Robotics: Advanced Concepts & Analysis

Module 2 - Elements of robots: joints, links, actuators & sensors

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- Homogeneous Transformation

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- Elements of a robot – Actuators & Transmission

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- Problems, References, and Suggested Reading

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Position of a Rigid Body

Cartesian coordinates of a point in $\Re^3$

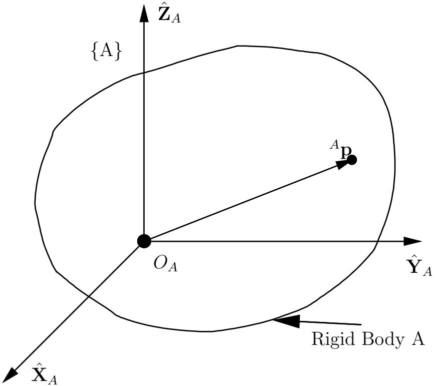

Figure 1: Position of point P denoted by ${}^A\mathbf{p}$

- Position of a point of interest such as centre of mass/gravity
- Right-handed coordinate system specified by
  - Origin O_A
  - Set of 3 axis – $\hat{\mathbf{X}}_A$, $\hat{\mathbf{Y}}_A$ and $\hat{\mathbf{Z}}_A$
  - Label to keep track of coordinate systems – $\{A\}$
- Point ${}^A\mathbf{p}$ with Cartesian coordinates $(p_x, p_y, p_z)^T$

$$ {}^A\mathbf{p} = p_x\hat{\mathbf{X}}_A + p_y\hat{\mathbf{Y}}_A + p_z\hat{\mathbf{Z}}_A = (p_x, p_y, p_z)^T \quad (1) $$

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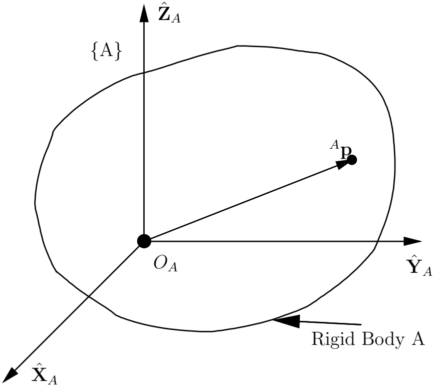

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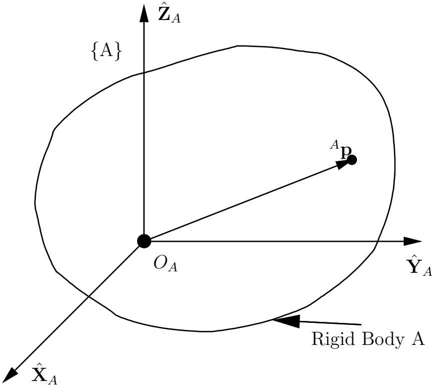

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Orientation of a Rigid Body

Coordinate systems

- Position of *one* point on the rigid body – not enough to describe in 3D space
- Orientation of a rigid body B with respect to $\{A\}$

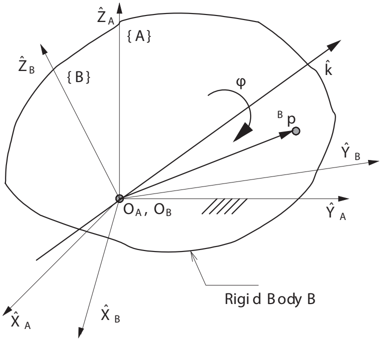

Figure 2: Orientation of a rigid body

- Attach coordinate system, $\{B\}$, to rigid body B
- Origin of $\{B\}$ coincident with origin of $\{A\}$ (see figure 2)
- Obtain description of $\{B\}$ with respect to $\{A\}$

ORIENTATION OF A RIGID BODY

DIRECTION COSINES

- Unit vectors $\hat{\mathbf{X}}_B$, $\hat{\mathbf{Y}}_B$, and $\hat{\mathbf{Z}}_B$, attached to B, can be described in $\{A\}$

$$\begin{aligned} {}^A\hat{\mathbf{X}}_B &= r_{11}\hat{\mathbf{X}}_A + r_{21}\hat{\mathbf{Y}}_A + r_{31}\hat{\mathbf{Z}}_A \\ {}^A\hat{\mathbf{Y}}_B &= r_{12}\hat{\mathbf{X}}_A + r_{22}\hat{\mathbf{Y}}_A + r_{32}\hat{\mathbf{Z}}_A \\ {}^A\hat{\mathbf{Z}}_B &= r_{13}\hat{\mathbf{X}}_A + r_{23}\hat{\mathbf{Y}}_A + r_{33}\hat{\mathbf{Z}}_A \end{aligned} \tag{2}$$

- $r_{ij}, i,j = 1,2,3$ are called *direction cosines*
  - $r_{11} = {}^A\hat{\mathbf{X}}_B \cdot \hat{\mathbf{X}}_A$
  - Magnitude of unit vectors are $1 \rightarrow r_{11}$ is cosine of angle between ${}^A\hat{\mathbf{X}}_B$ and $\hat{\mathbf{X}}_A$. *All* r_{ij}'s are cosines of angles.
- Define 3×3 *rotation matrix* ${}^A_B[R]$ with $r_{ij},\ i,j = 1,2,3$ as its elements. *Columns* of ${}^A_B[R]$ are ${}^A\hat{\mathbf{X}}_B$, ${}^A\hat{\mathbf{Y}}_B$, and ${}^A\hat{\mathbf{Z}}_B$.
- ${}^A_B[R]$ *completely* describes all three coordinate axis of $\{B\}$ with respect to $\{A\}$
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Orientation of a Rigid Body

Direction cosines

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Orientation of a Rigid Body

Properties of rotation matrix ${}^A_B[R]$

- The vector ${}^B\mathbf{p}$ in rigid body B can be described in $\{A\}$ by

$$ {}^A\mathbf{p} = {}^A_B[R]\,{}^B\mathbf{p} \tag{3} $$

- The columns of the rotation matrix are unit vectors, orthogonal to one another $\rightarrow {}^A\hat{\mathbf{X}}_B \cdot {}^A\hat{\mathbf{X}}_B = {}^A\hat{\mathbf{Y}}_B \cdot {}^A\hat{\mathbf{Y}}_B = {}^A\hat{\mathbf{Z}}_B \cdot {}^A\hat{\mathbf{Z}}_B = 1$, and ${}^A\hat{\mathbf{X}}_B \cdot {}^A\hat{\mathbf{Y}}_B = {}^A\hat{\mathbf{Y}}_B \cdot {}^A\hat{\mathbf{Z}}_B = {}^A\hat{\mathbf{Z}}_B \cdot {}^A\hat{\mathbf{X}}_B = 0$
  - The determinant of the rotation matrix is $+1$[1] .
  - ${}^A_B[R]^T\,{}^A_B[R] = [U]$, where $[U]$ is a 3×3 identity matrix.
  - Inverse of ${}^A_B[R]$ is same as transpose $\rightarrow {}^B_A[R] = {}^A_B[R]^{-1} = {}^A_B[R]^T$
- Three eigenvalues of ${}^A_B[R]$ – $+1$, $e^{\pm\imath\phi}$, where $\imath = \sqrt{-1}$, and $\phi = \cos^{-1}(\frac{r_{11}+r_{22}+r_{33}-1}{2})$
- The eigenvector corresponding to $+1$ is $\hat{\mathbf{k}} = (1/2\sin\phi)[r_{32} - r_{23},\ r_{13} - r_{31},\ r_{21} - r_{12}]^T$

[1] Rotation matrices have determinant as $+1$. For reflection, determinant is -1

Orientation of a Rigid Body

Properties of rotation matrix ${}^{A}_{B}[R]$

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Orientation of a Rigid Body

Properties of rotation matrix (contd.)

- Rotation axis $\hat{\mathbf{k}}$ *fixed* in $\{A\}$ **and** $\{B\} \rightarrow {}^A\hat{\mathbf{k}} = {}^A_B[R]\,{}^B\hat{\mathbf{k}} = 1\,{}^B\hat{\mathbf{k}}$
  First equality is transformation of a vector from $\{B\}$ to $\{A\}$ and the second equality follows from equation (3) and the definition of an eigenvector.
- Elements of ${}^A_B[R]$ in terms of $(k_x, k_y, k_z)^T$ and angle ϕ

$$
\begin{aligned}
r_{11} &= {k_x}^2(1-\cos\phi)+\cos\phi \\
r_{12} &= k_x k_y(1-\cos\phi)-k_z\sin\phi \\
r_{13} &= k_z k_x(1-\cos\phi)+k_y\sin\phi \\
r_{21} &= k_x k_y(1-\cos\phi)+k_z\sin\phi \\
r_{22} &= {k_y}^2(1-\cos\phi)+\cos\phi \\
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r_{32} &= k_y k_z(1-\cos\phi)+k_x\sin\phi \\
r_{33} &= {k_z}^2(1-\cos\phi)+\cos\phi
\end{aligned}
\tag{4}
$$

PROPERTIES OF ROTATION MATRIX (CONTD.)

- Rotation axis $\hat{\mathbf{k}}$ *fixed* in $\{A\}$ **and** $\{B\}$ $\rightarrow$ ${}^{A}\hat{\mathbf{k}} = {}^{A}_{B}[R]^{B}\hat{\mathbf{k}} = 1^{B}\hat{\mathbf{k}}$
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\end{aligned}
\tag{4}
$$

- Rotation axis $\hat{\mathbf{k}}$ is parallel to $\hat{\mathbf{X}}_A$ and hence to $\hat{\mathbf{X}}_B$ $\rightarrow$ Rotation about $\hat{\mathbf{X}}$ axis.

$$ {}^{A}_{B}[R] = [R(\hat{\mathbf{X}}, \phi)] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix} \tag{5} $$

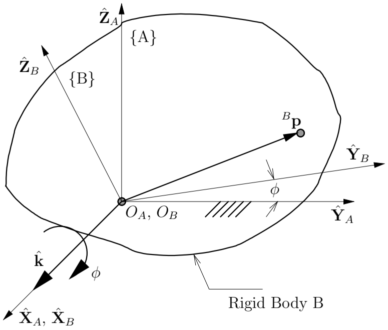

Figure 3: Rotation about $\hat{\mathbf{X}}$ by angle ϕ

- Rotation about $\hat{\mathbf{X}}$ shown in figure 3.

