

# Optimum Design of a Compliant Foot for a Quadruped

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## ABSTRACT

15 Improving the mechanical design of a quadruped in order to im-  
16 prove its performance and efficiency is an area of active research.  
17 The design of the feet with compliant element, such as a spring, can  
18 play an important role in the performance of a quadruped robot.  
19 In this work, we obtain the optimum value of the spring stiffness  
20 which results in efficient hopping behavior of a single leg using  
21 an evolutionary strategy. We start with an initial set of controller  
22 gain parameters for the hip and knee actuators along with the foot  
23 spring stiffness. The performance of each candidate solution was  
24 evaluated based on the maximum hop height. The algorithm is able  
25 to efficiently search the high-dimensional solution space and find  
26 the optimal control parameters, resulting in an improved hopping  
27 mechanism. The optimized controller gain parameters with a spe-  
28 cific range of foot spring stiffness values showed that the compliant  
29 element at the foot helps the leg hop higher than a rigid foot.  
30

## KEYWORDS

32 Quadruped, Single-leg hopping, CMA-ES Evolutionary Strategy,  
33 Compliant foot  
34

## ACM Reference Format:

36 Pramod Pal, Anubhab Dasgupta, Shishir Kolathaya, and Ashitava Ghosal.  
37 2023. Optimum Design of a Compliant Foot for a Quadruped. In *Proceedings*  
38 of ACM Conference (Conference'17). ACM, New York, NY, USA, 7 pages.  
39 <https://doi.org/XXXXXX.XXXXXXX>

## 1 INTRODUCTION

42 Legged robots have gained a lot of interest and significance in  
43 recent years due to their potential applications in many different  
44 fields. Legged robots can climb, walk, and jump, making them suited  
45 for activities requiring movement across challenging or uncertain  
46 environments [4, 7]. Legged robots can mimic human movements,  
47 making them perfect for activities that require interaction with  
48 people, such as search and rescue operations or helping those who  
49

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57 Conference'17, July 2017, Washington, DC, USA  
58 © 2023 Association for Computing Machinery.  
59 ACM ISBN 978-x-xxxx-xxxx-x/YY/MM...\$15.00  
60 <https://doi.org/XXXXXX.XXXXXXX>

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75 have mobility impairments [18]. Designing a quadruped robot ne-  
76 cessitates a broad range of considerations, including the design  
77 of the legs, actuators, control systems, power supply, and overall  
78 structure. The leg design must offer stability and mobility, with the  
79 ideal number and configuration of joints, actuation type, and foot  
80 design depending on the intended use and environment. Therefore,  
81 it is essential to investigate foot design that may provide stability,  
82 terrain adaptability, increased energy efficiency, and less impact  
83 force of the robot movement on the environment, making it safer  
84 to use in populous or sensitive locations. Recent years have seen  
85 an increase in research and development of compliant feet which  
86 are intended to be flexible and deform under external stresses and  
87 this is expected to improve the robot's performance across various  
88 terrains and situations.

89 Legged locomotion experiences energy losses for three main  
90 reasons: transmission losses due to friction, actuator losses due  
91 to heating, and system-environment interaction losses. Reducing  
92 the first two losses requires a new actuator design and electronics,  
93 which can be system-specific and challenging to implement [24].  
94 More sophisticated series elastic actuators (SEAs) can solve the  
95 latter issue. By placing compliant parts in series with the actuator,  
96 SEAs reduce the ground impact force. However, SEAs are typically  
97 more expensive, complicated, and power-consuming than conven-  
98 tional actuators. They also have limited bandwidth and force range  
99 and can have limited accuracy due to mechanical compliance and  
100 control electronics [15]. Other forms of advanced actuation, like  
101 quasi-direct drive (QDD) actuators with a lower gear ratio, provide  
102 direct torque input and exhibit good control. However, these motors  
103 are expensive and have poor torque output with increased Joule  
104 heating [11].

