Obstacle avoidance for hyper-redundant snake robots and one dimensional flexible bodies using optimization

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Abstract— *Robots with snake topology and large number of redundant degrees of freedom are extensively studied due to increased flexibility in motion planning. Such robots can navigate through narrow passages, avoid obstacles and find use in search and rescue. It is also finding increasing use in fields of medical robotics and surgery in the form of actuated endoscopes. This paper proposes a method to avoid obstacle for such snake robots using an optimization based approach. The path of the leading end of the snake robot and the obstacle field is assumed to be known a priori and the obstacles are assumed to be bounded by smooth and differentiable surfaces. The obstacle avoidance algorithm uses only the task space variables and it is shown that the entire length of the snake robot can avoid all the obstacles while executing the prescribed motion. It is also shown that motion of the snake robot is more natural looking as the motion of the individual links die down along the length of the snake robot. Numerical simulation results are shown to illustrate the effectiveness of the algorithm in two- and three-dimensional space.*

Keywords: Extended body, Hyper-redundant, motion planning, obstacle avoidance, optimization

I. Introduction

Motion planning of hyper-redundant serial 'snake' robots has been an active research area in the robotics community since the 1980's. One of key feature in any hyper-redundant robot is the existence of an infinite number of solutions for the inverse kinematic problem and this in turn has been profitably used for optimization of a suitable objective function of the robot motion variables, avoidance of singularities in its workspace and in obstacle avoidance (see, the review paper by Klein and Huang [1], textbook by Nakamura [2] and the references contained therein). More recently, there is a renewed interest due to the increasing applications of hyper-redundant snake robots for search and rescue in a cluttered disaster area or in the use of the developed motion planning tools and techniques for realistic simulations in medical robotics (realistic endoscopy/ surgery and suturing simulation) and in animation industry to realistically simulate motion of flexible one-dimensional objects such as strings, hair etc. This paper presents a novel optimization based algorithm for obstacle avoidance of hyper-redundant snake robots and flexible one-dimensional objects in two- and three-dimensional space.

The problem of obstacle avoidance for point bodies or mobile objects and robots approximated by a point has been studied in depth and many solutions have been proposed (see the review paper by Hwang et al. [3] and the references contained therein). The various methods can be broadly classified into three main categories depending on their approaches. The first approach involve mapping obstacle information and geometry into the workspace of a robot and partitioning the space into free and occupied spaces and then finding a obstacle free path for the robot in this space (see, for example, references [4], [5]). A second approach is based on a method termed as the dynamic window approach (see, for example, [6]) where graph theoretic constructions and search is used solve the so-called find path problem. A third extensively used approach uses artificial potentials (see Khatib [7] and later extensions of the concept by others [8], [9]) where a repulsive virtual force field is generated around the obstacle locations and then a minimal potential energy path is computed. The computed path effectively avoids the obstacles and can be directly implemented in the control algorithm and hardware used by the robot. In addition, to the three mentioned approaches, due to the need for effective and real-time obstacle avoidance for mobile and stationary robots, researchers have attempted to use Voronoi diagrams [10], artificial neural networks [11], polyhedral interference detection based on computational geometry [12], reinforcement learning algorithms [13], dynamic programming [14] and optimal control (see, for example, [15], [16], [17], [18]).

Unlike motion planning for point objects, there is less work on obstacle avoidance algorithms for entire manipulators, redundant or otherwise. In redundant manipulators, the main approach is to carefully choose one of the infinitely many solutions such that interference with the obstacles is avoided. Freund [19] used redundancy to trace a spatial trajectory avoiding obstacles whereas Nakamura [20] proposed an algorithm to avoid obstacles by placing restriction on joint angles indirectly while using the pseudo-inverse of the manipulator Jacobian to resolve the redundancy. In the configuration space based approach (see, for example, [21], [22], [23]) the spatial description

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and geometry of the obstacles and the robot manipulator are used to partition the configuration space of the manipulator into interference free zones. These are in turn used for motion planning and obstacle avoidance. Obstacle avoidance for a redundant robot have also been attempted with instantaneous Jacobian [26], artificial neural networks [24] and optimal control [25].