- Rotation about $\hat{\mathbf{Y}}$ and $\hat{\mathbf{Z}}$

$$[R(\hat{\mathbf{Y}}, \phi)] = \begin{pmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{pmatrix} \tag{6}$$

$$[R(\hat{\mathbf{Z}}, \phi)] = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{7}$$

- Rotation matrices in equations (5) through (7) called *simple* rotations.

- Two successive rotations:
  1. Initially B is coincident with $\{A\}$.
  2. First rotation relative to $\{A\}$. After first rotation $\{A\} \rightarrow \{B_1\}$.
  3. Second rotation relative to $\{B_1\}$. After second rotation $\{B_1\} \rightarrow \{B\}$.

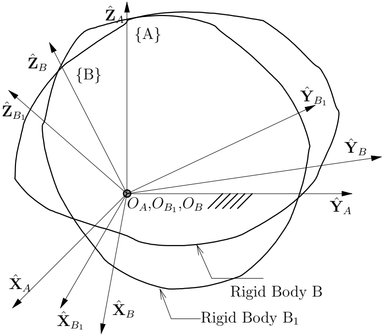

Figure 4: Successive rotation

- Resultant rotation: ${}^A_B[R] = {}^A_{B_1}[R]\ {}^{B_1}_B[R]$
  Note order of matrix multiplication
- Resultant of n rotations –
  ${}^A_B[R] = {}^A_{B_1}[R]\ {}^{B_1}_{B_2}[R] \quad \ldots \quad {}^{B_{n-1}}_B[R]$
- Matrix multiplication is non commutative –
  ${}^A_{B_1}[R]\ {}^{B_1}_B[R] \neq {}^{B_1}_B[R]\ {}^A_{B_1}[R]$.

Orientation of a Rigid Body

Successive rotations

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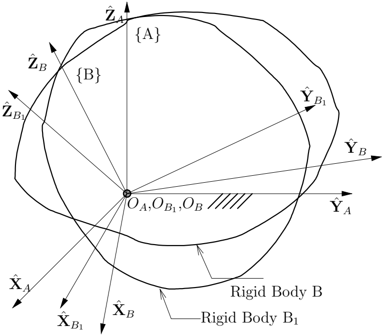

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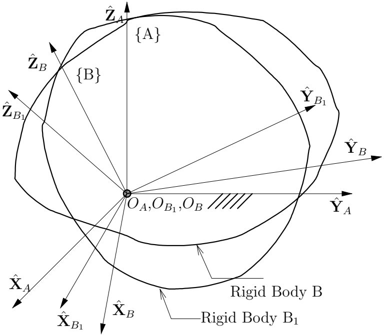

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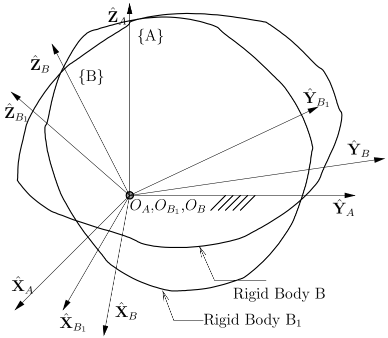

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- Orientation described by 3 independent parameters $\rightarrow$ Three rotations completely describe orientation of a rigid body.
- Three successive rotations about *axes fixed to moving body*
  - Rotations about three distinct axes: 6 combinations – X-Y-Z, X-Z-Y, Y-Z-X, Y-X-Z, Z-X-Y & Z-Y-X
  - Rotations about two distinct axes: 6 combinations – X-Y-X, X-Z-X, Y-X-Y, Y-Z-Y, Z-X-Z, & Z-Y-Z
- Rotations about *axes fixed in space* – 12 possible combinations for 3 and 2 distinct axes.
- *Minimal* representation of orientation of rigid body – only three parameters (angles) and *no* constraints.
- Three angles also called *Euler angles*.

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  - Rotations about three distinct axes: 6 combinations – X-Y-Z, X-Z-Y, Y-Z-X, Y-X-Z, Z-X-Y & Z-Y-X
  - Rotations about two distinct axes: 6 combinations – X-Y-X, X-Z-X, Y-X-Y, Y-Z-Y, Z-X-Z, & Z-Y-Z
- Rotations about *axes fixed in space* – 12 possible combinations for 3 and 2 distinct axes.
- *Minimal* representation of orientation of rigid body – only three parameters (angles) and *no* constraints.
- Three angles also called *Euler angles*.

Orientation of a Rigid Body

Representation of orientation by three angles

- Orientation described by 3 independent parameters → Three rotations completely describe orientation of a rigid body.
- Three successive rotations about *axes fixed to moving body*
  - Rotations about three distinct axes: 6 combinations – X-Y-Z, X-Z-Y, Y-Z-X, Y-X-Z, Z-X-Y & Z-Y-X
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- Three angles also called *Euler angles*.

- Rotation about $\hat{\mathbf{X}}$ – ${}^{A}_{B_1}[R] = [R(\hat{\mathbf{X}}, \theta_1)] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{pmatrix}$
- Rotation about $\hat{\mathbf{Y}}$ – ${}^{B_1}_{B_2}[R] = [R(\hat{\mathbf{Y}}, \theta_2)] = \begin{pmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{pmatrix}$

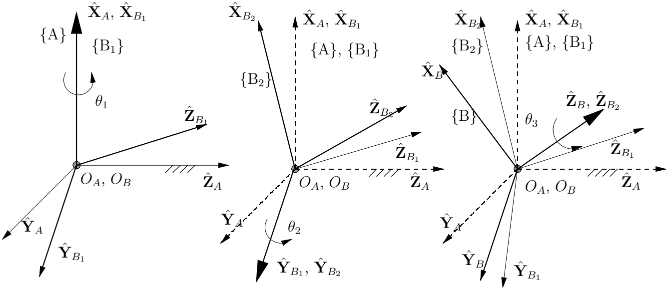

Figure 5: X–Y–Z Euler angles

- Rotation about $\hat{\mathbf{Z}}$ – ${}^{B_2}_{B}[R] = [R(\hat{\mathbf{Z}}, \theta_3)] = \begin{pmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
- Resultant rotation – ${}^{A}_{B}[R] = {}^{A}_{B_1}[R]\ {}^{B_1}_{B_2}[R]\ {}^{B_2}_{B}[R] =$
  $$\begin{pmatrix} c_2c_3 & -c_2s_3 & s_2 \\ s_1s_2c_3 + s_3c_1 & -s_1s_2s_3 + c_3c_1 & -s_1c_2 \\ -c_1s_2c_3 + s_3s_1 & c_1s_2s_3 + c_3s_1 & c_1c_2 \end{pmatrix}$$
- Note: c_i, s_i denote $\cos\theta_i$ and $\sin\theta_i$, respectively.
- ${}^{A}_{B}[R]$ gives *orientation* of $\{B\}$ given X–Y–Z angles.
- ${}^{A}_{B}[R]$ will be *different* if *order* of rotations is different.

X–Y–Z EULER ANGLES (CONTD.)

- Given ${}^A_B[R]$, find X–Y–Z Euler angles

Algorithm $r_{ij} \Rightarrow \theta_i$ **for** X–Y–Z **rotations**

If $r_{13} \neq \pm 1$, *then*

$$\theta_2 = \text{Atan2}(r_{13}, \pm\sqrt{(r_{11}^2 + r_{12}^2)})$$
$$\theta_1 = \text{Atan2}(-r_{23}/\cos\theta_2, r_{33}/\cos\theta_2)$$
$$\theta_3 = \text{Atan2}(-r_{12}/\cos\theta_2, r_{11}/\cos\theta_2)$$

Else

If $r_{13} = 1$, *then*

$$\theta_1 = \text{Atan2}(r_{21}, r_{22}),\ \theta_2 = \pi/2,\ \theta_3 = 0$$

If $r_{31} = -1$, *then*

$$\theta_1 = -\text{Atan2}(r_{21}, r_{22}),\ \theta_2 = -\pi/2,\ \theta_3 = 0$$

- Atan2(y, x): four-quadrant arc-tangent function (see function Atan2 in Matlab®) – θ_1, θ_2, θ_3 in the range $[-\pi, \pi]$.
- Two sets of values of θ_1, θ_2 and θ_3
- If θ_2 is $\pm\pi/2 \rightarrow \theta_1$ and θ_2 not *unique* – $\theta_1 \pm \theta_3$ can be found.
- *Singularities* in Euler angle representation.

Orientation of a Rigid Body

Z–Y–Z Euler angles

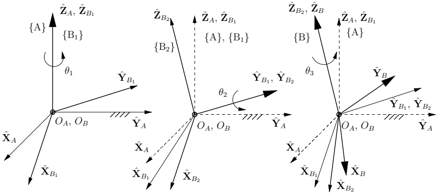

Figure 6: *Z–Y–Z* Euler angles

$$
\begin{aligned}
{}^A_B[R] &= \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{pmatrix} \begin{pmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c_1c_2c_3 - s_1s_3 & -c_1c_2s_3 - s_1c_3 & c_1s_2 \\ s_1c_2c_3 + c_1s_3 & -s_1c_2s_3 + c_1c_3 & s_1s_2 \\ -s_2c_3 & s_2s_3 & c_2 \end{pmatrix}
\end{aligned} \tag{8}
$$

Algorithm $r_{ij} \Rightarrow Z$–Y–Z Euler angles

If $r_{33} \neq \pm 1$, then

$$\theta_2 = \mathrm{Atan2}(\pm\sqrt{(r_{31}^2 + r_{32}^2)}, r_{33})$$
$$\theta_1 = \mathrm{Atan2}(r_{23}/\sin\theta_2, r_{13}/\sin\theta_2)$$
$$\theta_3 = \mathrm{Atan2}(r_{32}/\sin\theta_2, -r_{31}/\sin\theta_2)$$

Else

If $r_{33} = 1$, then

$$\theta_1 = \theta_2 = 0,\ \theta_3 = \mathrm{Atan2}(-r_{12}, r_{11})$$

If $r_{33} = -1$, then

$$\theta_1 = 0,\ \theta_2 = \pi,\ \theta_3 = \mathrm{Atan2}(r_{12}, -r_{11})$$

- Two possible sets of Z–Y–Z Euler angles $(\theta_1, \theta_2, \theta_3)$ for a given ${}^A_B[R]$.
- $r_{33} = \pm 1 \rightarrow$ singularity $\rightarrow \theta_1 \pm \theta_3$ can be found.
- For unique θ_1 and θ_3 when $r_{33} = \pm 1 \rightarrow$ choose $\theta_1 = 0$.

