OPTIMAL PARAMETER ESTIMATION OF DAMPING PARTICLES INSERTED IN A HONEYCOMB SANDWICH CELLS

Nazeer Ahmad*[1], Ranganath R[1], Ashitava Ghosal[2]

[1]ISRO Satellite Centre (ISAC), Old Airport Road, Bangalore 560017, India
[2]Indian Institute of Science, Department of Mechanical Engineering, Bangalore 560012, India

Abstract

Damping characteristic of a honeycomb sandwich laminate is reported to improve when the empty cells of the honeycomb core are filled with damping particles. The enhancement of the damping is a result of the transfer of kinetic energy from the host structure to the damping particles through impact and friction mechanism. The energy of the damping particles is dissipated by inelastic collision and friction among themselves and by collisions with the walls of the honeycomb cell. The impact damping depends on several parameters, prominent amongst them are the material and geometric properties of the damping particles, cell volume to filled-particle volume ratio, area and the location of the area of the structure treated with damping particles and level and type of excitation. The effect of these parameters can be studied using the discrete element method (DEM) where the equations of motion of each particle is established by considering the contact forces and moments from the surrounding particles and boundary contacts and the motion of each particle and its interaction with surrounding boundaries is tracked and energy dissipation through impact and friction is computed. The use of DEM for large structures, where the number of particles is very large, is inefficient and impractical. In this work DEM is applied on a small coupon of honeycomb sandwich and a spline model for equivalent viscous damping is obtained. Then the model is applied to maximize the equivalent damping coefficient using sequential quadratic programming (SQP) subject to a limit on the ratio of mass of the damping particles to host structure, and the area at which to apply the damping particles

1. INTRODUCTION

Honeycomb sandwich composite are widely used in aerospace industries owing to its light weight and excellent mechanical properties. The natural damping of honeycomb sandwich composites is very small which results in excessive vibration responses at resonance frequencies. Taking advantage of the porous nature of the core, the damping can be improved by inserting the granular particles [1, 2]. This technique of enhancing damping is called particle impact damping. This method of enhancing the damping is extremely simple and requires effectively least modification of host structure. The particle impact dampers (PID) are insensitive to environment, low cost and effective over wide temperature and frequency range. The energy dissipation mechanism in PID is highly nonlinear and depends on a host of parameters: size, shape, number and material of the particles; enclosure relative geometry and material; mass ratio; volumetric packing fraction; location of PD with respect to structural mode; and the level of response. The dissipation of energy takes place mainly through inter-particle and particle-

* nazeeralig@gmail.com

wall impact and friction. The dominant mode of dissipation depends on the relative proportion of different parameters. It is extremely difficult to evolve a model to capture the entire interactions taking place. Most of the researchers have tried to quantify the damping enhancement due granular particles experimentally [3-5]. Therefore, extensive study of its dependence on the important parameters is not reported. The modelling of the dynamics of the damping particles coupled with the dynamics of the host structures is done using the DEM. The DEM is based on Newtonian mechanics where equation of motion of each particle is established by considering the contact forces and moments from the surrounding particles and boundary contacts. Thus, it allows the exploration of interactions taking place when the vibration of host structure is transferred to damping particles and thereafter dissipated as heat and sound. Generally, the honeycomb sandwich panels used are large and thus number of damping particles are required to effect the damping characteristic. The DEM computation involves integration of motion of each particle, along with motion of the host structures. The solution process involves detection of contact and thus generation of new set of equations at each step of time as the contact forces are required to solve motion of particles and structures. For the larger problems where millions of particle motion is required to be solved coupled with the dynamics of the structure, the DEM becomes inefficient. Therefore, in this work, DEM is applied on a honeycomb sandwich coupon of smaller size filled with damping particle. The coupon is subjected to vibration loads and the energy dissipated is estimated and further its dependence on the other parameter is studied. An equivalent damper which dissipates same amount of energy per cycle is estimated which could be readily integrated like a proof mass actuator enabling prediction of structural responses without solving the DEM. A spline multivariable model of the equivalent viscous damper is obtained and optimization is performed to maximize the value of equivalent viscous damping coefficient.

2. MATHEMATICAL FORMULATIONS

To quantify the dissipation of energy by damping particles filled in the cells on honeycomb, a small square shaped coupon as shown in Fig. 1, is considered.