However, for hyper-redundant manipulators or one dimensional flexible objects modeled as hyper-redundant robots with large number of links and degrees of freedom, almost all of the above mentioned approaches are not useful. This is due to demand for large computational power for real-time simulation and visualization of the motion. To overcome this problem, Reznik and Lumelsky [27] proposed the use of a classical curve called the tractrix and combined this with an iterative obstacle avoidance algorithm based on active sensing of the environment. They claim efficient real-time simulation for hyper-redundant robots and obstacles in two- and three-dimensions. Subsequently, Choset [28] proposed a follow-the-leader approach for obstacle avoidance combined with generalized Voronoi graph. Chirikjian and Burdick [29] proposed an approach using differential geometry to constraint the manipulator into obstacle free zones called tunnels. In reference [30], authors have proposed a tractrix based motion planning algorithm based on optimization. In this paper, we extend this approach to obstacle avoidance with obstacles modeled as smooth differentiable surfaces and using a constrained Lagrangian during optimization. The tractrix based approach is known to be computationally efficient with a $\mathcal{O}(n)$ complexity and numerical simulations show that obstacle avoidance for hyper-redundant manipulators, with large n, is efficient in this tractrix based obstacle avoidance approach.

The paper is organized as follows. Section II describes the constrained Lagrangian formulation of the obstacle avoidance problem for an extended body. The constrained optimization requires that the obstacles are represented by smooth differentiable surfaces and in section II, we describe in brief representation of objects using super-quadrics. Section III presents simulation results for snake robot moving in a two- and three-dimensional space with obstacles represented by super-quadrics. Section IV presents the conclusions work and directions for future work.

II. Constrained Lagrangian Approach

As shown in [30], the tractrix motion is the solution to the Cartesian velocity minimization under length preservation constraint. For a flexible one dimensional object of length L, the optimization problem constructed is as follows.

$$
\begin{aligned}
&\underset{x(s,t),y(s,t)}{Min\ I} \ : \int_0^L\int_0^T \sqrt{\Big(\frac{dT_x}{dt}+\frac{\partial x}{\partial t}\Big)^2+\Big(\frac{dT_y}{dt}+\frac{\partial y}{\partial t}\Big)^2}\,dtds \\
&Subject\ to \\
&\Lambda(t):\ A=\int_0^L\Big(\sqrt{\Big(\frac{\partial x}{\partial s}\Big)^2+\Big(\frac{\partial y}{\partial s}\Big)^2}-1\Big)ds=0 \\
&Data:\ x(s,0),y(s,0),T_x(t),T_y(t),x(0,t)=0,y(0,t)=0
\end{aligned}
\tag{1}
$$

The above global optimization problem can be broken down into smaller optimization problems on discretized rigid link kinematic chain approximation of the flexible object. If the discretized form of the flexible object made of $(n-1)$ links is $P=[p_1\ p_2\ \dots\ p_n]^T=[X,Y]$ where $X=[x_1(t)\ x_2(t)\ \dots x_n(t)]^T$ and $Y=[y_1(t)\ y_2(t)\ \dots y_n(t)]^T$, then the reduced set of optimization problems are as follows.

$$
\begin{aligned}
&\underset{p_{i+1}(t_2))}{Min\ I} \ : \big(\Delta(T_x+x_{i+1})\big)^2+\Big(\Delta(T_y+\Delta y_{i+1})\Big)^2 \\
&Subject\ to \\
&\lambda_i:\ \sqrt{(x_{i+1}(t_2)-x_i(t_2))^2+(y_{i+1}(t_2)-y_i(t_2))^2}=L \\
&Data:\ p_i(t_m),p_{i+1}(t_1),T_x(t_m),T_y(t_m),p_1(t_1)=(0,0)^T \\
&\forall i\in[1,n-1]\ \&\ \forall\, t_1\in[0\dots T],\ t_2=t_1+\Delta t
\end{aligned}
\tag{2}
$$

It may be noted that here $m=1,2$.