- Euler parameters (see Kane et al., 1983, McCarthy, 1990): 4 parameters derived from $\hat{\mathbf{k}} = (k_x, k_y, k_z)^T$ and angle ϕ
  - 3 parameters – $\varepsilon = \hat{\mathbf{k}} \sin\phi/2$
  - fourth parameter – $\varepsilon_4 = \cos\phi/2$
  - One constraint – ${\varepsilon_1}^2 + {\varepsilon_2}^2 + {\varepsilon_3}^2 + {\varepsilon_4}^2 = 1$
- Quaternions (see p. 185 Arfken, 1985, Wolfram website): 4 parameters
  - 'Sum' of a scalar q_0 and a vector $(q_1, q_2, q_3)^T$ – $Q = q_0 + q_1\hat{\mathbf{i}} + q_2\hat{\mathbf{j}} + q_3\hat{\mathbf{k}}$
  - $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are the unit vectors in $\Re^3$.
  - Product of two quaternion also a quaternion
  - Inverse of quaternion *exists*.
  - $q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \rightarrow$ unit quaternion – represent orientation of a rigid body in $\Re^3$

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- Orientation of a rigid body in $\Re^3$ is specified by 3 independent parameters.
- Various representation of orientation with their own advantages and disadvantages
  - Rotation matrix ${}^A_B[R]$ – 9 r_{ij}'s + 6 constraints $\rightarrow$ Too many variables and constraints but ideal for analysis.
  - Axis $(k_x, k_y, k_z)^T$ and angle ϕ – 4 parameters + one constraint $k_x^2 + k_y^2 + k_z^2 = 1$ $\rightarrow$ Useful for insight and extension to screws, twists and wrenches.
  - Euler angles: 3 parameters and *zero* constraints$\rightarrow$ Minimal representation but suffer from problem of *singularities* and *sequence* must be known.
  - Euler parameters and quaternions: 4 parameters +1 1 constraint $\rightarrow$ Similar *but not exactly* same as axis-angle form, no singularities and used extensively in motion planning.
- Can convert from one representation to any other representation.

ORIENTATION OF A RIGID BODY

SUMMARY

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Combined Translation and Orientation of a Rigid Body

Transformation between coordinate systems

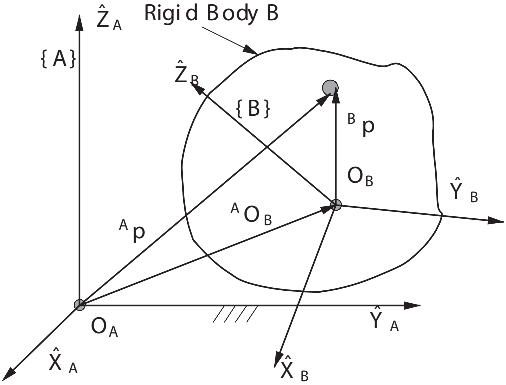

Figure 7: General transformation

- $\{A\}$ and $\{B\}$, O_A and O_B not coincident
- Orientation of $\{B\}$ with respect to $\{A\}$ – ${}^A_B[R]$
- ${}^A\mathbf{p} = {}^A\mathbf{O}_B + {}^A_B[R]{}^B\mathbf{p}$
- ${}^A\mathbf{O}_B$ locates O_B with respect to O_A.

OUTLINE

1 CONTENTS

2 LECTURE 1

- Mathematical Preliminaries
- Homogeneous Transformation

3 LECTURE 2

- Elements of a robot – Joints
- Elements of a robot – Links

4 LECTURE 3

- Examples of D-H Parameters & Link Transformation Matrices

5 LECTURE 4

- Elements of a robot – Actuators & Transmission

6 LECTURE 5

- Elements of a Robots – Sensors

7 MODULE 2 – ADDITIONAL MATERIAL

- Problems, References, and Suggested Reading

- Combined translation and orientation ${}^{A}\mathbf{P} = {}^{A}_{B}[T]\,{}^{B}\mathbf{P}$
- ${}^{A}\mathbf{P}$ and ${}^{B}\mathbf{P}$ are 4×1 *homogeneous* coordinates (see Appendix 1) constructed by concatenating a '1' to ${}^{A}\mathbf{p}$ and ${}^{B}\mathbf{p}$
  ${}^{A}\mathbf{P} = [{}^{A}\mathbf{p} \mid 1]^{T}$ and ${}^{B}\mathbf{P} = [{}^{B}\mathbf{p} \mid 1]^{T}$
- 4×4 homogeneous transformation matrix ${}^{A}_{B}[T]$ is formed as

$$ {}^{A}_{B}[T] = \begin{pmatrix} & {}^{A}_{B}[R] & & {}^{A}\mathbf{O}_{B} \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (9) $$

- In computer graphics and computer vision $\rightarrow$ last row is used for perspective and scaling and not [0 0 0 1].
- Upper left 3×3 matrix is identity matrix $\rightarrow$ pure translation.
- Top right 3×1 vector is zero $\rightarrow$ pure rotation.

4×4 Transformation Matrix

Introduction

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4×4 TRANSFORMATION MATRIX

PROPERTIES OF ${}^A_B[T]$

- Inverse of ${}^A_B[T]$ is given by
  $${}^A_B[T]^{-1} = \begin{pmatrix} & {}^A_B[R]^T & & -{}^A_B[R]^T\, {}^A\mathbf{O}_B \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
- Two successive transformations $\{A\} \rightarrow \{B_1\} \rightarrow \{B\}$ gives ${}^A_B[T] = {}^A_{B_1}[T]\; {}^{B_1}_B[T]$.
  $${}^A_B[T] = \begin{pmatrix} & {}^A_{B_1}[R]^{B_1}_B[R] & & {}^A_{B_1}[R]^{B_1}\mathbf{O}_B + {}^A\mathbf{O}_{B_1} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
- n successive transformations ${}^A_B[T] = {}^A_{B_1}[T]\; {}^{B_1}_{B_2}[T] \; \ldots \; {}^{B_{n-1}}_B[T]$

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Properties of ${}^A_B[T]$ (Contd.)

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- Eigenvectors for $+1$ is $\hat{\mathbf{k}}$ – no other eigenvector !
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- General motion of rigid body as a *twist* – **rotation about a line and translation along the line**.
  - Direction of the line: $(k_x, k_y, k_z)^T \rightarrow 2$ independent parameters.
  - Rotation about the line: angle $\phi \rightarrow 1$ independent parameter.
  - Location of the line in $\Re^3$: $(\hat{\mathbf{k}}, \mathbf{Y} \times \hat{\mathbf{k}})$ where $\mathbf{Y} = \frac{([U] - {}^A_B[R]^T)\,{}^A\mathbf{O}_B}{2(1-\cos\phi)}$.

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- Rigid body B *conceptually identical* to a coordinate system $\{B\}$.
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- Position of rigid body $\rightarrow$ position of a point of interest on rigid body with respect to coordinate system $\{A\} \rightarrow$ 3 Cartesian coordinates: ${}^A\mathbf{p} = (p_x, p_y, p_z)^T$.
- Orientation described in many ways: 1) by 3×3 rotation matrix ${}^A_B[R]$, 2) $(\phi, \hat{\mathbf{k}})$ or angle-axis form, 3) 3 Euler angles, 4) Euler parameters & quaternions.
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- A joint imposes constraints on the links it connects.
  - 2 free rigid bodies have $6+6$ degrees of freedom.
  - Hinge joint connecting two rigid bodies – $6+1$ degrees of freedom.
  - Hinge joint imposes 5 constraints – hinge joint allows 1 relative (rotary) degree of freedom.
- Degree of freedom of a joint in 3D space: $6-m$ where m is the number of constraint imposed.
- Serial manipulators $\rightarrow$ *all joints actuated* $\rightarrow$ one- degree-of-freedom joints used.
- Parallel and hybrid manipulators $\rightarrow$ some joints *passive* $\rightarrow$ multi-degree-of-freedom joints used.

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Representation of joints

Types of joints

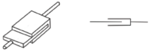
PAIR	SYMBOL		REPRESENTAION
SCREW	H	1	
REVOLUTE	R	1	
PRISMATIC	P	1	
CYLINDRIC	C	2	
SPHERICAL	S	3	
PLANAR PAIR	E	3	
HOOKE JOINT	T	2	

Figure 8: – Types of joints

REPRESENTATION OF JOINTS*

CONSTRAINTS IMPOSED BY A ROTARY JOINT*

- Two rigid bodies, $\{i-1\}$ and $\{i\}$, connected by a rotary (R) joint (see figure 9)
- $\{i\}$ can rotate by (unknown) angle θ_i about $\hat{\mathbf{k}}$ with respect to $\{i-1\}$ – **2 constraints** in ${}^0_i[R] = {}^0_{i-1}[R]\ {}^{i-1}_i[R(\hat{\mathbf{k}}, \theta_i)]$

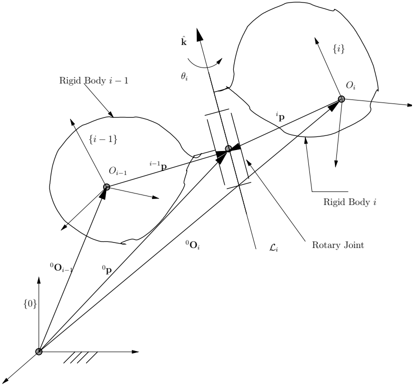


Figure 9: A rotary joint

- For a common point P on the rotation axis along line $\mathscr{L}_i$ ${}^0\mathbf{p} = {}^0\mathbf{O}_{i-1} + {}^0_{i-1}[R]{}^{i-1}\mathbf{p} = {}^0\mathbf{O}_i + {}^0_i[R]{}^i\mathbf{p}$ – **3 constraints**

Representation of joints*

Constraints imposed by a rotary joint in a loop*

- Rotary joint in a loop – 2 ends $\{L\}$ and $\{R\}$.