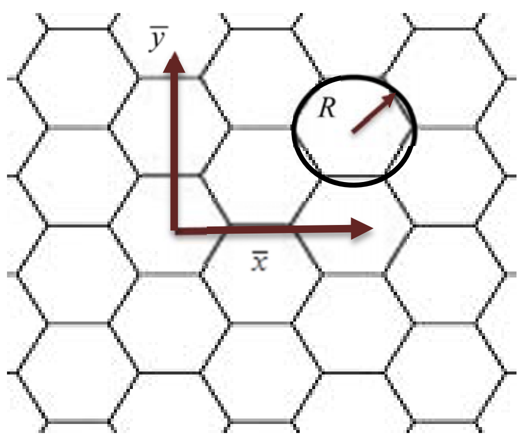

Figure 1. Honeycomb sandwich coupon and local coordinate system

A local coordinate system with origin at the geometric center of the cell, x axis along the L - direction of the core and y axis is along the W – direction of the core and parallel to global axis is chosen. As the coupon is small in dimension, it can be assumed that the walls of the honeycomb cells do not rotate or deform when the coupon is vibrated in transverse direction. The equations of the cell-walls in local coordinate system can written as:

$$\bar{z} \pm \frac{h}{2} = 0$$
$$\bar{y} \pm \frac{\sqrt{3}}{2} R = 0$$
$$\bar{x} + \frac{\bar{y}}{\sqrt{3}} \pm R = 0 \tag{1}$$
$$\bar{x} - \frac{\bar{y}}{\sqrt{3}} \pm R = 0$$

where h is the thickness of the core and R is the radius of circumscribing circle, shown in Fig. 1. As the damping particles are filled in the cells, they are constrained to move inside the cells when the coupon is vibrated, as shown in Fig. 2. The damping particles in the cells, collide and rub with the walls of the cells and face-sheets as well as between themselves.

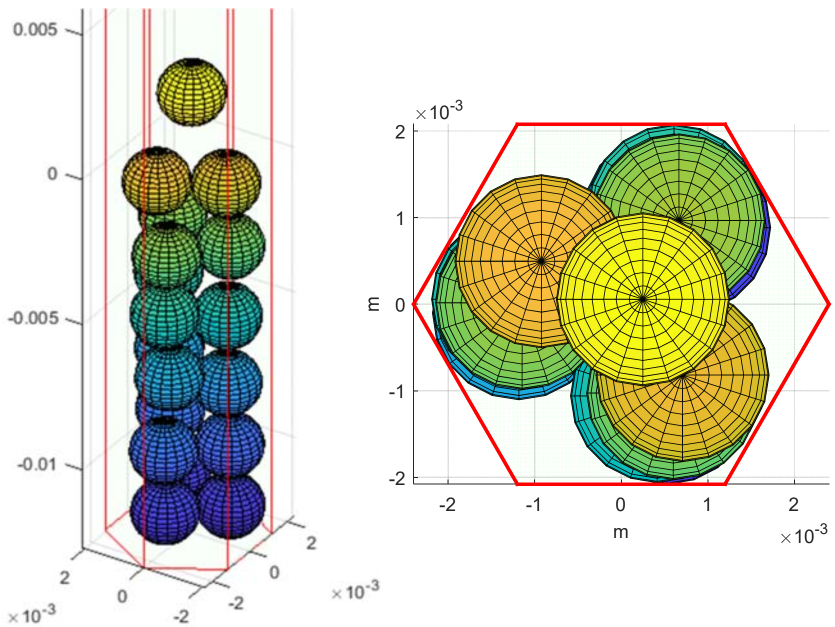

Figure 2. Damping particles in the cell

The equation of motion of the coupon can be written as:

$$m_c \ddot{\mathbf{w}} + c\dot{\mathbf{w}} = \mathbf{F}^{\mathrm{d}} + \mathbf{F}^{\mathrm{e}} \tag{2}$$

where $\ddot{w}$ and $\dot{w}$ are acceleration and velocity of the coupon in z direction, respectively. Mass of the coupon is represented by m_c whereas $\mathbf{F}^{\mathrm{d}}$ and $\mathbf{F}^{\mathrm{e}}$ are the particle damping force and external excitation force, respectively. The particle damping force $\mathbf{F}^{\mathrm{d}}$ is the summation of the force in excitation direction due to the impact and rubbing of the damping particles with walls of the cell. If the cell i contains n particles which are in contact with the cell wall and there are N cells in the coupon then the impact damping force on the coupon can be written as:

$$\mathbf{F}^{\mathrm{d}} = \sum_{j=1}^{N}\sum_{i=1}^{n}\mathbf{f}_{\mathrm{ij}}^{\mathrm{d}} \tag{3}$$