Given any circle $\mathcal{C}\in\mathbb{R}^2$ (or it's topological equivalent curve), by Jordan-Brouwer theorem([31]), $\mathbb{R}^2-\mathcal{C}$ has two components, and *interior*($\mathcal{I}$) and an *exterior*($\mathcal{E}$) with $\mathcal{C}$ bounding both. For example, given a circular obstacle with center $P_c:(x_c,y_c)$ and radius R, the classification of a point $P_i:(x_i,y_i)$ into the three partition sets can be done as follows.

$$
(x_i-x_c)^2+(y_i-y_c)^2-R^2\begin{cases}>0 & \Rightarrow P_i\in\mathcal{E}\\ <0 & \Rightarrow P_i\in\mathcal{I}\\ =0 & \Rightarrow P_i\in\mathcal{C}\end{cases}
\tag{3}
$$

The Jordan curve theorem has also been extended to $\mathbb{R}^3$ wherein any topologically equivalent sphere ($\mathcal{S}$) partition the points in space into points in the interior set($\mathcal{I}$), points in the exterior($\mathcal{E}$) and points on the surface of sphere($\mathcal{S}$). The partitioning is based on the value of the implicit representation of the obstacle boundary $f(\boldsymbol{P})=0,\ \boldsymbol{P}\in\mathbb{R}^2$ or $\mathbb{R}^3$. For example, for the circular obstacle in equation (3), $f(\boldsymbol{P})=(x_i-x_c)^2+(y_i-y_c)^2-R^2\forall\boldsymbol{P}\in\mathbb{R}^2$), the general classification for spatial point $\boldsymbol{P}\in\mathbb{R}^3$ or $\mathbb{R}^2$ for this obstacle is as follows.

$$
\begin{aligned}
\mathcal{E}=&\{\boldsymbol{P}|f(\boldsymbol{P})>0\}\\
\mathcal{I}=&\{\boldsymbol{P}|f(\boldsymbol{P})<0\}\\
\mathcal{C}=&\{\boldsymbol{P}|f(\boldsymbol{P})=0\}
\end{aligned}
\tag{4}
$$

If the obstacle implicit boundary representation $f(\boldsymbol{P})=0$ is differentiable, then such obstacles can be incorporated as constraints in optimization problem and classical optimization algorithms like gradient-based methods can be used. In case of a single obstacle with implicit boundary representation $f(\boldsymbol{P}) = 0$, the modified optimization problem for obstacle avoidance takes the following form.

$$\begin{aligned}
&\underset{p_{i+1}(t_2))}{Min\ I}\ :\ \big(\Delta(T_x + x_{i+1})\big)^2 + \Big(\Delta(T_y + \Delta y_{i+1})\Big)^2 \\
&Subject\ to \\
&\lambda_i :\ \sqrt{(x_{i+1}(t_2) - x_i(t_2))^2 + (y_{i+1}(t_2) - y_i(t_2))^2} = L \qquad (5)\\
&\beta :\ f(\boldsymbol{P}) > 0 \\
&Data :\ p_i(t_m), p_{i+1}(t_1), T_x(t_m), T_y(t_m), p_1(t_1) = (0,0)^T \\
&\forall i \in [1, n-1]\ \&\ \forall\, t_1 \in [0 \ldots T],\ t_2 = t_1 + \Delta t
\end{aligned}$$

More generally, if there are multiple interfering obstacles $\mathcal{O}_j,\ 1 \le j \le p$ each with an exterior $\mathcal{E}_j$, then the intersection of all the individual exteriors gives the permissible space for motion planning, namely $\mathcal{E} = \bigcap_{j=1}^{p} \mathcal{E}_j$. Hence, the most general form of the optimization problem for obstacle avoidance in presence of multiple obstacles $\mathcal{O}_j$ each having differentiable implicit representation $f_j(\boldsymbol{P}) = 0$ is as follows.

$$\begin{aligned}
&\underset{p_{i+1}(t_2))}{Min\ I}\ :\ \big(\Delta(T_x + x_{i+1})\big)^2 + \Big(\Delta(T_y + \Delta y_{i+1})\Big)^2 \\
&Subject\ to \\
&\lambda_i :\ \sqrt{(x_{i+1}(t_2) - x_i(t_2))^2 + (y_{i+1}(t_2) - y_i(t_2))^2} = L \qquad (6)\\
&\beta_j :\ f_j(\boldsymbol{P}) > 0\ \forall\, j \in [1, p] \\
&Data :\ p_i(t_m), p_{i+1}(t_1), T_x(t_m), T_y(t_m), p_1(t_1) = (0,0)^T \\
&\forall i \in [1, n-1]\ \&\ \forall\, t_1 \in [0 \ldots T],\ t_2 = t_1 + \Delta t
\end{aligned}$$

In this paper, we restrict the obstacle shapes to a class of objects known as super-quadrics..

