ME 261:Worksheet on Complex Analysis.

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1. For the function $f(z) = \frac{(z^2-1)-\sin(z-1)}{(z-1)}$, find the residue at $z = 1$.
2. Consider $J = \oint_C f(z)\, dz$, where C is the counter clockwise unit circle with its center at the origin. For $f(z) = \frac{e^z}{z^2-\frac{1}{4}}$, find the value of J.
3. Given the complex function $f(z) = \frac{\cos(z)}{z^5}$, find the residue at the only singular point.
4. Compute the integral $\int_C z\, dz$ along the straight line from the origin to $(2+2i)$.
5. Compute the integral $\int_C z\, dz$ along a closed contour C, that is composed of a straight line from the origin to $2\sqrt{2}i$. And then along a circular arc from $2\sqrt{2}i$ to $(2+2i)$ with centre at the origin. And then from (2+2i) to the origin.
6. Compute the integral $\int_C z\, dz$ along a closed contour C, that is composed of three straight lines: 0 to 2, 2 to $(2+2i)$ and $(2+2i)$ to 0 in the anti-clockwise direction.
7. Consider $J = \oint_C \frac{\cos(z)}{z^5} dz$. C is the contour defined by a counter clockwise circle $|z-0.4| = 0.5$. Find the value of J.
8. Find the value of the integral $J = \oint_C \frac{5z^2+17}{z^3-2z^2+4z-8} dz$ for three different contours C given below. C is a counter clockwise circle given by
   (a) $|z-3| = 3$
   (b) $|z-(1+i)| = 3$
   (c) $|z| = 3$
9. Find the value of the integrals given below using contour integration
   (a) $I = \int_0^\infty \frac{1}{\sqrt{5x}(1+25x^2)} dx$
   (b) $I = \int_0^\infty \frac{1}{\sqrt[3]{x}(1+x^2)} dx$
   (c) $I = \int_{-1/a}^{1/a} \frac{\sqrt{1-a^2x^2}}{1+a^2x^2} dx$
   (d) $I = \int_0^2 \frac{1}{\sqrt{x(2-x)}} dx$
   (e) $I = \int_0^\infty \frac{x^{1/3}}{(x+2)(x+1)} dx$
10. The Bromwich integral $f(t) = \mathcal{L}^{-1}(F(s)) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} F(s)e^{st} ds$. Find $f(t)$ when $F(s)$ is
    (a) $F(s) = \frac{1}{s-3}$
    (b) $F(s) = \frac{3}{s^2+9}$
    (c) $F(s) = \frac{s}{s^2+4}$
    (d) $F(s) = \frac{2}{s^3}$
    (e) $F(s) = \frac{5+s}{s^2+1}$