105 Consequently, it is more practical to use a simple passive spring  
106 to store energy during the foot's contact and then release it during  
107 the other phases of the leg's motion. This not only increases the  
108 stability but also increases the efficiency of the legged robot [5].  
109 The linear springs may be designed to meet the needs of a given  
110 application by considering parameters such as the robot's mass, size,  
111 and operating environment. It may also be engineered to produce  
112 damping to modify the rate at which the foot returns to its original  
113 position after deformation. This plays a crucial role in limiting  
114 the transfer of vibrations from the ground to the robot's frame.  
115 Linear springs are a cost-effective solution due to their versatility,  
116 durability, low material cost, and low manufacturing complexity.  
117 They are also adaptable to robots of varying sizes and weights  
118 because of their scalability. This paper will examine the effect of

117 compliant feet that use linear springs and aim to determine optimal  
 118 control of the actuators along with the optimal foot stiffness.

119 The paper is organized as follows: in the rest of this section,  
 120 we review the relevant literature. We present the leg design and  
 121 its kinematics in section 2. We present the MuJoCo simulation  
 122 parameters, control strategy, and optimization algorithms used for  
 123 training the single-leg robot for hopping in section 3. We present  
 124 the numerical simulation results in section 4, with the conclusion  
 125 in section 5.

## 127 1.1 Related Work

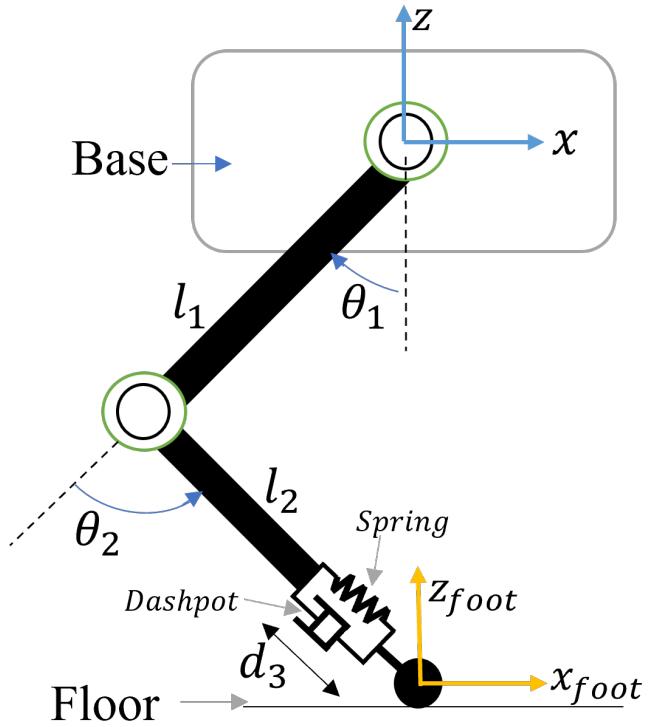
129 Raibert was an early innovator in legged robotics, and his research  
 130 on compliant feet for robot legs impacted the development of modern  
 131 mono-ped, biped, and quadruped robots. In the studies by him  
 132 and his co-workers, they focused on using passive dynamics to  
 133 improve legged movement by introducing compliant features into  
 134 the design of the foot. This allowed the robots to adapt to uneven  
 135 terrain and recover from disturbances with less effort (see, for  
 136 example, [26]). Buehler and co-workers developed hopping and  
 137 quadruped running robots by incorporating springs in the robot's  
 138 leg axis or hip joint [1, 21]. Kostamo et al. introduced a novel method  
 139 for reducing bounce between a robotic leg and the floor using a  
 140 semi-active responsive foot [16]. Hyon and Mita demonstrated that  
 141 the spring converts the robot's kinetic energy to elastic potential  
 142 energy and stores impulse energy for the next step [14]. The 'KOLT'  
 143 leg design by Palmer et al. showed minimal impact loss, low inertia,  
 144 variable stiffness, and an energy storage system [20]. Focchi  
 145 demonstrated the effect of variable spring stiffness by plotting the  
 146 torque produced by the impact in the knee joint when the leg hits  
 147 the floor [8].