The total impact damping force is obtained by summing the forces arising from the impact of particles and walls of the cells. A typical contact force due to impact of particle i with particle j or with the wall can be written as

$$\mathbf{f}_{\mathrm{ij}} = \mathbf{f}_{\mathrm{ij}}^{\mathrm{n}} + \mathbf{f}_{\mathrm{ij}}^{\mathrm{t}} \tag{4}$$

where $\mathbf{f}_{\mathrm{ij}}^{\mathrm{t}}$ and $\mathbf{f}_{\mathrm{ij}}^{\mathrm{n}}$ are the tangential and normal components of the contact force, respectively. The normal contact force $\mathbf{f}_{\mathrm{ij}}^{\mathrm{n}}$ is modeled by the Hertz's nonlinear dissipative model which was proposed by Tsuji et al. [6] and has been used by researchers [7-9] in a particle damping problems with reasonably good results. The normal contact force can be written as.

$$\mathbf{f}_{\mathrm{ij}}^{\mathrm{n}} = -\left(k_n\left(\delta_{ij}\right)^{3/2} + \alpha\sqrt{m_{ij}^{*}k_n}\left(\delta_{ij}\right)^{1/4}\dot{\delta}_{ij}\right)\mathbf{n}_{\mathrm{ij}} \tag{5}$$

where k_n is the Hertz's constant, δ_{ij} is the indentation, shown in Fig. 3, and α is the damping constant which is a nonlinear function of the normal coefficient of restitution e_n [6] and given as

$$\alpha = -\ln\left(e_n\right)\sqrt{\frac{5}{\ln\left(e_n\right)^2 + \pi^2}} \tag{6}$$

The equivalent mass m_{ij}^{*} in Eq. (5) is defined as

$$m_{ij}^{*} = \frac{m_i m_j}{m_i + m_j} \tag{7}$$

where m_i is the mass of the particel i .The Coulomb's law has been used by many researcher in DEM [7, 8] to model the tangential contact force $\mathbf{f}_{\mathrm{ij}}^{t}$ for predicting vibration responses and is given as.

$$\mathbf{f}_{\mathrm{ij}}^{\mathrm{t}} = -\mu\left|\mathbf{f}_{\mathrm{ij}}^{\mathrm{n}}\right|\frac{\mathbf{V}_{\mathrm{ij}}^{\mathrm{t}}}{\left|\mathbf{V}_{\mathrm{ij}}^{\mathrm{t}}\right|} \tag{8}$$

where μ is the coefficient of friction and $\mathbf{V}_{\mathrm{ij}}^{\mathrm{t}}$ is the relative tangential velocity of contact point. Let $\mathbf{v}_{\mathrm{i}}$ and $\mathbf{v}_{\mathrm{j}}$ be the linear velocity of the center of mass and $\boldsymbol{\omega}_{\mathrm{i}}$ and $\boldsymbol{\omega}_{\mathrm{j}}$ be the angular velocity of particles i and j, respectively. Then the relative velocity at the center of contact area of particle i with respect to particle j can be written as

$$\mathbf{V}_{\mathrm{ij}} = \mathbf{v}_{\mathrm{i}} - \mathbf{v}_{\mathrm{j}} + \left(r_{\mathrm{i}}\boldsymbol{\omega}_{\mathrm{i}} + r_{\mathrm{j}}\boldsymbol{\omega}_{\mathrm{j}}\right)\times\mathbf{n}_{\mathrm{ij}} \tag{9}$$

and the tangential component $\mathbf{V}_{\mathrm{ij}}^{\mathrm{t}}$ can be written as

$$\mathbf{V}_{\mathrm{ij}}^{\mathrm{t}} = \mathbf{V}_{\mathrm{ij}} - \left(\mathbf{V}_{\mathrm{ij}}.\mathbf{n}_{\mathrm{ij}}\right)\mathbf{n}_{\mathrm{ij}} \tag{10}$$