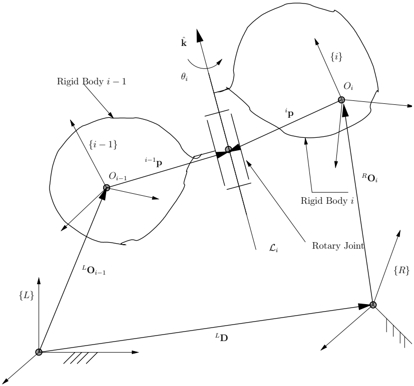


Figure 10: A rotary joint in a loop

- Two constraint: ${}^L_i[R] = {}^L_{i-1}[R]\,{}^{i-1}_i[R(\hat{\mathbf{k}}, \theta_i)] = {}^L_R[R]^R_i[R]$
- Three constraints: ${}^L\mathbf{p} = {}^L\mathbf{O}_{i-1} + {}^L_{i-1}[R]^{i-1}\mathbf{p} = {}^L\mathbf{D} + {}^R\mathbf{O}_i + {}^R_i[R]^i\mathbf{p}$

REPRESENTATION OF JOINTS*

CONSTRAINTS IMPOSED BY A PRISMATIC JOINT*

- Two rigid bodies, $\{i-1\}$ and $\{i\}$, connected by a sliding/prismatic (P) joint
- 3 constraints in ${}^0_i[R] = {}^0_{i-1}[R]$
- $\{i\}$ can slide by d_i, along $\mathscr{L}_i$, with respect to $\{i-1\}$

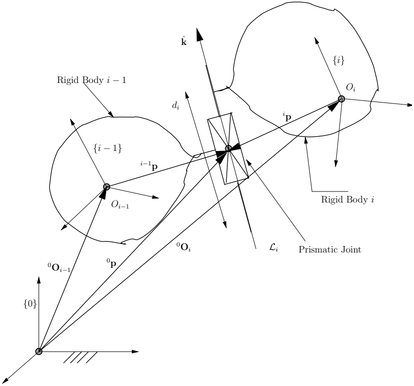


Figure 11: A prismatic joint

- 2 constraints (d_i is a variable): ${}^0\mathbf{O}_{i-1} + {}^0_{i-1}[R]^{i-1}\mathbf{p} + d_i\hat{\mathbf{k}} = {}^0\mathbf{O}_i + {}^0_i[R]^i\mathbf{p}$

- Spherical(S) or ball and socket joint allows three rotations.
- S joint can be represented as 3 intersecting rotary(R) joints.

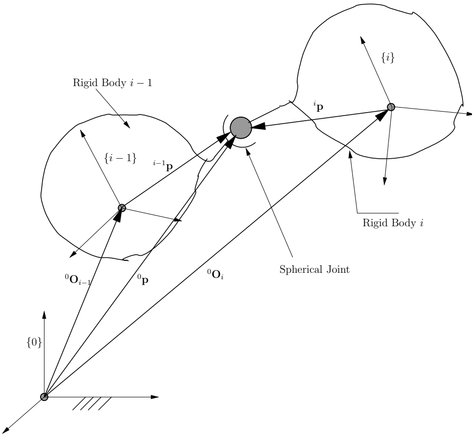


Figure 12: A spherical joint

- 3 constraints: $^{0}\mathbf{p} = ^{0}\mathbf{O}_{i-1} + ^{0}_{i-1}[R]^{i-1}\mathbf{p} \;\; = ^{0}\mathbf{O}_{i} + ^{0}_{i}[R]^{i}\mathbf{p}$

- Common in many parallel manipulators.
- Equivalent to two intersecting rotary (R) joints $\rightarrow$ Two degrees of freedom $\rightarrow$ 4 constraints.
- For a common point P at the intersection of two rotation axes ${}^0\mathbf{p} = {}^0\mathbf{O}_{i-1} + {}^0_{i-1}[R]\,{}^{i-1}\mathbf{p} = {}^0\mathbf{O}_i + {}^0_i[R]\,{}^i\mathbf{p} \rightarrow$ 3 constraints.
- ${}^0_i[R] = {}^0_{i-1}[R]\;{}^{i-1}_i[R(\hat{\mathbf{k}}_1, \theta_{1_i})][R(\hat{\mathbf{k}}_2, \theta_{2_i})] \rightarrow$ 1 constraint.
- θ_{1_i} and θ_{2_i} – unknown rotations about $\hat{\mathbf{k}}_1$ and $\hat{\mathbf{k}}_2$

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Constraints imposed by a spherical-spherical (S-S) joint pair

- The S-S pair appear in many parallel manipulators.
- Distance between two spherical joint is *constant*.

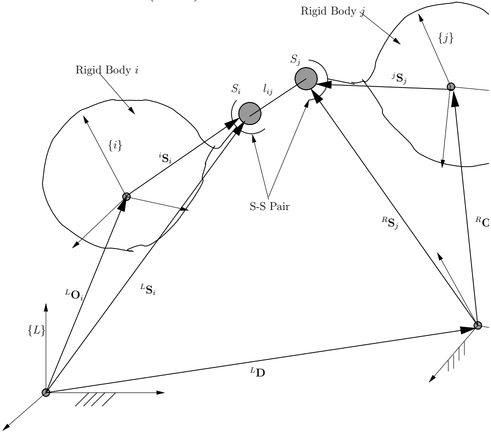

Figure 13: A S-S pair in a loop

- One constraint: $({}^L\mathbf{S}_i - ({}^L\mathbf{D} + {}^R\mathbf{S}_j)) \cdot ({}^L\mathbf{S}_i - ({}^L\mathbf{D} + {}^R\mathbf{S}_j)) = l_{ij}^2$, l_{ij} is a constant.

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- A rigid body in $\Re^3$ has 6 degrees of freedom $\rightarrow$ 3 rotation + 3 translation $\rightarrow$ 6 parameters (see Lecture 1)
- For links connected by rotary (R) and prismatic (P), possible to use 4 parameters – Denavit-Hartenberg (D-H)parameters (see Denavit & Hartenberg, 1955).
- 4 parameters since lines related to rotary(R) and prismatic (P) joint axis are used.
- For multi- degree-of-freedom joints $\rightarrow$ use equivalent number of one-degree-of-freedom joints.

Representation of links

Introduction

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- A word about *Conventions* – several ways to derive D-H parameters!
- Convention used:
  1. Coordinate system $\{i\}$ is attached to the link i.
  2. Origin of $\{i\}$ lies on the joint axis i – link i is *"after"* joint i.
  3. *"after"* for serial manipulators – numbers increasing from fixed $\{0\} \rightarrow$ link 1 $\{1\} \rightarrow \ldots \rightarrow$ free end $\{n\}$.
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Representation of links

The D-H parameters – assignment of coordinate axis

- Three intermediate links – $\{i-1\}$, $\{i\}$ and $\{i+1\}$.
- Rotary joint axis – labeled $\hat{\mathbf{Z}}_{i-1}$, $\hat{\mathbf{Z}}_i$ and $\hat{\mathbf{Z}}_{i+1}$

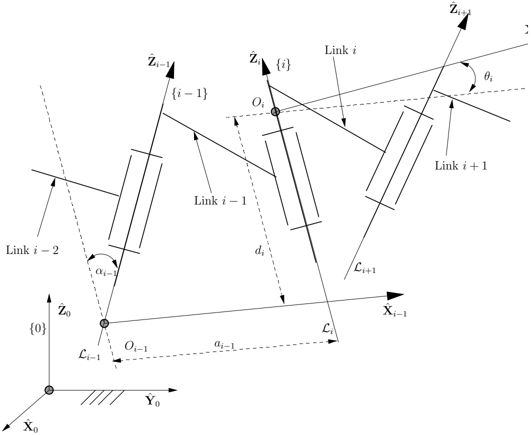

Figure 14: Intermediate links and D-H parameters

REPRESENTATION OF LINKS

THE D-H PARAMETERS – ASSIGNMENT OF COORDINATE AXIS (CONTD.)

- For coordinate system $\{i-1\}$
  - $\hat{\mathbf{Z}}_{i-1}$ is along joint axis $i-1$
  - $\hat{\mathbf{X}}_{i-1}$ is **chosen** along the *common* perpendicular between lines $\mathscr{L}_{i-1}$ and $\mathscr{L}_i$
  - $\hat{\mathbf{Y}}_{i-1} = \hat{\mathbf{Z}}_{i-1} \times \hat{\mathbf{X}}_{i-1}$ – Right-handed coordinate system.
  - The origin O_{i-1} is the point of intersection of the mutual perpendicular line and the line $\mathscr{L}_{i-1}$.
- For coordinate system $\{i\}$: $\hat{\mathbf{Z}}_i$ is along the joint axis i, $\hat{\mathbf{X}}_i$ is along the common perpendicular between $\hat{\mathbf{Z}}_i$ and $\hat{\mathbf{Z}}_{i+1}$, and the origin of $\{i\}$, O_i, is the point of intersection of the line along $\hat{\mathbf{X}}_i$ and line along $\hat{\mathbf{Z}}_i$ (see Figure 14).

Representation of Links

The D-H parameters – Assignment of coordinate axis (Contd.)