## 148 2 LEG DESIGN AND CONSTRAINTS FOR 149 HOPPING

152 The leg design has the following main components: the length of  
 153 the links, the shape of the links, hip and knee joint placements,  
 154 the actuator power ratings, and the foot design consisting of the  
 155 shape of the part touching the ground and the added compliance  
 156 mechanism. The ideal leg length for a jumping robot will vary  
 157 depending on various elements, such as the robot's size and weight,  
 158 the power of its actuators, and the environment in which it will  
 159 be used—including surface friction coefficients, the incline of the  
 160 surface, and so on. We have chosen to keep the shape of the links  
 161 straight to keep the structure simple for analysis in the simulation.  
 162 Both the actuators for the hip and knee joints were fixed to the  
 163 top part of the hip link. The shank link is driven by a belt and  
 164 pulley drive with a gear ratio of 1:2. For power transmission from  
 165 the motor to the links, we have used a planetary gearbox with a  
 166 gear ratio of 1:6. The foot is designed by adding a linear spring and  
 167 damper which connects the lower extreme of the shank link to the  
 168 foot part as shown in Fig. 1.

### 171 2.1 Selection of Link Lengths

172 In general, longer link lengths might give the robot a higher me-  
 173 chanical advantage since they enable a longer stride and more



175 176 177 178 179 180 181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198 199 200 201 202 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223 224 225 226 227 228 229 230 231  
**Figure 1: Schematic diagram of the single leg with compliant foot**

leverage. Larger link lengths will increase stability during the leap, but they will also require more power to get the robot off the ground and more sophisticated control algorithms to keep it there. As the literature shows, longer legs could potentially be more vulnerable to harm or failure [2]. We decided to use economical 360 KV brushless motors in our hardware and this limited the torque values. To make our system simple, we assumed equal links of 0.17 m length, which is equivalent to a medium size dog leg length [9, 27]. The base frame of the leg is allowed to move vertically in the Z-direction. The base of the leg is connected to bearing shafts with the help of linear bearing, as shown in Fig. 1, thus constraining the base motion in the Z-direction.

## 2.2 Kinematic Modelling and Tracking Trajectory Generation

The leg is a critical component of any quadruped or a walking robot. A well-designed leg should be able to absorb sudden impact force to prevent its parts from breaking. This may be accomplished by simply attaching a compliant link to the foot, which can minimize the force of contact and simultaneously store and transfer energy while executing various gaits. The single leg, as seen in Fig. 1, consists of a base, thigh, shank, and foot link joined by two active revolute joints and one passive prismatic joint. The lengths of the thigh and shank links are represented by  $l_1$  and  $l_2$ ; the rotations

of the links are represented by the angles  $\theta_1$  (from vertical) and  $\theta_2$  (from the direction along thigh link) and their range is listed in Table 1. The forward kinematics equations provide the Cartesian locations that can be tracked by the foot tip and are given as

$$x_{foot} = -l_1 \sin(\theta_1) - l_2 \sin(\theta_1 + \theta_2) - d_3 \sin(\theta_1 + \theta_2) \quad (1)$$

$$z_{foot} = -l_1 \cos(\theta_1) - l_2 \cos(\theta_1 + \theta_2) - d_3 \cos(\theta_1 + \theta_2) \quad (2)$$

where  $d_3$  denotes the linear motion of the spring (see Fig. 1).

The cyclic trajectory is made linear by constraining one of the coordinates to be a fixed point, and the resulting reference path is used to approximate the foot end-point trajectory to make the leg hop in one place. The end-point trajectory is given by

$$x = 0, \quad z = -0.174 + 0.026 * \sin(\phi)$$

where the angle  $\phi$  is used to divide the trajectory into 200 equally spaced points.

There are three joint variables in a leg, and to obtain  $\theta_1$ ,  $\theta_2$ , and  $d_3$  for a given  $(x, z)$ , a redundancy resolution scheme needs to be used. In this work, we have used a sequential least squares programming (SLSQP) algorithm, originally implemented by Kraft [17] (see also [10]). We use the quantity  $\|q\|^2$  as the objective function in the optimization, where  $q$  denotes the vector of the hip angle, knee angle, and the change in length divided by the original length of the spring-damper system, to obtain the inverse kinematics solution of the compliant legged system.