To evaluate the contact forces on the wall of the cell, the kinematics of impacting particle need to be known. The equation of motion of the particle i that is in contact with n_1 number of surrounding particles and n_2 points with cell walls can be written as:

$$m_{\mathrm{pi}}\ddot{\mathbf{p}}_{\mathrm{i}} = -m_{\mathrm{pi}}\mathbf{g} + \sum_{j=1}^{n_1}\mathbf{f}_{\mathrm{ij}} + \sum_{w=1}^{n_2}\mathbf{f}_{\mathrm{iw}} \tag{11}$$

$$\mathbf{I}_i\ddot{\boldsymbol{\Phi}}_i = \sum_{j=1}^{n_1}\left(r_i - \frac{\delta_{ij}}{2}\right)\mathbf{n}_{\mathrm{ij}}\times\mathbf{f}_{\mathrm{ij}} + \sum_{j=1}^{n_2}\left(r_i - \delta_{iw}\right)\mathbf{n}_{\mathrm{iw}}\times\mathbf{f}_{\mathrm{iw}} \tag{12}$$

where m_{pi}, r_i, and $\mathbf{I}_i$ are the mass, radius and mass moment of inertia of the particle i, respectively; $\mathbf{p}_i$ is the position vector of the center of mass and $\mathbf{\Phi}_i$ the angular displacement of the particle i. The unit vectors $\mathbf{n}_{ij}$ and $\mathbf{n}_{iw}$ point from the center of the particle i towards the center of the particle j and towards the point of contact with cell wall. The acceleration due to gravity is represented by $\mathbf{g}$ and $\mathbf{f}_{ij}$ and $\mathbf{f}_{iw}$ are the forces on particle i due to interaction of particle j and by the wall of the cell, respectively. The normal relative displacements of the center of mass of the particle i with respect to the particle j when they are in contact with each other or with the wall are represented by, δ_{ij} and δ_{iw}, respectively.

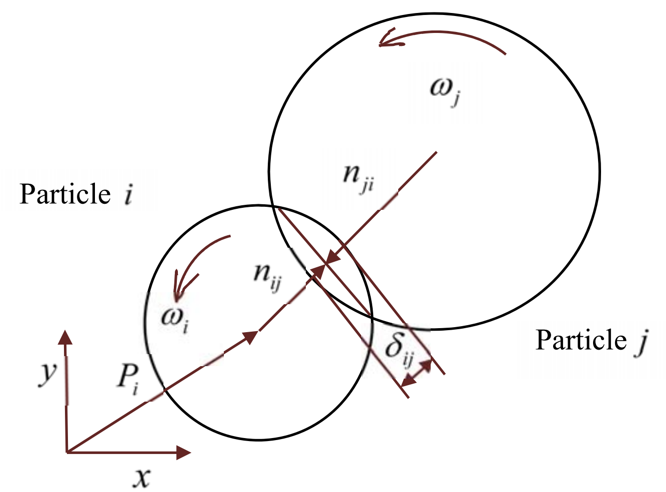

Figure 3. Impact of two damping particles

The energy dissipated by the damping particle in normal contact and coulomb friction can be written as:

$$E_d = \sum_{k=1}^{N_c} \int_0^{t_c} \left(\alpha \sqrt{m_{ij}^{*} k_n} \delta^{1/4} \dot{\delta}_{ij} \dot{\delta}_{ij} + \mathbf{f}_{ij}^{t} . \mathbf{V}_{ij}^{t} \right) dt \tag{13}$$

where E_d is the energy dissipated by damping particles, t_c is the contact duration and N_c is the number of contacts. The equivalent viscous damping coefficient can be obtained from the Eq. (13) as.

$$C_{eq} = \frac{E_d}{\pi \omega W^2} \tag{14}$$

where C_{eq} is the equivalent viscous damping coefficient, ω is the excitation frequency and W is amplitude of the displacement.

3. COMPUTING DAMPING USING DEM

An aluminium honeycomb sandwich coupon of dimension 100 mm x 100 mm is considered for evaluation of energy dissipation using DEM and subsequently computing the equivalent viscous damping constants. The coupon has 441 cells and each cell can accommodate 36 damping particles when filled to 100%. Acrylic damping particle are filled in the cells of honeycomb. The geometric and material properties of the damping particles and honeycomb sandwich is given in Table 3 and Table 4, respectively.