A. Differentiable Super-ellipses and Super-ellipsoids

Super-quadrics were first invented by Piet Hein [32, Chapter 18] and studied by Barr [33]. They are defined as follows.

$$\left(\left|\frac{x}{a_1}\right|^{\frac{2}{\epsilon_1}} + \left|\frac{y}{a_2}\right|^{\frac{2}{\epsilon_1}}\right)^{\frac{\epsilon_1}{\epsilon_2}} + \left|\frac{z}{a_3}\right|^{\frac{2}{\epsilon_2}} = 1 \qquad (7)$$

To limit to cases of curves/surfaces with well defined gradients, this paper restricts to super-ellipsoids of the following form.

$$\begin{aligned}
&\left(\left(\left(\frac{x}{a_1}\right)^2\right)^{\frac{1}{\epsilon_1}} + \left(\left(\frac{y}{a_2}\right)^2\right)^{\frac{1}{\epsilon_1}}\right)^{\frac{\epsilon_1}{\epsilon_2}} + \left(\left(\frac{z}{a_3}\right)^2\right)^{\frac{1}{\epsilon_2}} = 1 \\
&0 \le \epsilon_1 \le 1,\ 0 \le \epsilon_2 \le 1
\end{aligned} \qquad (8)$$

The family of manifolds generated by equation (8) includes circles, ellipses, rectangles, cylinders, cuboids, cubes, ellipsoids, spheres etc. Some of shapes generated with their parameters are shown in figure 1. It may be noted that some shapes though they have the same exponent parameters, differ in scaling along the axes, namely the variables $a_i^{\prime}s$. Examples are spheres and ellipsoids, cubes and cuboids and this family of curves have equation (8) as their exterior-interior partition function.

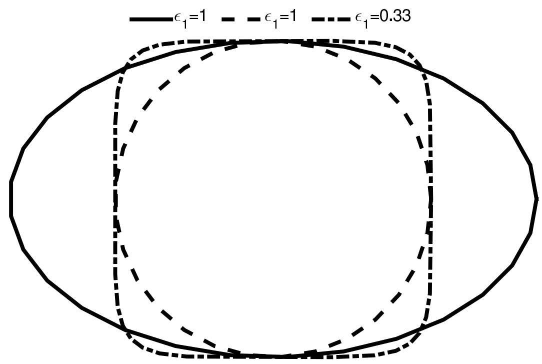


Fig. 1: Super-ellipses

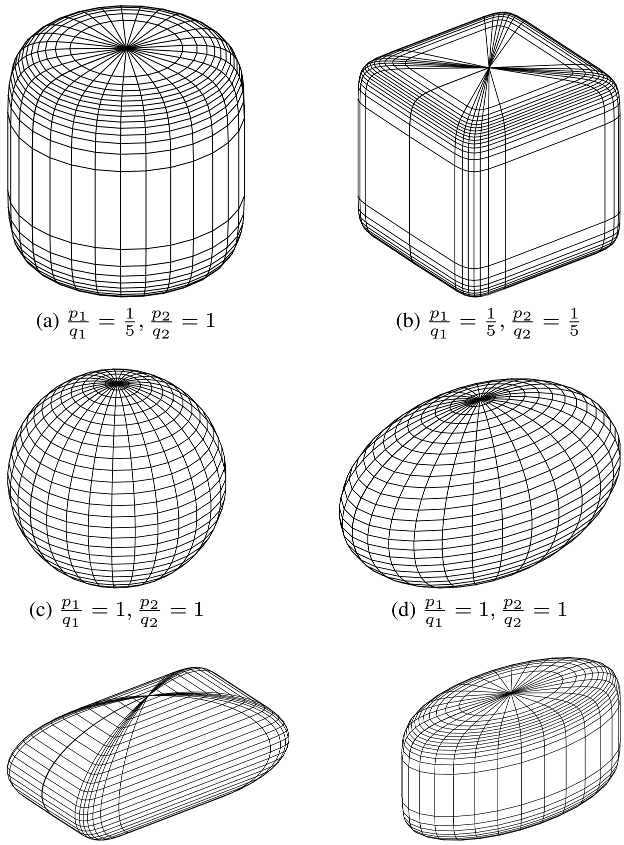
III. Simulation, Results and Discussion

In this section, the numerical simulation[1] results are presented for a chosen one-dimensional(1D) flexible object whose leading end is moved along a generic trajectory in two- and three-dimensional space. In all the simulations, the initial configuration of the object is chosen to be a straight line although there is no restriction on initial configuration.