Joint Variable	Joint name	Type of joint	Range
$\theta_1$	hip	Revolute	0 to 1.6 radian
$\theta_2$	knee	Revolute	-2.1 to 0 radian
$d_3$	foot	Prismatic	0 to 0.012 m

Table 1: Joint variables motion range

### 3 MUJOCO SIMULATION AND PARAMETERS USED

To study the effect of linear spring at foot, we made use of the open AI MuJoCo simulation environment [25]. MuJoCo captures contact dynamics with state-of-the-art capabilities. It is a popular open-source program for robotic system simulation and is simple to use. To communicate with the simulator and get experimental data, we used the MuJoCo-py module. In reference [6], MuJoCo is shown to be the quickest and most accurate for robotics-related system. The model shown in the Fig. 2 represents the single leg that has been designed using the Solidworks program, and then the URDF (Unified Robotics Description Format) is created with the help of an add-on module. Taking reference from the URDF file for the position and orientation of the links, an XML file world body tag is made. Using XML API documentation, we first make the ground plane on which the single leg will jump and then added other leg elements along with the base frame.

To correctly depict the robot's behavior using the MuJoCo simulation, a number of parameters must be specified. These parameters

consist of model parameters, which determine the physical qualities of the simulation's objects, such as mass, size, and location. Simulation parameters, such as the time step size and the number of iterations, govern the simulation itself. Control parameters, such as actuator forces and torques, define how the robot is controlled. The visualization factors, such as camera position and lighting conditions, dictate how the simulation can be shown. Solver parameters can be used to control the numerical solver, which is utilized to solve the equations of motion and the stability and precision of the simulation can be controlled with the use of these parameters. Environment parameters describe the attributes of the simulated environment, such as the coefficients of gravity and friction. All of these parameters work together to form a realistic depiction of the robot and its environment, enabling the realistic simulation and assessment of robotic systems.

In our simulations, we assigned a gravity that allows the single leg to fall to make contact with the ground when it hits the ground plane. In MuJoCo, a variety of actuators are available. We have used the proportional and derivative controller gain of a motor to simulate the behavior of a spring and damper as an added compliance mechanism to the foot of the single-legged robot. We have used a timestep of 0.001 which provides better accuracy and stability. For the solver, we have used Newton's method. A 4<sup>th</sup>-order Runge-Kutta method is used as an integrator, known to be better than the Euler method for the energy-conserving system both in terms of stability and accuracy [23]. The total mass of the single leg is 3 kg. At all joints, we have chosen a damping of 1 N s/m. The torque is limited in both hip and knee joints to 7 N-m.

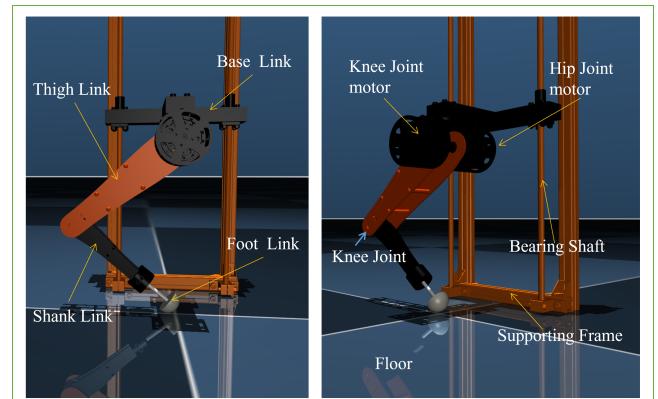
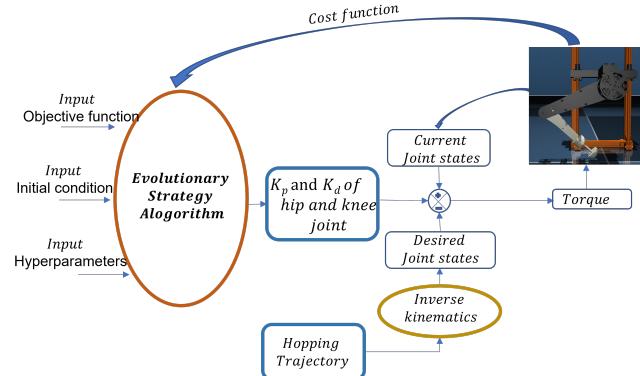