Table 1 Properties of damping particles

Properties	Units	Aluminum	Acrylic
Radius	mm	1	1.25
Density	kg/m3	2850	1180
Young's modulus	N/m2	70 x 109	2.84 x 109
Poisson's ratio	-	0.33	0.402
Material pairs	-	Coefficient of sliding friction	Normal restitution coefficient
Aluminium – aluminium	-	0.50	0.85
Acrylic – acrylic	-	0.096	0.70
Acrylic – aluminium	-	0.14	0.70

Table 2 Properties of honeycomb coupon

Properties	Units	Face-sheet (AA 2024 T3)	Honeycomb core (CR 3/16-5056-0.0007-P-32)
Thickness	mm	0.25	25.4
Density	kg/m3	2800	32.1
Young's modulus	N/m2	72 x 109	$E_{xx} = E_{yy} = E_{zz} = 10000$
Poisson's ratio		0.33	$\nu_{xy} = \nu_{yz} = \nu_{xz} = 0.3$
Shear modulus	N/m2	-	$G_{xy} = 10000$ $G_{yz} = 0.89 \times 10^8$ $G_{xz} = 1.85 \times 10^8$
Diameter of inscribing circle of hexagonal cell	mm	-	4.76

The energy dissipated by the damping particles, when the coupon is excited with a constant amplitude sinusoidal acceleration of amplitude **g** and frequency ω, is computed using Eq. (13) for different fill fractions r. The equivalent viscous damping C_{eq} is computed using Eq. (14). Table 3 shows the computed values of equivalent viscous damping for different combinations of excitation and fill fractions.

Table 3 Equivalent viscous damping coefficient computed using DEM

ω	g	r	Ceq	ω	g	r	Ceq	ω	g	r	Ceq
50	1	25	0.0009	50	5	25	1.0425	50	10	25	0.6449
50	1	50	0.0072	50	5	50	0.4936	50	10	50	0.9719
50	1	75	0.0169	50	5	75	0.6959	50	10	75	1.4471
50	1	90	0.0282	50	5	90	1.4492	50	10	90	1.7163
100	1	25	0.0036	100	5	25	1.5000	100	10	25	1.4048
100	1	50	0.0247	100	5	50	0.9654	100	10	50	1.7566
100	1	75	0.0784	100	5	75	1.5746	100	10	75	2.6750
100	1	90	0.1314	100	5	90	1.6865	100	10	90	2.8160
500	1	25	2.9428	500	5	25	8.6277	500	10	25	4.6368
500	1	50	6.9911	500	5	50	14.0934	500	10	50	8.1613
500	1	75	14.9472	500	5	75	19.9140	500	10	75	11.5414
500	1	90	25.1659	500	5	90	24.8507	500	10	90	15.0455
750	1	25	0.8868	750	5	25	11.3041	750	10	25	5.4388
750	1	50	14.7637	750	5	50	22.4851	750	10	50	13.5106
750	1	75	68.4725	750	5	75	32.1501	750	10	75	17.7975
750	1	90	111.4007	750	5	90	38.9846	750	10	90	20.0345
1000	1	25	3.6385	1000	5	25	17.8389	1000	10	25	7.0294
1000	1	50	30.4235	1000	5	50	29.1117	1000	10	50	17.3705
1000	1	75	123.5922	1000	5	75	46.1753	1000	10	75	24.6060
1000	1	90	168.3460	1000	5	90	50.8938	1000	10	90	27.7986

4.1 Interpolation model for C_{eq}

The equivalent viscous damping coefficients C_{eq} computed using DEM and given in Table 3 shows that the C_{eq} depend prominently on excitation frequency, packing fraction and level of excitation. A multivariate spline is used to obtain the functional relation between the variables and C_{eq} to obtain the intermediates values and to perform the optimization described in subsection 4.2.