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Representation of Links

The D-H parameters for link i

- *Twist angle* α_{i-1} – Note subscript: Twist angle for link i has subscript $i-1$!.
  - Angle between joints axis $i-1$ and i & measured about $\hat{\mathbf{X}}_{i-1}$ using right-hand rule (see figure 14).
  - Signed quantity between 0 and $\pm\pi$ radians.
- *Link length* a_{i-1} – Note subscript: Link length for link i is a_{i-1}!.
  - Distance between joints axis $i-1$ and i & measured along $\hat{\mathbf{X}}_{i-1}$ (see figure 14)
  - Always a positive quantity.
- *Link offset* d_i.
  - Measured along $\hat{\mathbf{Z}}_i$ **from** $\hat{\mathbf{X}}_{i-1}$ **to** $\hat{\mathbf{X}}_i$ – can be positive or negative.
  - Joint i is rotary $\rightarrow d_i$ is constant.
  - Joint i is prismatic $\rightarrow d_i$ is the *joint variable* (see figure 14).
- *Rotation angle* θ_i.
  - Angle between $\hat{\mathbf{X}}_{i-1}$ and $\hat{\mathbf{X}}_i$ measured about $\hat{\mathbf{Z}}_i$ using right-hand rule – between 0 and $\pm\pi$ radians.
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Representation of Links

The D-H parameters for link i – Special cases

- Two consecutive joints axis parallel $\rightarrow$ infinitely many common perpendiculars.
  - $\alpha_{i-1} = 0, \pi$, a_{i-1} distance along *any* of the common perpendiculars.
  - Joint i is rotary (R) $\rightarrow$ d_i is taken as zero.
  - Joint i is prismatic (P) $\rightarrow$ θ_i is taken as zero.
  - Consecutive P joints and parallel $\rightarrow$ two d's not independent!
- Consecutive joints axis intersecting $\rightarrow$ two choices of common perpendicular and $a_{i-1} = 0$.
- First link $\{1\}$: choice of $\hat{\mathbf{Z}}_0$ and thereby $\hat{\mathbf{X}}_1$ is arbitrary.
  - R joint $\rightarrow$ choose $\{0\}$ and $\{1\}$ coincident & $\alpha_{i-1} = a_{i-1} = 0$, $d_i = 0$.
  - P joint $\rightarrow$ choose $\{0\}$ and $\{1\}$ parallel & $\alpha_{i-1} = a_{i-1} = \theta_i = 0$.
- Last link $\{n\}$: $\hat{\mathbf{Z}}_{n+1}$ not defined.
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Representation of links

The D-H parameters for link i – Special cases

- Two consecutive joints axis parallel $\rightarrow$ infinitely many common perpendiculars.
  - $\alpha_{i-1} = 0, \pi$, a_{i-1} distance along *any* of the common perpendiculars.
  - Joint i is rotary (R) $\rightarrow$ d_i is taken as zero.
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- Consecutive joints axis intersecting $\rightarrow$ two choices of common perpendicular and $a_{i-1} = 0$.
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- The 4×4 transformation matrix **for link** i with respect to link $i-1$ is

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{}^{i-1}_{i}[T] = \begin{pmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & a_{i-1} \\ s_{\theta_i} c_{\alpha_{i-1}} & c_{\theta_i} c_{\alpha_{i-1}} & -s_{\alpha_{i-1}} & -s_{\alpha_{i-1}} d_i \\ s_{\theta_i} s_{\alpha_{i-1}} & c_{\theta_i} s_{\alpha_{i-1}} & c_{\alpha_{i-1}} & c_{\alpha_{i-1}} d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{10}
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- ${}^{i-1}_{i}[T]$ is a function *only* one joint variable – d_i or θ_i.
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Summary

- Two elements of robots – links and joints.
- Joints allow relative motion between connected links $\rightarrow$ joints impose constraints.
  - Serial robots mainly use one- degree-of-freedom rotary (R) and prismatic (P) *actuated* joints.
  - Parallel and hybrid robots use *passive* multi- degree-of-freedom joints and *actuated* one- degree-of-freedom joints.
- One degree-of-freedom R and P joints represented by lines along joint axis – $\hat{Z}$ is along joint axis.
- Formulation of constraints imposed by various kinds of joints.
- Link is a rigid body in 3D space $\rightarrow$ represented by 4 Denavit-Hartenberg (D-H) parameters.
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OUTLINE

1. CONTENTS
2. LECTURE 1
   - Mathematical Preliminaries
   - Homogeneous Transformation
3. LECTURE 2
   - Elements of a robot – Joints
   - Elements of a robot – Links
4. LECTURE 3
   - Examples of D-H Parameters & Link Transformation Matrices
5. LECTURE 4
   - Elements of a robot – Actuators & Transmission
6. LECTURE 5
   - Elements of a Robots – Sensors
7. MODULE 2 – ADDITIONAL MATERIAL
   - Problems, References, and Suggested Reading

- Steps to obtain D-H parameters for a n link serial manipulator (see Lecture 2 for details)
  1. Label joint axis from 1 (fixed) to n free end.
  2. Assign $\hat{\mathbf{Z}}_i$ to joint axis i, $i = 1, 2, ..., n$.
  3. Obtain mutual perpendiculars between lines along $\hat{\mathbf{Z}}_{i-1}$ and $\hat{\mathbf{Z}}_i$ – $\hat{\mathbf{X}}_{i-1}$.
  4. Origin O_{i-1} on joint axis $i-1$.
  5. Handle special cases – a) consecutive joint axes parallel and perpendicular, b) first and last link
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- For each link i obtain ${}^{i-1}_{i}[T]$ using equation (10).
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- All 3 rotary joint axis $\hat{\mathbf{Z}}_i$ parallel and pointing out.
- $\{0\}$ – $\hat{\mathbf{Z}}_0$ is pointing out, $\hat{\mathbf{X}}_0$ and $\hat{\mathbf{Y}}_0$ pointing to the right and top, respectively.
- Origin O_0 is coincident with O_1 shown in figure.

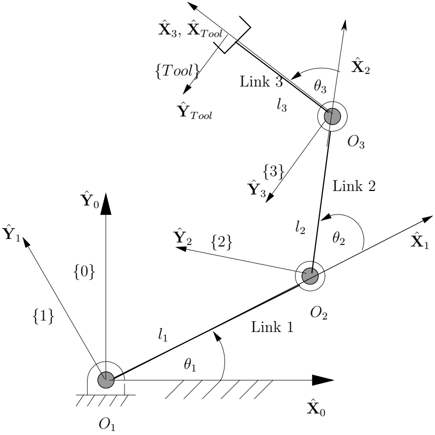

Figure 15: The planar 3R manipulator

- For $\{1\}$ origin O_1 and $\hat{\mathbf{Z}}_1$ are coincident with O_0 and $\hat{\mathbf{Z}}_0$.
- $\hat{\mathbf{X}}_1$ and $\hat{\mathbf{Y}}_1$ are coincident with $\hat{\mathbf{X}}_0$ and $\hat{\mathbf{Y}}_0$ when θ_1 is zero.
- $\hat{\mathbf{X}}_1$ is along the mutual perpendicular between $\hat{\mathbf{Z}}_1$ and $\hat{\mathbf{Z}}_2$.
- $\hat{\mathbf{X}}_2$ is along the mutual perpendicular between $\hat{\mathbf{Z}}_2$ and $\hat{\mathbf{Z}}_3$.
- For $\{3\}$, $\hat{\mathbf{X}}_3$ is aligned to $\hat{\mathbf{X}}_2$ when $\theta_3 = 0$.
- O_2 is located at the intersection of the mutual perpendicular along $\hat{\mathbf{X}}_2$ and $\hat{\mathbf{Z}}_2$.
- O_3 is chosen such that d_3 is zero.
- The origins and the axes of $\{1\}$, $\{2\}$, and $\{3\}$ are shown in figure 15.

The D-H Table

From the assigned axes and origins, the D-H parameters are as follows:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	l_1	0	θ_2
3	0	l_2	0	θ_3

- l_1 and l_2 are the link lengths and $\theta_i,\ i = 1,2,3$ are rotary joint variables (see figure 15).
- Length of the end-effector l_3 does not appear in the table – Recall: D-H parameters are till the origin which is at the beginning of a link!!
- For end-effector/parallel jaw gripper, attach frame, $\{Tool\}$, as follows:
  - Axis of $\{Tool\}$ parallel to $\{3\}$ $\rightarrow$ ${}^{3}_{Tool}[R]$ is identity matrix.
  - Origin of $\{Tool\}$ at the mid-point of the parallel jaw gripper, at a distance of l_3 from O_3 along $\hat{\mathbf{X}}_3$.

- Substitute elements of *first row* of D-H table in equation (10) to get

$$ {}^{0}_{1}[T] = \begin{pmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

- The *second row* of the D-H table gives ${}^{1}_{2}[T] = \begin{pmatrix} c_2 & -s_2 & 0 & l_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

- Finally, *third row* of D-H table gives ${}^{2}_{3}[T] = \begin{pmatrix} c_3 & -s_3 & 0 & l_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

- The ${}^{3}_{Tool}[T]$ is obtained as ${}^{3}_{Tool}[T] = \begin{pmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- To obtain ${}^{0}_{3}[T]$ multiply ${}^{0}_{1}[T]\ {}^{1}_{2}[T]\ {}^{2}_{3}[T]$ and get

$$
{}^{0}_{3}[T] = \begin{pmatrix} c_{123} & -s_{123} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{123} & c_{123} & 0 & l_1 s_1 + l_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

- To obtain ${}^{0}_{Tool}[T]$, multiply ${}^{0}_{3}[T]\ {}^{3}_{Tool}[T]$

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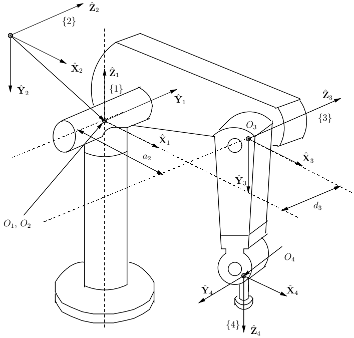
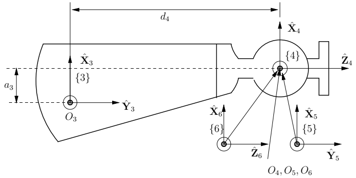
{}^{0}_{Tool}[T] = \begin{pmatrix} c_{123} & -s_{123} & 0 & l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ s_{123} & c_{123} & 0 & l_1 s_1 + l_2 s_{12} + l_3 s_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

The PUMA 560 – six- degree-of-freedom manipulator with all rotary joints – figure 16 shows assigned coordinate systems.