Figure 2: Single leg with compliant foot in MuJoCo environment

Fig. 3 shows the block diagram of simulation. We first find the optimal proportional ( $k_p$ ) and derivative gains ( $k_d$ ) for hip and knee joints motor with the help of a data-driven evolutionary algorithm – see section 3.2 below for the details of the three different evolutionary strategies that were attempted. These gains are applied to the errors of the joint state to yield torque commands. The torques are given to the drive motors to yield new joint states and the base position. The base position helps to compute the cost function, and the evolutionary strategy algorithm maximizes the cost function by maximizing the jump. We trained the model for a range of spring

349 stiffness to get optimal  $k_p$  and  $k_d$  for the hip and knee joint motor.  
 350 We then chose the spring stiffness which gives the maximum cost  
 351 function value.



367 **Figure 3: Schematic diagram of single leg with compliant  
 368 foot**

### 3.1 Control Strategy

373 In the simulation, torque is used as the control input. The control  
 374 system uses the proportional plus derivative (PD) strategy to cal-  
 375 culate the necessary torque output based on the current position  
 376 and velocity of the system as well as the desired position and ve-  
 377 locity – see Fig. 2. The PD control continues to be one of the most  
 378 popular strategies despite significant advancements in nonlinear  
 379 control systems and development of model-based controllers. PD  
 380 controllers draw more community interest since they don't require  
 381 models (and its inherent uncertainty) and are simpler to develop.  
 382 In walking robots like bipeds, PD controllers have been widely  
 383 employed, and research has shown that these controllers are locally  
 384 stable in a variety of circumstances. The PD controller for the three  
 385 actuators can be written as

$$387 \tau_1 = k_{p1}e + k_{d1}\dot{e}, \quad \tau_2 = k_{p2}e + k_{d2}\dot{e}, \quad \tau_3 = k_{p3}e + k_{d3}\dot{e}$$

388 where,  $\tau_1, \tau_2, \tau_3$ , are the torques  $k_{p1}, k_{p2}, k_{p3}$  are the proportional  
 389 gains,  $k_{d1}, k_{d2}, k_{d3}$  are the derivative gains,  $e$  is error in current  
 390 position and desired position and  $\dot{e}$  is rate of change of error, re-  
 391 spectively.

### 3.2 Evolutionary Algorithms

394 We have implemented three distinct evolutionary strategies to find  
 395 the optimal controller gains for the single-legged system. A brief  
 396 description of each of them is given below:

- 397 • **Covariance Matrix Adaptation Evolutionary Strategy:**  
 398 We have implemented the work of Hansen et al. [13]. In the  
 399 CMA-ES evolution strategy, new candidate solutions are  
 400 sampled according to a multivariate normal distribution in  
 401  $\mathbb{R}^n \times \mathbb{R}^n$ . In our case  $n = 6$  comprising of the hip, knee, and  
 402 foot  $k_p, k_d$  values. We treat the problem as a black box opti-  
 403 mization problem, and thus the optimization is model-free.

407 We sample controller gains from the multivariate normal  
 408 distribution and perform a mutation operation. Mutation  
 409 amounts to adding a random vector, a perturbation with  
 410 zero mean. Pairwise dependencies between the variables in  
 411 the distribution are represented by a covariance matrix. The  
 412 covariance matrix adaptation (CMA) is a method to update  
 413 the covariance matrix of this distribution, which gives the  
 414 next distribution with an updated covariance matrix and  
 415 mean, from which the following iteration's controller gains  
 416 will be sampled from. CMA-ES uses the following as its  
 417 equation for sampling:

$$418 \mathbf{k} \leftarrow \mathbf{m} + \sigma \mathbf{k}' \sim \mathcal{N}(\mathbf{m}, \sigma^2 \mathbf{C})$$

419 where  $\mathbf{k}' \sim \mathcal{N}(0, \mathbf{C})$ . Following the above, the covariance  
 420 matrix  $\mathbf{C}$  is updated, and the mean vector  $\mathbf{m}$  is updated  
 421 using linear weighted recombination followed by step-size  
 422 control.