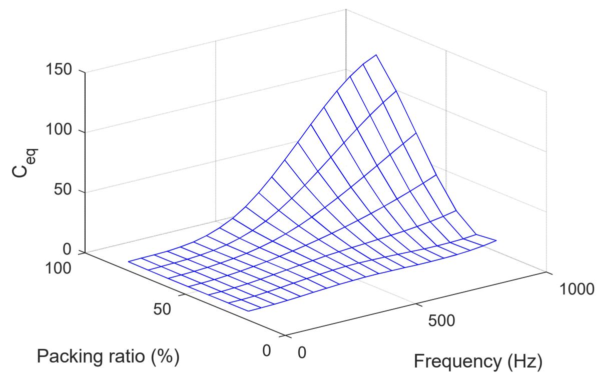

Figure 4. C_{eq} with sine acceleration input of 2g

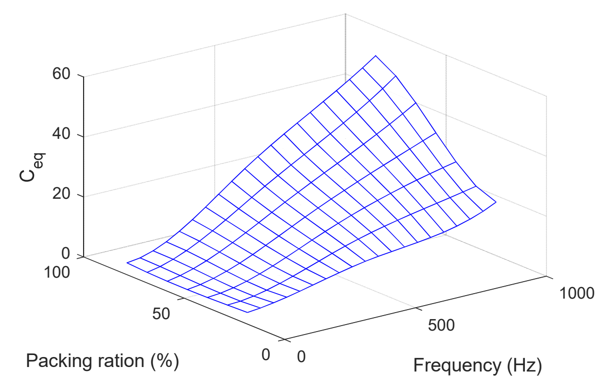

Figure 5. C_{eq} with sine acceleration input of 5g

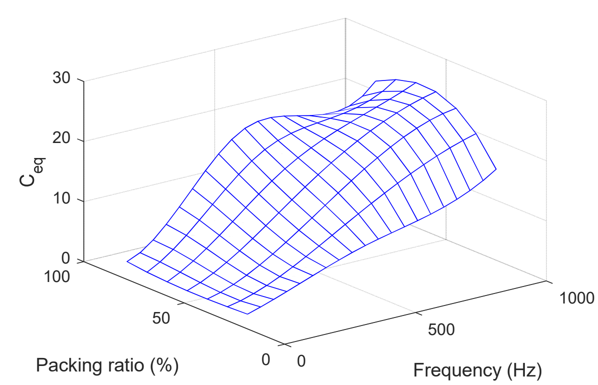

Figure 6. C_{eq} with sine acceleration input of 8g

Figures 4-6 show that the C_{eq} distribution when acceleration amplitude is increased from 2 g to 8 g. It can be seen that for lower input excitation the C_{eq} increases with packing ratio and frequency. However, at higher level of around 8g it tends to increase reaching maximum around 500 Hz and there after decreases for high packing fractions, more than 80%.

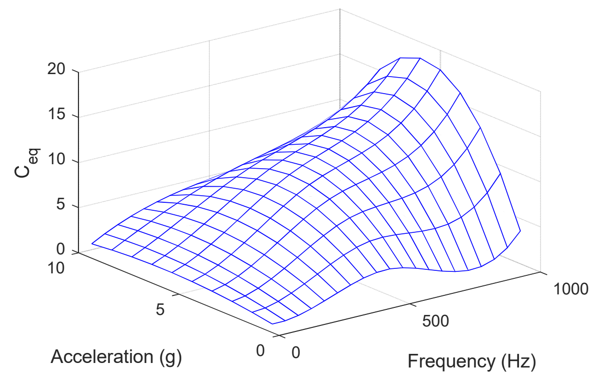

Figure 7. C_{eq} with 25% packing fraction

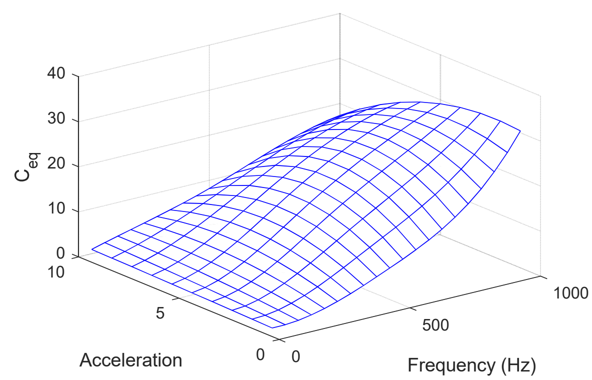

Figure 8. C_{eq} with 50% packing fraction

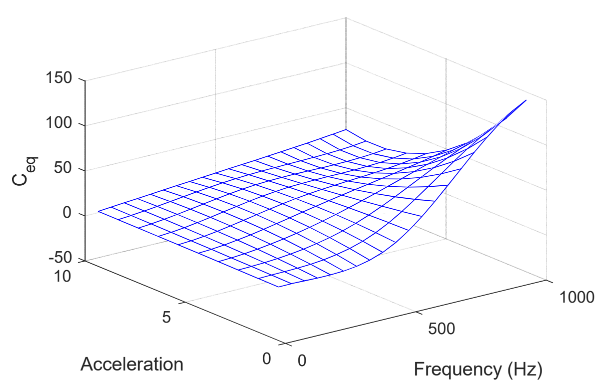

Figure 9. C_{eq} with 80% packing fraction

4.2 Optimization

The equivalent viscous damping constant per cell of the honeycomb can be written as a function of four variables as,