In first simulation done in two-dimensional (2D) space, a 1D object of length 5 units is discretized into 30 rigid segments connected by rotary joints yielding a hyper-redundant system with 30 degrees of freedom. The leading end is moved along an arbitrarily chosen trajectory in steps of 0.05 length units for 540 steps. The arbitrary path is shown in figure 3. Along the path, eight arbitrary snapshot locations are chosen denoted by ① to ⑧. The initial configuration (at snapshot ①) of the 1D flexible object is shown in blue. The configuration of the 1D flexible object at each of the 8 snapshot locations of the leading end are shown in figure 4. As seen here, the obstacles are avoided and motion is minimized as one moves away from the leading end of the flexible object. In other words, tractrix motion is followed in obstacle free spaces and algorithm automatically switches to obstacle avoidance once the objects are encountered.

In the second simulation, the motion planning of the 1D flexible object is done in for an arbitrarily chosen three-dimensional (3D)motion of the leading end. Here too, the initial configuration of the object is chosen as a straight line. A flexible 1D object of length 30 units is discretized

[1] All simulations were done using the commercial software Matlab [35] on a Pentium quad core PC with 16Gb RAM running Linux operating system.



(a) $\frac{p_1}{q_1} = \frac{1}{5}, \frac{p_2}{q_2} = 1$ (b) $\frac{p_1}{q_1} = \frac{1}{5}, \frac{p_2}{q_2} = \frac{1}{5}$

(c) $\frac{p_1}{q_1} = 1, \frac{p_2}{q_2} = 1$ (d) $\frac{p_1}{q_1} = 1, \frac{p_2}{q_2} = 1$

(e) $\frac{p_1}{q_1} = 1, \frac{p_2}{q_2} = \frac{1}{7}$ (f) $\frac{p_1}{q_1} = \frac{1}{3}, \frac{p_2}{q_2} = 1$

Fig. 2: Super-ellipsoids

into 40 rigid segments with two degree of freedom joints connecting the segments. The leading end is subjected to an arbitrarily chosen motion discretized in to steps of 0.2 length units and the total motion is for 400 steps. The trajectory is in an obstacle field with 7 obstacles of type superquadrics discussed earlier. It can be seen that the extended body avoids obstacles optimally by gracing them tangentially. This demonstrates the efficacy of the algorithm.

IV. Conclusions

This paper presents a new approach to the problem of obstacle avoidance for extended bodies using an extension to task space velocity minimization tractrix based approach. The presented approach yields natural looking motion while optimally avoiding obstacles. An important feature of the proposed algorithm is that it is purely kinematics and geometry based algorithm – it does not use any dynamics or potential field type of constructs. The framework is very general as any obstacle shape modeled by union of objects modeled by first order differentiable implicit equation can be directly incorporated. The numerical results demonstrates that the algorithm is able to efficiently avoid obstacles while maintaining the nature of tractrix motion diminishing property wherever possible. This approach presented in this paper has been demonstrated to obstacle avoidance in 2D and 3D space and works for obstacle avoidance of extended bodies like snakes, ropes, surgical suturing simulation and in motion planning of hyper-redundant manipulators.

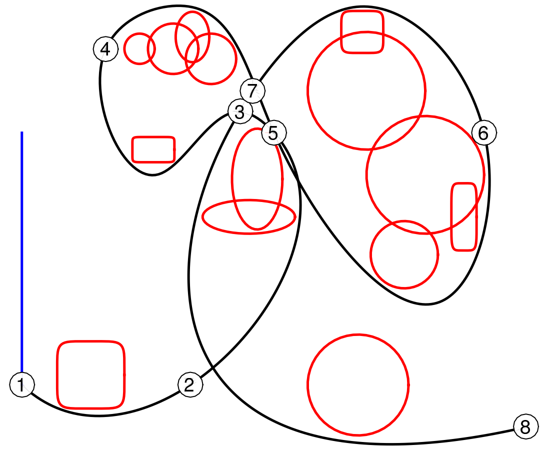


Fig. 3: Trajectory for 2D simulation with snapshot locations and initial configuration of the flexible 1D object

Acknowledgment

This work was funded in part by the Robert Bosch Center for Cyber Physical Systems (RBCCPS) at the Indian Institute of Science, Bangalore.