- 423 • **Elitist Genetic Algorithm:** For comparison, we imple-  
 424 mented the Elitist Genetic Algorithm, where the method  
 425 keeps the best individuals of a generation alive until some  
 426 better individual arrives. This ensures a monotonic evolu-  
 427 tion of the cost function. It uses mutation and cross-over  
 428 operations to optimize the objective, which is the cost func-  
 429 tion of the environment setup. Bhandari showed EGA con-  
 430 verges to the global optimal solution with any choice of  
 431 initial population [3], Raji showed that it only takes a small  
 432 number of fitness values to converge [22]. In this work, we  
 433 observed that the CMA-ES algorithm performed better than  
 434 this algorithm in most simulations.

- 435 • **Cartesian Genetic Programming:** We have also imple-  
 436 mented Cartesian Genetic Programming [19], which shows  
 437 the usage of a particular tree-based functional expression  
 438 encoding to represent the parameters to be optimized –  
 439 in this case the controller gains. Cross-over and mutation  
 440 operations are then performed to update the encoding pa-  
 441 rameters, which finally yields the updated set of controller  
 442 gains. When compared, CMA-ES outperformed Cartesian  
 443 Genetic Programming, and therefore, we selected the CMA-  
 444 ES algorithm for the training.

445 The CMA-ES is particularly useful if the reward function is ill-  
 446 conditioned or non-convex over the sample space and hence the  
 447 choice of using it over several other evolutionary algorithms. It  
 448 is known in the literature that the CMA-ES is a prominent algo-  
 449 rithm best suited for non-convex problems with an uneven sample  
 450 space [12], the reason being that CMA-ES uses a normal distribution  
 451 from which gains are sampled and they have the largest entropy  
 452 for a given sample space.

### 3.3 Training

453 We have used an evolutionary training method that finds the opti-  
 454 mal gains of the controller by using a model-free objective function.  
 455 The objective function has been chosen to depend solely on the  
 456 jump height from the ground of the base slider. The motivation  
 457 for choosing this as an objective, and not something like the range  
 458 of jump, is because local maximization of the range is possible

even if the base slides down to the ground and moves up minimally. The selection of this objective function is also motivated by the fact that it is a convex function in  $h$  and that the speed of CMA-ES search is not greatly slowed by its use. In this work, we chose  $Ke^{-(\mathbb{H}-h)}$  as an objective to be maximized, where  $K$  is a non-zero positive constant,  $\mathbb{H}$  represents an upper limit on the jumping height (assumed to be much bigger than what the robot can jump realistically), and  $h$  is the height of the base-slider in the step in which it is getting computed. We took 4000 steps and calculated the objective as  $\max(R_i) \in [2000 \rightarrow 4000]$ , where  $R_i$  is the objective function value at the  $i$ -th iteration of the training. This preserves the nature of the step-wise objective without affecting the end goal of the training.

## 4 RESULTS AND DISCUSSION

In this section, we present some of the main simulation results. We also present the training results and the usage of the trained controller gains to analyze the hopping behavior.

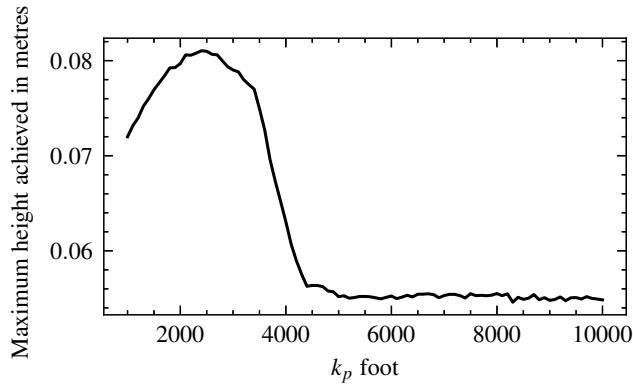


Figure 4: Maximum base height achieved with varying foot spring constant for fixed values of controller gains obtained after training