$$\frac{C_{eq}}{N} n_c = f\left(\omega, r, \ddot{w}, n_c\right) \tag{15}$$

The function f is a spline interpolant derived based on DEM data. It can be seen that the C_{eq} per unit area or per cell is depending on the excitation acceleration and frequency which is characteristic of the operating environment. An optimization problem can be defined as:

$$\begin{aligned} &\mathit{maximize}\ : && \frac{C_{eq}}{N} n_c = f\left(\omega, r, \ddot{w}, n_c\right) \\ &\mathit{Subject\ to}: && \frac{9}{25} n_c r - n_b = 0 \\ & && lb\left(\omega, r, \ddot{w}, n_c\right) \\ & && ub\left(\omega, r, \ddot{w}, n_c\right) \end{aligned} \tag{16}$$

Since the damping particles add mass to the structure, a fixed mass of the damping particles, not more than 10% mass of the structure is desirable. Fixing the mass of the damping particles limits the number of damping particles to be used and an upper and lower bound for N_c is established. When each cell contains small number of damping particles that is the packing fraction r is low, the number of cell filled are more, and therefore it is spread over a large area. The inverse relation between total mass of the damping particle and packing fraction can be obtained as:

$$\frac{9}{25} n_c r - n_b = 0 \tag{17}$$

Optimization was performed in the MATLAB using *fmincon* subroutine with sequential programming algorithm option. A maximization result is obtained by minimizing the negative of objective function using the said subroutine. Three different operating environment defined by frequency bands 250 – 300 Hz, 450 – 500 Hz and 750 – 800 Hz and corresponding excitation levels 5 – 6 g, 1 – 2 g, and 4 – 6 g respectively. Figures 10 – 12 show the convergence with respect to iteration number for different starting vectors. A large number of starting vectors were tried to capture the global maxima. It was seen that algorithm would frequently converge to local optimal values. Figure 10 shows the convergence to one local and one global maxima. Only global optima iteration histories are plotted in Figures 11 and 12.

The converged results for three load cases are given in Table 4. The fist load case shows that the maximum equivalent viscos damping is achieved at lower packing ration of 25% which will cover 1764 cells of the honeycomb while third load case shows that optima value is obtained at 92% filling ratio covering 496 cells. The optimal values vary depending upon the operating conditions, and therefore choice of area and fill fraction should be made on average values of operating frequency and excitation amplitude.

Table 4 Mechanical Properties

Operating environment	r (%)	n_c	C_{eq} (N/m/s)
250 – 300 Hz, 5 – 6 g	25	1764	20.6
450 – 500 Hz, 1 – 2 g	42	1050	35.3
750 – 800 Hz, 4 – 6 g	89	496	160.7