(a) The PUMA 560 manipulator



(b) PUMA 560 - forearm and wrist

Figure 16: The PUMA 560 manipulator

The D-H parameters for the PUMA 560 manipulator (see figure 16) are

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$-\pi/2$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-\pi/2$	a_3	d_4	θ_4
5	$\pi/2$	0	0	θ_5
6	$-\pi/2$	0	0	θ_6

Note: θ_i, $i = 1, 2, ..., 6$ are the six joint variables.

- Substituting elements of row #1 of D-H table and using equation (10)

$$ {}^0_1[T] = \begin{pmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

- From row # 2 – ${}^1_2[T] = \begin{pmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

- From row # 3 – ${}^2_3[T] = \begin{pmatrix} c_3 & -s_3 & 0 & a_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

- From row # 4 – $^{3}_{4}[T] = \begin{pmatrix} c_4 & -s_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s_4 & -c_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- From row # 5 – $^{4}_{5}[T] = \begin{pmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- From row # 6 – $^{5}_{6}[T] = \begin{pmatrix} c_6 & -s_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_6 & -c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

- $\{3\}$ with respect to $\{0\}$

$$ {}^0_3[T] = {}^0_1[T]{}^1_2[T]{}^2_3[T] = \begin{pmatrix} c_1 c_{23} & -c_1 s_{23} & -s_1 & a_2 c_1 c_2 - d_3 s_1 \\ s_1 c_{23} & -s_1 s_{23} & c_1 & a_2 s_1 c_2 + d_3 c_1 \\ -s_{23} & -c_{23} & 0 & -a_2 s_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

- $\{6\}$ with respect to $\{3\}$ ${}^3_6[T] = {}^3_4[T]{}^4_5[T]{}^5_6[T] =$

$$ \begin{pmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_4 c_6 & -c_4 s_5 & a_3 \\ s_5 c_6 & -s_5 s_6 & c_5 & d_4 \\ -s_4 c_5 c_6 - c_4 s_6 & s_4 c_5 s_6 - c_4 c_6 & s_4 s_5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

- Can obtain any required link transformation matrix once D-H table known!

- $\{3\}$ with respect to $\{0\}$

$$
{}^0_3[T] = {}^0_1[T]{}^1_2[T]{}^2_3[T] = \begin{pmatrix} c_1 c_{23} & -c_1 s_{23} & -s_1 & a_2 c_1 c_2 - d_3 s_1 \\ s_1 c_{23} & -s_1 s_{23} & c_1 & a_2 s_1 c_2 + d_3 c_1 \\ -s_{23} & -c_{23} & 0 & -a_2 s_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

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$$
\begin{pmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_4 c_6 & -c_4 s_5 & a_3 \\ s_5 c_6 & -s_5 s_6 & c_5 & d_4 \\ -s_4 c_5 c_6 - c_4 s_6 & s_4 c_5 s_6 - c_4 c_6 & s_4 s_5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

- Can obtain any required link transformation matrix once D-H table known!

- $\{3\}$ with respect to $\{0\}$

$$ {}^0_3[T] = {}^0_1[T]{}^1_2[T]{}^2_3[T] = \begin{pmatrix} c_1 c_{23} & -c_1 s_{23} & -s_1 & a_2 c_1 c_2 - d_3 s_1 \\ s_1 c_{23} & -s_1 s_{23} & c_1 & a_2 s_1 c_2 + d_3 c_1 \\ -s_{23} & -c_{23} & 0 & -a_2 s_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

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$$ \begin{pmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_4 c_6 & -c_4 s_5 & a_3 \\ s_5 c_6 & -s_5 s_6 & c_5 & d_4 \\ -s_4 c_5 c_6 - c_4 s_6 & s_4 c_5 s_6 - c_4 c_6 & s_4 s_5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

- Can obtain any required link transformation matrix once D-H table known!

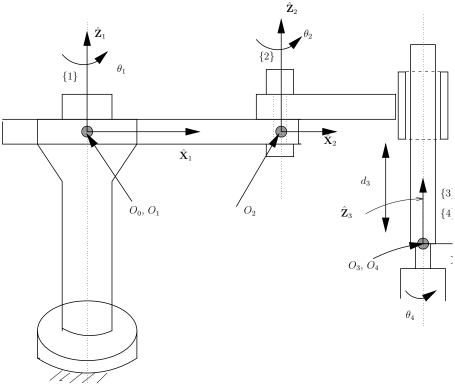
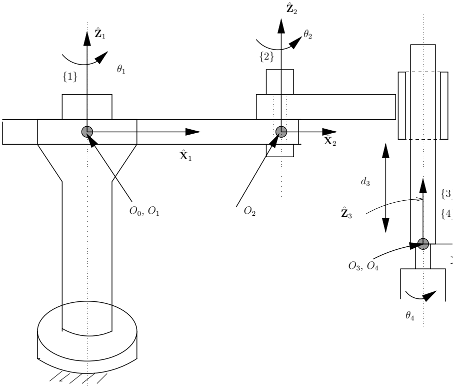


Figure 17: A SCARA manipulator

- SCARA – *S*elective *C*ompliance *A*ssembly *R*obot *A*rm
- Very popular for robotic assembly
- Capability of desired compliance and rigidity in selected directions.
- Three rotary(R) joint and one prismatic (P) joint.
- A 4 degree-of-freedom manipulator



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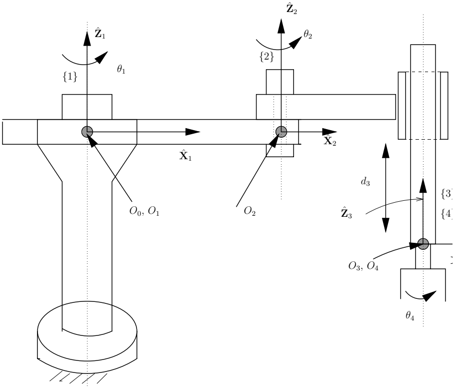

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- Very popular for robotic assembly
- Capability of desired compliance and rigidity in selected directions.
- Three rotary(R) joint and one prismatic (P) joint.
- A 4 degree-of-freedom manipulator

- {0} and {1} have same origin & Origins of {3} and {4} chosen at the base of the parallel jaw gripper.
- Directions of $\hat{\mathbf{Z}}_3$ chosen pointing upward (see figure 17).
- Note: Actual SCARA manipulator has ball-screw at the third joint – assume P joint.
- D-H Table for SCARA robot

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	a_1	0	θ_2
3	0	a_2	$-d_3$	0
4	0	0	0	θ_4

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2	0	a_1	0	θ_2
3	0	a_2	$-d_3$	0
4	0	0	0	θ_4

Using equation (10) and the D-H table, link transforms can be obtained as

$$
{}^0_1[T] = \begin{pmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad {}^1_2[T] = \begin{pmatrix} c_2 & -s_2 & 0 & a_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^2_3[T] = \begin{pmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad {}^3_4[T] = \begin{pmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

The transformation matrix ${}^0_4[T]$ is

$$
{}^0_4[T] = {}^0_1[T]{}^1_2[T]{}^2_3[T]\;{}^3_4[T] = \begin{pmatrix} c_{124} & -s_{124} & 0 & a_1c_1 + a_2c_{12} \\ s_{124} & c_{124} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & -d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

- Extend idea of D-H parameters to a closed-loop mechanism/parallel manipulator.
- Key idea – "break" a parallel manipulator into serial manipulators.
- Obtain D-H parameters for serial manipulators.
  - Several ways to "break"
  - Choose one that leads to "simple" serial manipulators
- During analysis – combine serial manipulators using constraints

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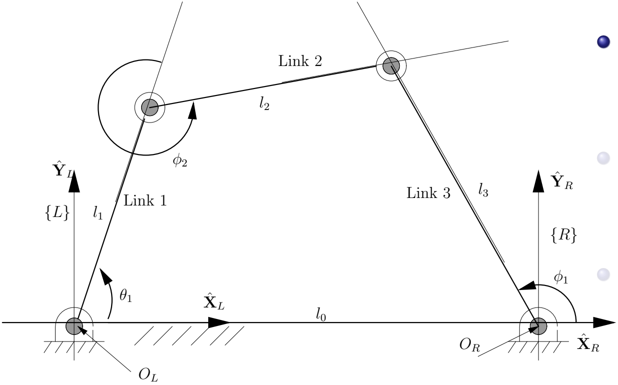
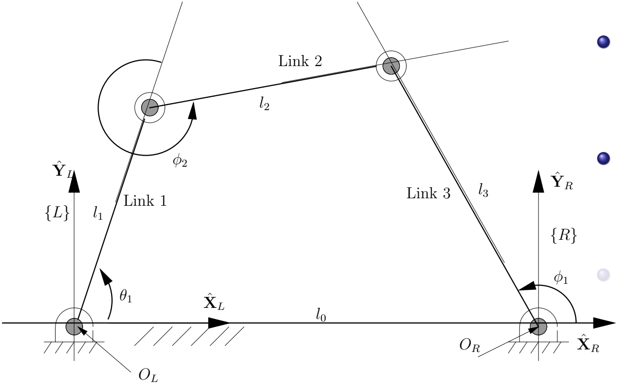


Figure 18: A planar four-bar mechanism

- Rotary joints 1 and 4 fixed to ground at *two* places.
- "Break" 4-bar mechanism into *two* serial manipulators.
- Break at joint 3 $\rightarrow$ A 2R planar manipulator + a 1R manipulator



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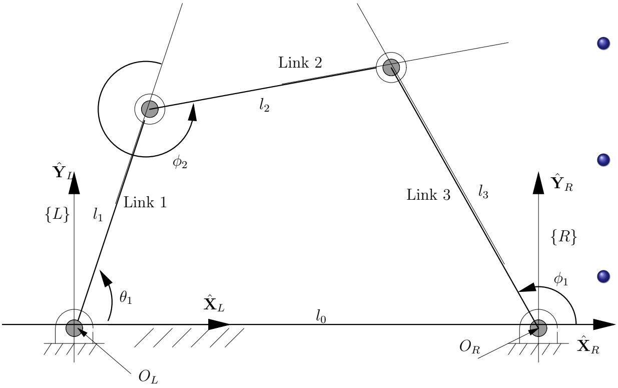

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- For 2R planar manipulator – D-H parameter with respect to $\{L\}$

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	l_1	0	ϕ_2

- For 1R planar manipulator – D-H parameters with respect to $\{R\}$

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	ϕ_1

- The constant transform ${}^{L}_{R}[T]$ is known.