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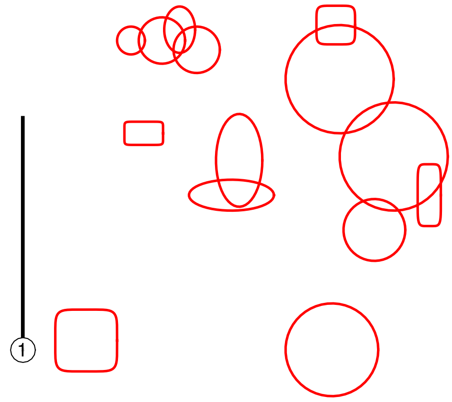
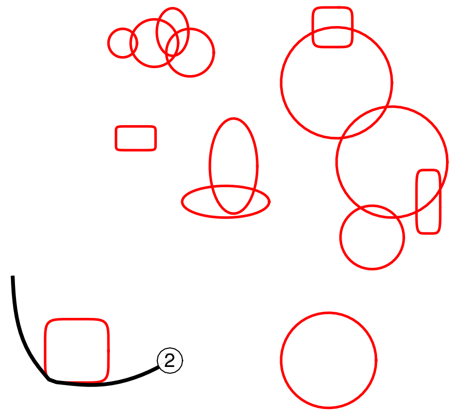
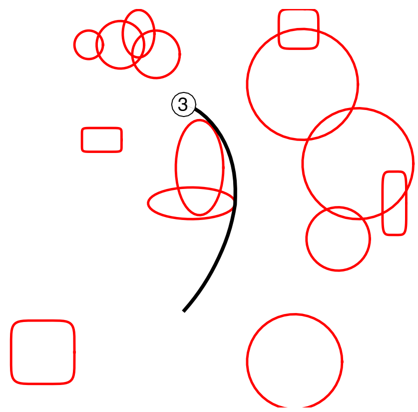


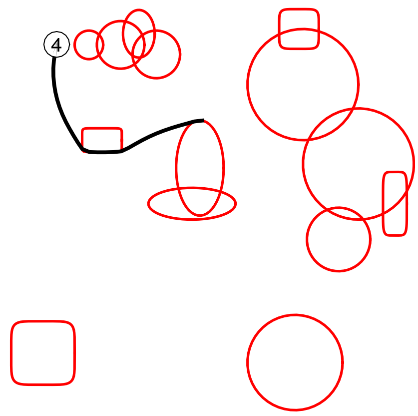
(a) Snapshot at ①



(b) Snapshot at ②

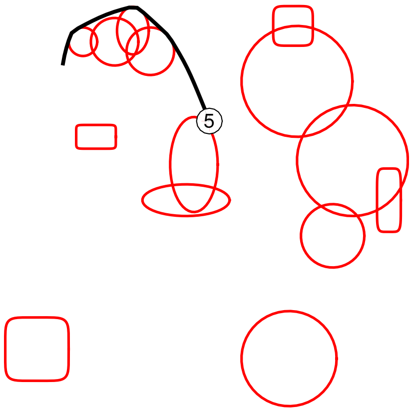


(c) Snapshot at ③

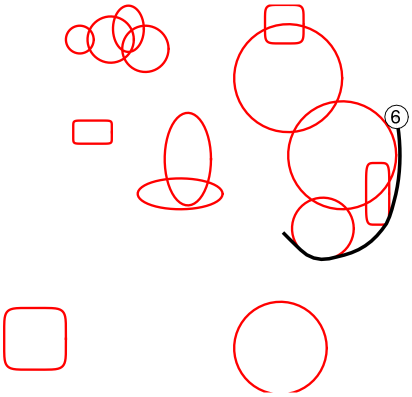


(d) Snapshot at ④

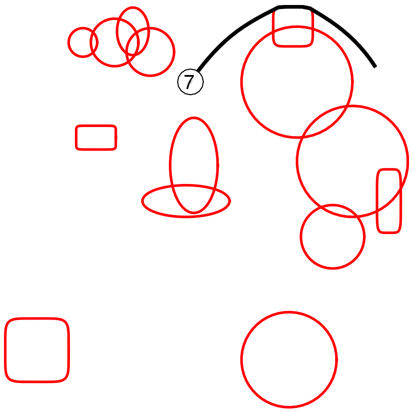
Fig. 4: Motion snapshots ① to ④ for 2D simulation