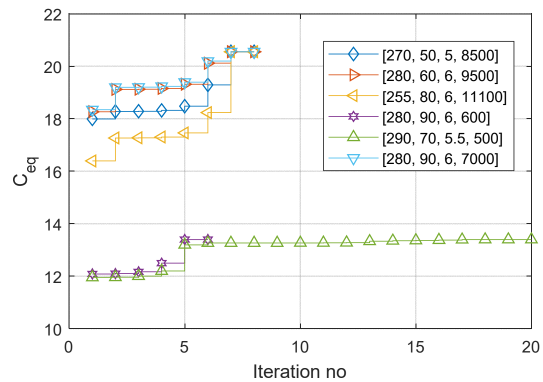

Figure 10. Iteration number vs objective function for load case – 1

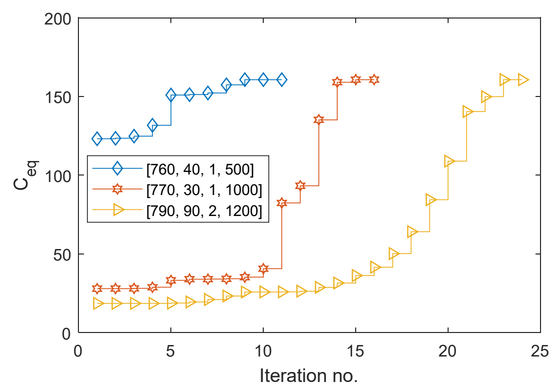

Figure 11. Iteration number vs objective function for load case – 2

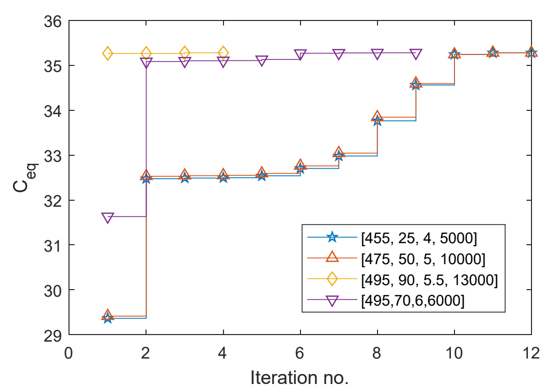

Figure 12. Iteration number vs objective function for load case – 3

5. CONCLUSIONS

The equivalent viscous damping coefficient corresponding to the energy dissipated by the damping particles filled in a small coupon of honeycomb is studied with discrete element method and optimal values of the parameters on which the dissipation depends are obtained. The coupon is subjected to different levels of constant amplitude harmonic acceleration in frequency band of 50 – 1000 Hz with varying amount of damping particles in the cell. Equivalent viscous damping coefficient, obtained by equating the energy dissipated due to impact and friction, is found to depend on: fill fraction, amplitude and frequency of the input acceleration. A multivariate interpolation model of equivalent damping coefficient is worked out using spline interpolant. The interpolant was used to study the variation of equivalent viscous damping at intermediate values of variables and carryout optimization. Optimum values of the fill fraction and the number of cells to be filled with damping particles varies widely with operating conditions. A lower value of fill fraction at lower excitation level and lower frequency gives maximum damping coefficient while a higher value of fill fraction is more appropriate filling strategy for high frequency environments.

ACKNOWLEDGMENTS

This research work is partially funded by ISRO-IISc Space Technology Cell (ISTC/MME/AG/394). The authors would like to thank ISRO Satellite Center, Bangalore, for the support.

REFERENCES

[1] N. Ahmad, R. Ranganath, and A. Ghosal, "Modeling and experimental study of ahoneycomb beamfilled with damping particles," *Journal of Soundand Vibration,* vol. 391, pp. 20–34, 2017.

[2] N. Ahmad, R. Ranganath, and A. Ghosal, "Assessment of particle damping device for large laminated structures under acoustic excitations," *Proceedings of 14th ISAMPE National Conference on Composites (INCCOM-14)*, Hyderabad, 2016.

[3] C. Wong, M. Daniel, and J. Rongong, "Energy dissipation prediction of particle dampers," *Journal of Sound and Vibration,* vol. 319, pp. 91-118, 2009.

[4] G. Michon, A. Almajid, and G. Aridon, "Soft hollow particle damping identification in honeycomb structures," *Journal of Sound and Vibration,* vol. 332, pp. 536-544, 2013.

[5] M. Gharib and S. Ghani, "Free vibration analysis of linear particle chain impact damper," *Journal of Sound and Vibration,* vol. 332, pp. 6254-6264, 2013.

[6] Y. Tsuji, T. Tanaka, and T. Ishida, "Lagrangian numerical simulation of plug flow of cohesionless particles in a horizontal pipe," *Powder Technology,* vol. 71, pp. 239-250, 1992.

[7] M. Saeki, "Impact Damping with Granular Materials in a Horizontally Vibrating System," *Journal of Sound and Vibration,* vol. 251, pp. 153-161, 2002.

[8] M. Saeki, "Analytical study of multi-particle damping," *Journal of Sound and Vibration,* vol. 281, pp. 1133-1144, 2005.

[9] X. Fang, J. Tang, and H. Luo, "Granular damping analysis using an improved discrete element approach," *Journal of Sound and Vibration,* vol. 308, pp. 112-131, 2007.