Parallel Manipulators: three-degree-of-freedom parallel manipulator

Description

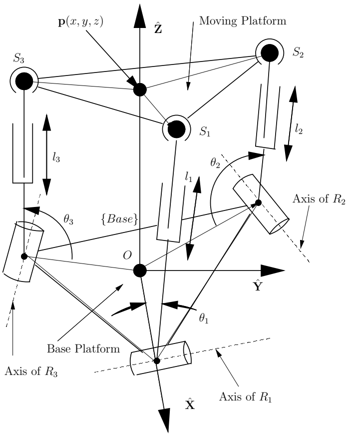
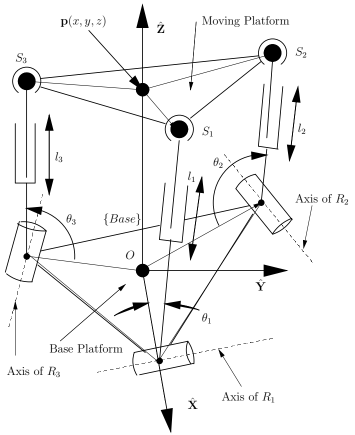


Figure 19: A three- degree-of-freedom parallel manipulator

- Spatial 3- degree-of-freedom parallel manipulator.
- Top platform + fixed bottom platform.
- Three legs – each leg R-P-S configuration.
- Three P joints actuated.
- First proposed by Lee and Shah (1988) as a parallel wrist.

PARALLEL MANIPULATORS: THREE-DEGREE-OF-FREEDOM PARALLEL MANIPULATOR

DESCRIPTION



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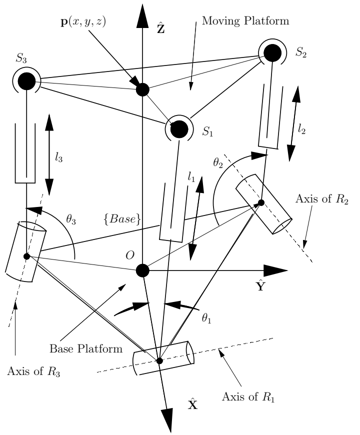

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- Top platform + fixed bottom platform.
- Three legs – each leg R-P-S configuration.
- Three P joints actuated.
- First proposed by Lee and Shah (1988) as a parallel wrist.

- D-H parameter for first leg with respect to $\{L_1\}$

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	ϕ_1
2	$-\pi/2$	0	l_1	0

- D-H parameter for all R-P-S leg same except the reference coordinate system.
- $\{L_1\}$, $\{L_2\}$, and $\{L_3\}$ are at the three rotary joints R_1, R_2, and R_3, respectively.
- $\{Base\}$ is located at the centre of the base platform & ${}^{Base}_{L_i}[T]$, $i = 1, 2, 3$, are constant and known.
- Note: The angle θ_1 shown in figure is same as $\pi/2 - \phi_1$.

Link transformation matrices

- ${}^{L_1}_{1}[T] = \begin{pmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad {}^{1}_{2}[T] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- ${}^{2}_{S_1}[T]$ is an identity matrix – $\{S_1\}$ is located at the centre of the spherical joint and parallel to $\{2\}$.
- ${}^{Base}_{S_1}[T] = {}^{Base}_{L_1}[T]{}^{L_1}_{1}[T]{}^{1}_{2}[T]{}^{2}_{S_1}[T]$
- Location of spherical joint S_1 with respect to $\{Base\}$ from ${}^{Base}_{S_1}[T]$ ${}^{Base}\mathbf{S}_1 = (b - l_1 \cos\theta_1, 0, l_1 \sin\theta_1)^T$, b is the distance of R_1 from the origin of $\{Base\}$ (see figure 19).
- Location of S_2 and S_3
  ${}^{Base}\mathbf{S}_2 = (-\frac{b}{2} + \frac{1}{2} l_2 \cos\theta_2, \frac{\sqrt{3}b}{2} - \frac{\sqrt{3}l_2}{2}\cos\theta_2, l_2 \sin\theta_2)^T$
  ${}^{Base}\mathbf{S}_3 = (-\frac{b}{2} + \frac{1}{2} l_3 \cos\theta_3, -\frac{\sqrt{3}b}{2} + \frac{\sqrt{3}l_3}{2}\cos\theta_3, l_3 \sin\theta_3)^T$

Parallel Manipulators: six- degree-of-freedom parallel manipulator

Description

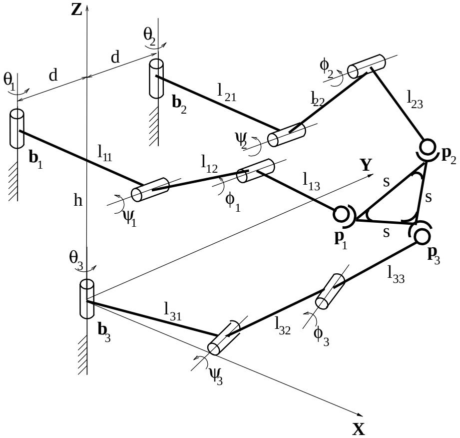
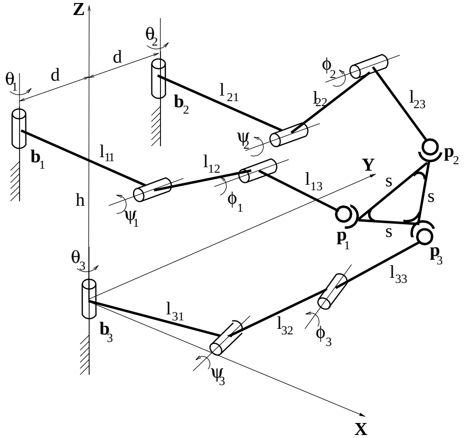


Figure 20: A six- degree-of-freedom parallel (hybrid) manipulator

- Moving platform connected to fixed base by three legs.
- Each leg R-R-R &S at top.
- Model of a three-fingered hand (Salisbury, 1982) gripping an object with point contact and no-slip.
- Each finger has three joints and modeled as R-R-R chain.

PARALLEL MANIPULATORS: SIX- DEGREE-OF-FREEDOM PARALLEL MANIPULATOR

DESCRIPTION



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- Each leg R-R-R &S at top.
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- Each finger has three joints and modeled as R-R-R chain.

- D-H parameters for R-R-R chain.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$\pi/2$	l_{11}	0	ψ_1
3	0	l_{12}	0	ϕ_1

- D-H parameter does not contain last link length l_{13}.
- D-H parameters for three fingers with respect to $\{F_i\}$, $i = 1,2,3$ identical.
- Can obtain transformation matrix ${}^{F_i}_{p_i}[T]$ by matrix multiplication.

LINK TRANSFORMS (CONTD.)

- Position vector of spherical joint i
$$^{F_i}\mathbf{p}_i = \begin{pmatrix} \cos\theta_i(l_{i1} + l_{i2}\cos\psi_i + l_{i3}\cos(\psi_i + \phi_i)) \\ \sin\theta_i(l_{i1} + l_{i2}\cos\psi_i + l_{i3}\cos(\psi_i + \phi_i)) \\ l_{i2}\sin\psi_i + l_{i3}\sin(\psi_i + \phi_i) \end{pmatrix}$$
- With respect to $\{Base\}$, the locations of $\{F_i\}$, $i = 1,2,3$, are known and constant (see Figure (20))
$^{Base}\mathbf{b}_1 = (0, -d, h)^T$ $^{Base}\mathbf{b}_2 = (0, d, h)^T$ $^{Base}\mathbf{b}_3 = (0,0,0)^T$
- Orientation of $\{F_i\}$, $i = 1,2,3$, with respect to $\{Base\}$ are also known - $\{F_1\}$ and $\{F_2\}$ are parallel to $\{Base\}$ and $\{F_3\}$ is rotated by γ about the $\hat{\mathbf{Y}}$ (Not shown in figure!).
- The transformation matrices $^{Base}_{p_i}[T]$ is $^{Base}_{F_1}[T]^0_1[T]^1_2[T]^2_3[T]^3_{p_1}[T]$ – last transformation includes l_{13}.

- Position vector of spherical joint i

$$^{F_i}\mathbf{p}_i = \begin{pmatrix} \cos\theta_i(l_{i1} + l_{i2}\cos\psi_i + l_{i3}\cos(\psi_i + \phi_i)) \\ \sin\theta_i(l_{i1} + l_{i2}\cos\psi_i + l_{i3}\cos(\psi_i + \phi_i)) \\ l_{i2}\sin\psi_i + l_{i3}\sin(\psi_i + \phi_i) \end{pmatrix}$$

- With respect to $\{Base\}$, the locations of $\{F_i\}$, $i = 1, 2, 3$, are known and constant (see Figure (20))
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LINK TRANSFORMS (CONTD.)

- Position vector of spherical joint i
$$ {}^{F_i}\mathbf{p}_i = \begin{pmatrix} \cos\theta_i(l_{i1} + l_{i2}\cos\psi_i + l_{i3}\cos(\psi_i+\phi_i)) \\ \sin\theta_i(l_{i1} + l_{i2}\cos\psi_i + l_{i3}\cos(\psi_i+\phi_i)) \\ l_{i2}\sin\psi_i + l_{i3}\sin(\psi_i+\phi_i) \end{pmatrix} $$
- With respect to $\{Base\}$, the locations of $\{F_i\}$, $i = 1,2,3$, are known and constant (see Figure (20))
${}^{Base}\mathbf{b}_1 = (0,-d,h)^T$ ${}^{Base}\mathbf{b}_2 = (0,d,h)^T$ ${}^{Base}\mathbf{b}_3 = (0,0,0)^T$
- Orientation of $\{F_i\}$, $i = 1,2,3$, with respect to $\{Base\}$ are also known - $\{F_1\}$ and $\{F_2\}$ are parallel to $\{Base\}$ and $\{F_3\}$ is rotated by γ about the $\hat{\mathbf{Y}}$ (Not shown in figure!).
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- With respect to $\{Base\}$, the locations of $\{F_i\}$, $i = 1,2,3$, are known and constant (see Figure (20))
$^{Base}\mathbf{b}_1 = (0,-d,h)^T$ $^{Base}\mathbf{b}_2 = (0,d,h)^T$ $^{Base}\mathbf{b}_3 = (0,0,0)^T$
- Orientation of $\{F_i\}$, $i = 1,2,3$, with respect to $\{Base\}$ are also known - $\{F_1\}$ and $\{F_2\}$ are parallel to $\{Base\}$ and $\{F_3\}$ is rotated by γ about the $\hat{\mathbf{Y}}$ (Not shown in figure!).
- The transformation matrices $^{Base}_{p_i}[T]$ is $^{Base}_{F_1}[T]^0_1[T]^1_2[T]^2_3[T]^3_{p_1}[T]$ – last transformation includes l_{13}.