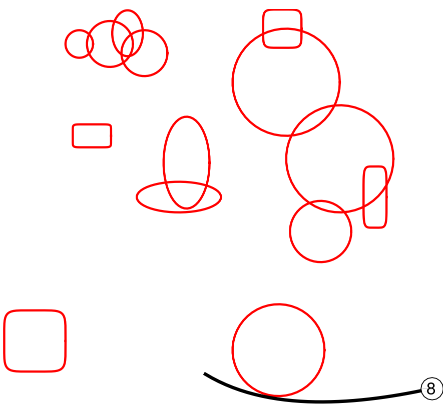
(e) Snapshot at ⑤



(f) Snapshot at ⑥



(g) Snapshot at ⑦



(h) Snapshot at ⑧

Fig. 4: Motion snapshots ⑤ to ⑧ for 2D simulation

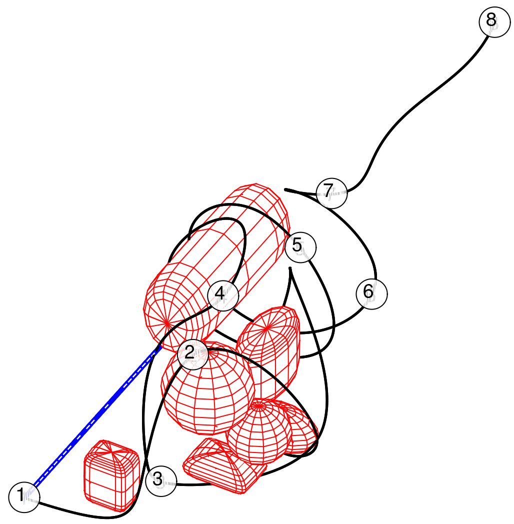
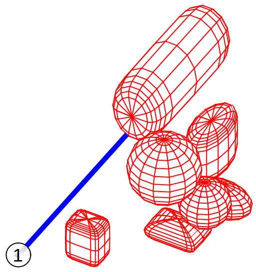
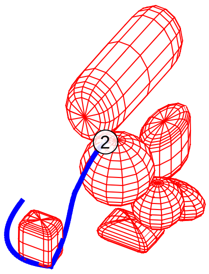


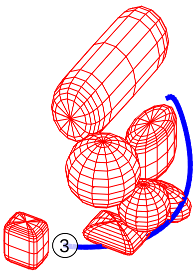
Fig. 5: Trajectory for 3D simulation with snapshot locations and initial configuration of the flexible 1D object



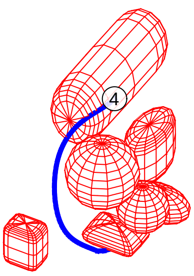
(a) Snapshot at ①



(b) Snapshot at ②

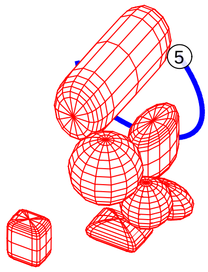


(c) Snapshot at ③

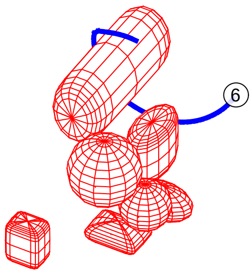


(d) Snapshot at ④

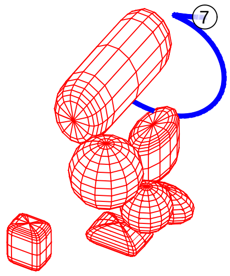
Fig. 6: Motion snapshots ① to ④ for 3D simulation



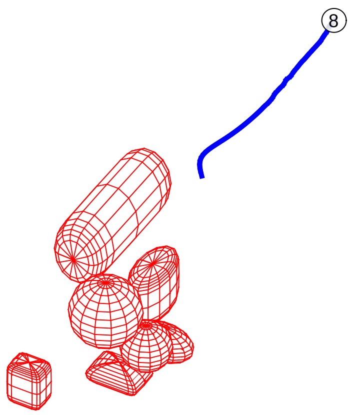
(e) Snapshot at ⑤



(f) Snapshot at ⑥



(g) Snapshot at ⑦



(h) Snapshot at ⑧

Fig. 6: Motion snapshots ⑤ to ⑧ for 3D simulation