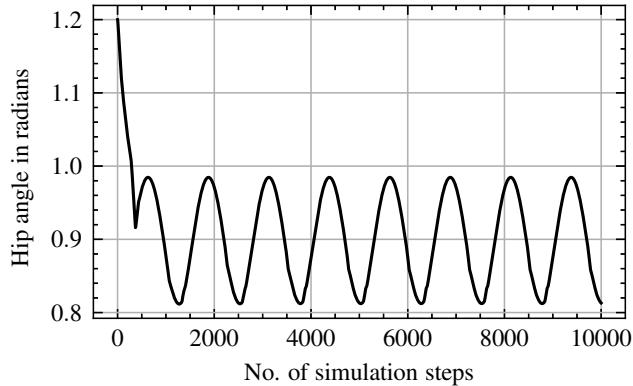


Figure 5: Hip angle with number of simulation time steps

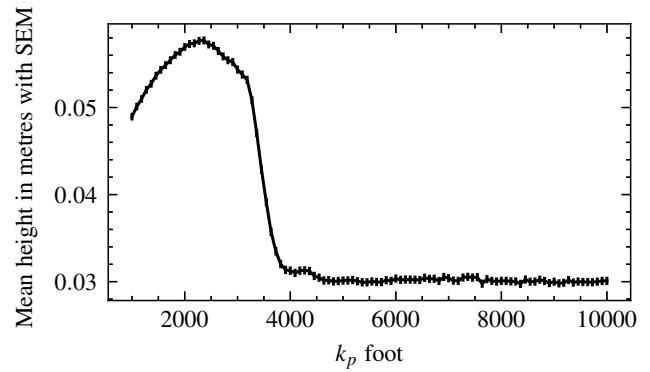


Figure 6: Mean height of base slider with standard error of the mean ( $\pm 10^{-4}$ )

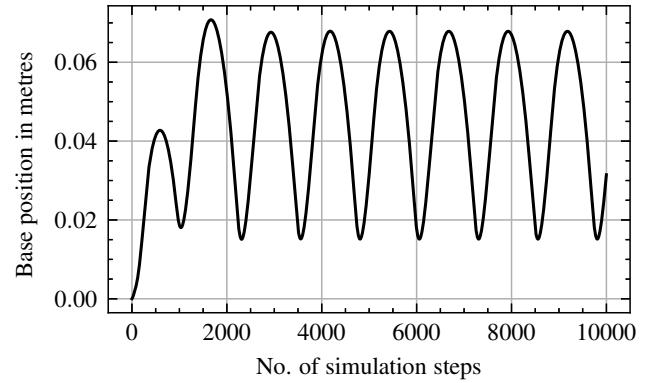


Figure 7: Height of base slider at 1000  $k_p$  foot

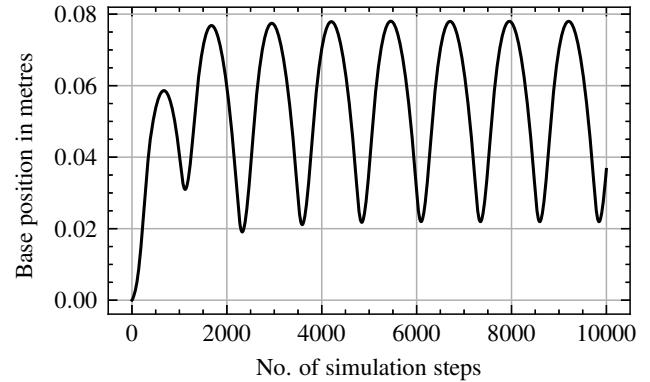


Figure 8: Height of base slider at 2400  $k_p$  foot

The single-leg hopping is simulated for a range of spring stiffness (1000 N/m to 10000 N/m) in MuJoCo shown in Fig. 4. This graph is generated by varying the stiffness of the foot joint, keeping values of controller gains and other parameters fixed obtained after training. This graph clearly shows that the maximum height achieved by

