grid can give a more refined solution if we specify lower and upper bounds on cross-section areas carefully.



Category 1

(a) Multiple force cases with fully prescribed boundary conditions over a fixed ground structure.

Here, we minimize the worst value (i.e., largest) strain energy of all the force cases (or load cases as it is sometimes called). No other change.

$$\min_{\text{Design variables}} \max(SE_1, SE_2, \dots, SE_m) \text{ where there are } m \text{ load cases.}$$

We had an elaborate discussion on it in the lecture on Sep. 24th.

(b) Single force case with variable boundary conditions over a fixed ground structure.

Here, we attach springs at each node of the boundary and make the spring constants additional optimization variables. Each spring constant has a lower bound that is nearly zero (spring is essentially absent) and upper bound (a very high value that fixes the DoF). But this will be too many variables. So, we used a continuous Gaussian distribution function, $k_{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, with only one variable, μ . This way, we can optimally locate the variable boundary condition. We begin with a large value of σ and slowly decrease it during the optimization process. This way, we do not bias the location of the spring.

We discussed this also on Sep. 24th in the class.

(c) Single force case with fully prescribed boundary conditions over a variable ground structure.

Here, we simply add the nodal coordinates as additional variables so that the grid gets distorted optimally.

(d) Single force case with fully prescribed boundary conditions using lattice units.

We discussed how an exhaustive ground structure can be avoided with only four lattice units (in 2D) and eight (in 3D). All the beams in a lattice unit have the same areas of cross-section. In 2D, the lattice units are a square, a cross, a plus, and a rhombus. We get away with only four variables per unit cell in the design domain. An alternative view of this is that we are actually choosing a “meta material” at each location in the design domain.

We discussed this at length on Sep. 24th.

Category 2

(a) Minimize strain energy for given volume.

You know this well.

(b) Minimize volume with stress constraints.

Quiz 14 elaborated this idea.

(c) Maximize fundamental natural frequency for given volume.

We discussed the Rayleigh quotient. You need to generate code for modal analysis.

(d) Maximize displacement of a degree of freedom with a constraint on strain energy for given volume.

We discussed mutual strain energy, which is numerically equal to the displacement of a DoF. It needs two load cases, one the applied load and the other a unit virtual load at the DoF where you want to know the displacement.