ME 260: Structural Optimization: Size, Shape, and Topology		
Assigned: Aug. 25, 2026	In-class quiz 5 (10 min)	Due: Aug. 25, 2026

Name:

Consider the following constrained minimization problem.

$\min_{x_1,x_2} f = x_1^4 - x_1^2 + x_2^2$

Subject to

$h = x_1^2 - 1 = 0$

Solve it by following the steps below to see if this problem has strong duality or not.

1. Write the Lagrangian.

   $L =$

2. KKT conditions.
   Write the gradient of the Lagrangian and equate to zero (the first of KKT conditions)
   $\nabla L = \begin{Bmatrix} \partial L/\partial x_1 = \\ \partial L/\partial x_2 = \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \Rightarrow \; x_2 =$
   and x_1 has two possibilities. Write them here by expressing x_1 in terms of λ.

   $x_1 =$
   or
   $x_1 =$

   Of course, the feasibility must hold.
   $h = x_1^2 - 1 = 0$
   Now, what are the feasible values of x_1?
   $x_1 =$
   or
   $x_1 =$

   Now, write optimum values of all these.

$x_1^* =$
$x_2^* =$
$\lambda^* =$
$f^* = p^*$ =minimum for the primal problem =

3. Now, write the dual problem by expressing the Lagrangian in terms of only λ. Notice that there are two expressions, one for $\lambda \leq 1$ and another for $\lambda > 1$.

$\max_{\lambda} L(\lambda) =$

4. What is the maximizing value of λ? And what is the maximum value of $L^* = d^* =$?

5. Is this duality strong or weak?