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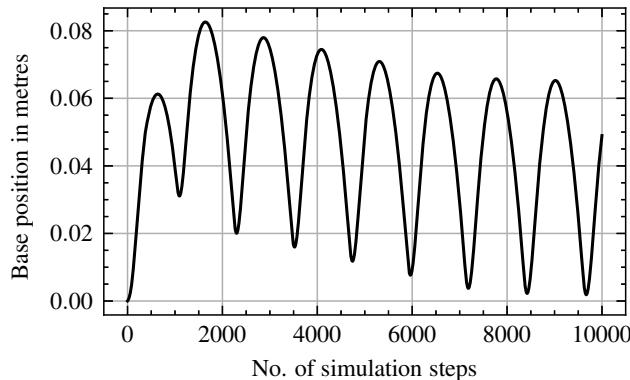
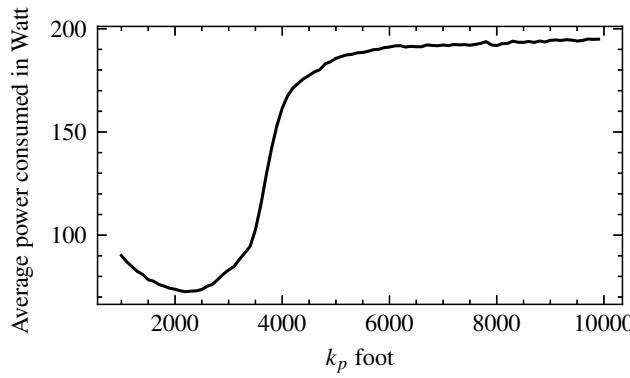
Figure 9: Height of base slider at 8000  $k_p$  foot

Figure 10: Average power consumed with foot stiffness

the base link keeps on increasing until an optimum stiffness of the foot link. After the optimal stiffness, the maximum base link height decreases and gets saturated at very high stiffness values (fixed foot). Fig. 5 shows the variation of the hip angle with the number of simulation time steps, where the initial transient behavior of jumping down from a higher hip angle to a periodic lower value is due to the system trying to position itself at the correct hopping coordinates. As we have seen in multiple simulation results, the periodic behavior is independent of the initial starting position of the leg, which demonstrates the stability and robustness of the controller with respect to initial joint states.

Fig. 8 shows the variation of the base position with simulation time steps and this demonstrates that the base varies periodically with almost the same amplitude. It gradually increases and stabilizes itself about a particular mean. Fig. 6 shows the mean height of the base with varying spring constant of the foot with the error scales representing the deviation of the cumulative mean. The increasing error scales at the end demonstrate the fact that the standard deviation increases with the foot  $k_p$  up to a particular value and then decreases again. As is evident from Fig. 6, the standard deviation achieves a peak value at a  $k_p$  of the foot at around 2400 which is the region of maximum hopping range for the particular value of the controller gains. Fig. 7 shows the base position keeping the foot

$k_p$  at 1000. The maximum height achieved by the base link is 0.071 m which is low compared to the optimal  $k_p$  of 2400 shown in Fig 8 where the height reached is 0.081 m. We also obtained the height achieved for a high  $k_p$  of 8000, and as shown in Fig. 9 a maximum value of 0.054 m was achieved. Fig. 10 shows the average power consumed as a function of foot stiffness and it can be seen that the power consumed is least for the optimum foot stiffness.

The overall the simulation results indicate that the trained controller is robust with respect to the initial position, and the base height from the ground gradually increases over time and stabilizes itself at a particular mean. There exists a local supremum in the maximum height achieved by the base slider as shown in Fig. 4 for the given inertial parameters and the trained controller gains, which demonstrates the existence of a particularly favorable range of foot spring-damper values for which the single leg demonstrates maximum hopping ability.

## 5 CONCLUSION

In this work, the effect of foot compliance on a single-legged hopping robot has been studied. The single leg has two motors and a spring at the feet, and the objective was to use evolutionary techniques to find the best combination of parameters for the highest vertical hop. MuJoCo a rigid body simulator, is used to study the effect of linear spring at the foot on the single-legged robot model. The primary finding of the study is that there is a stiffness and control gain range where the vertical jump is maximum. According to the simulation data, the optimum spring stiffness at the foot can enhance the highest point that can be reached by the base. This work is being extended to optimize the impact force and power consumption and to implement the findings on a physical hardware. The main goal is to apply this research to a quadruped, where stiffness will be added to each of the four legs to examine how the stiffness of the feet affects the quadruped's cost of transport.

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