- Extract the position vector ${}^{Base}\mathbf{p}_1$ from the last column of ${}^{Base}_{F_1}[T]$
$${}^{Base}\mathbf{p}_1 = {}^{Base}\mathbf{b}_1 + {}^{F_1}\mathbf{p}_1 =$$
$$\begin{pmatrix} \cos\theta_1(l_{11} + l_{12}\cos\psi_1 + l_{13}\cos(\psi_1 + \phi_1)) \\ -d + \sin\theta_1(l_{11} + l_{12}\cos\psi_1 + l_{13}\cos(\psi_1 + \phi_1)) \\ h + l_{12}\sin\psi_1 + l_{13}\sin(\psi_1 + \phi_1) \end{pmatrix}$$
- Similarly for second leg
$${}^{Base}\mathbf{p}_2 = \begin{pmatrix} \cos\theta_2(l_{21} + l_{22}\cos\psi_2 + l_{23}\cos(\psi_2 + \phi_2)) \\ d + \sin\theta_2(l_{21} + l_{22}\cos\psi_2 + l_{23}\cos(\psi_2 + \phi_2)) \\ h + l_{22}\sin\psi_2 + l_{23}\sin(\psi_2 + \phi_2) \end{pmatrix}$$
- For third leg
$${}^{Base}\mathbf{p}_3 = [R(\hat{\mathbf{Y}}, \gamma)] \begin{pmatrix} \cos\theta_3(l_{31} + l_{32}\cos\psi_3 + l_{33}\cos(\psi_3 + \phi_3)) \\ \sin\theta_3(l_{31} + l_{32}\cos\psi_3 + l_{33}\cos(\psi_3 + \phi_3)) \\ l_{32}\sin\psi_3 + l_{33}\sin(\psi_3 + \phi_3) \end{pmatrix}$$

LINK TRANSFORMS (CONTD.)

- Extract the position vector ${}^{Base}\mathbf{p}_1$ from the last column of ${}^{Base}_{F_1}[T]$
  ${}^{Base}\mathbf{p}_1 = {}^{Base}\mathbf{b}_1 + {}^{F_1}\mathbf{p}_1 =$
  $$\begin{pmatrix} \cos\theta_1(l_{11} + l_{12}\cos\psi_1 + l_{13}\cos(\psi_1+\phi_1)) \\ -d + \sin\theta_1(l_{11} + l_{12}\cos\psi_1 + l_{13}\cos(\psi_1+\phi_1)) \\ h + l_{12}\sin\psi_1 + l_{13}\sin(\psi_1+\phi_1) \end{pmatrix}$$
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Summary

- D-H parameters obtained for serial manipulators – planar 3R, PUMA 560, SCARA
- D-H parameters for closed-loop and parallel manipulators
  - "Break" parallel manipulator into serial manipulators.
  - Obtain D-H parameters for each serial chain.
  - Examples of 4-bar mechanism, 3- degree-of-freedom and 6-degree-of-freedom parallel manipulators.
- Can extract position vectors of point of interest & orientation of links from link transforms.
- Kinematic analysis, using the concepts presented here, discussed in Module 3 and Module 4.

Outline

1 Contents

2 Lecture 1
- Mathematical Preliminaries
- Homogeneous Transformation

3 Lecture 2
- Elements of a robot – Joints
- Elements of a robot – Links

4 Lecture 3
- Examples of D-H Parameters & Link Transformation Matrices

5 Lecture 4
- Elements of a robot – Actuators & Transmission

6 Lecture 5
- Elements of a Robots – Sensors

7 Module 2 – Additional Material
- Problems, References, and Suggested Reading

- Actuators are required to move joints, provide power and do work.
- Serial robot actuators must be of low weight – actuators of distal links need to be moved by actuators near the base.
- Parallel robots – often actuators are at the base.
- Actuators drive a joint through a *transmission* device
- Three commonly used types of actuators:
  - Hydraulic
  - Pneumatic
  - Electric motors

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Actuators for Robots

Introduction

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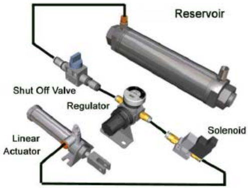
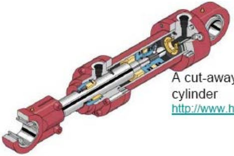
Actuators for Robots



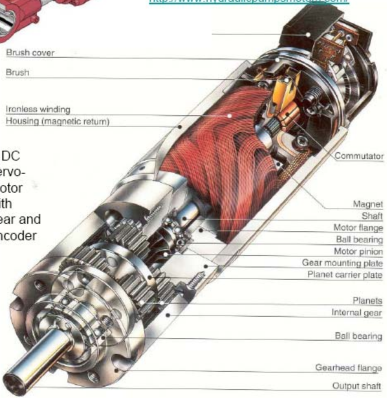
A typical arrangement of components in pneumatic set-up
http://www.gearseds.com/



A cut-away section of a hydraulic cylinder
http://www.hydraulicpumpsmotors.com/

Inside details of a pneumatic actuator
http://www.baltecorporation.com/

A DC servo-motor with gear and encoder



Source: http://www.hocdelam.org/vn/category/ho-tro/robotandcontrol/

Figure 21: Examples of actuators used in robots

Actuators for Robots
Hydraulic Actuators

- Early industrial robots driven by hydraulic actuators.
- Pump supplies high-pressure fluid (typically oil) to linear cylinders, rotary vane actuators or hydraulic motors at the joint!
- Large force capabilities.
- Large power-weight ratio – the pump, electric motor driving the pump, accumulator etc. stationary and not considered in the weight calculation!
- Control is by means of on/off solenoid valves or servo-valves controlled electronically.
- The entire system – electric motor, pump, accumulator, cylinders etc. – is bulky and often expensive – limited to 'big' robots.

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- Similar to hydraulic actuators – working fluid is air.
- Similar to hydraulic actuators, air is supplied from a compressor to cylinders and flow of air is controlled by solenoid or servo controlled valves.
- Less force and power capabilities.
- Less expensive than hydraulic drives.
- Chosen where electric drives are discouraged or for safety or environmental reasons such as in pharmaceutical and food packaging industries.
- Closed-loop servo-controlled manipulators have been developed for many applications.

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Pneumatic Actuators

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- Air used in pneumatic actuators is clean and safe.
- Oil in hydraulic actuator can be health and fire hazard especially if there is a leakage.
- Pneumatic actuators are typically light-weight, portable and faster.
- Air is compressible (oil is incompressible) and hence pneumatic actuators are 'harder' to control.
- Hydraulic actuator have the largest force/power density compared to *any* actuator.
- With compressors, accumulators and other components, the space requirement is larger than electric actuators.

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- Electric or electromagnetic actuators widely used in robots.
- Readily available in wide variety of shape, sizes, power and torque range.
- Very easily mounted and/or connected with transmission elements such as gears, belts and timing chains.
- Amenable for modern day digital control.
- Main types of electric actuators:
    - Stepper motors
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- Used in 'small' robots with small payload and "low" speeds.
- Stepper motor are of permanent magnet, hybrid or variable reluctance type.
- Actuated by a sequence of pulses – For a single pulse, rotor rotates by a known step such that poles on stator and rotor are aligned.
- Typical step size is 1.8° or 0.9°.
- Speed and direction can be controlled by frequency of pulses.
- Can be used in open-loop as *cumulative error* and maximum error is one step!
- Micro-stepping possible with closed-loop feedback control.

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Actuators for Robots

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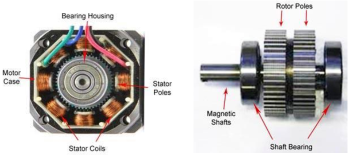
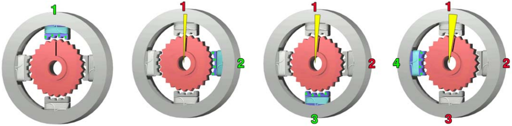


Parts of a Stepper Motor

http://www.engineersgarage.com/articles/stepper-motors



http://www.societyofrobots.com/member_tutorials/node/28

Variable Reluctance (VR) Stepper Motors
Number of teeth in the inner rotor (permanent magnet) is ***different*** than the number of teeth in stator.

1) Electro-magnet **1** is activated → Rotor rotates up such that nearest teeth line up.
2) Electro-magnet **1** is deactivated and **2** is turned on → Rotor rotates such that nearest teeth line up – rotation is by a ***step*** (designed amount) of typically **1.8 or 0.9 degrees**.
3) Electro-magnet **2** is deactivated and **3** is turned on → Rotor rotates by another step.
4) Electro-magnet **3** is deactivated and **4** is turned on and cycle repeated.

Permanent Magnet (PM) Stepper Motors – Similar to VR but rotor is radially magnetized.

Hybrid Stepper Motors – Combines best features of VR and PM stepper motors.

Figure 22: Stepper components and working principle

- Typically stepper motors have *two phases*.
- Four stepping mode
  - Wave drive – Only one phase/winding is on/energized → Torque output is smaller.
  - Full Step drive – Both phases are on at the same time → rated performance.
  - Half Step drive – Combine wave and full-step drive → angular movement half of first two.
  - Micro-stepping – Current is varying continuously → smaller than 1.8 or 0.9 degree step size, lower torque.
- Choice of a stepper motor based on:
  - Load, friction and inertia – higher load can cause slipping!
  - Torque-speed curve and quantities such as holding torque, pull-in and pull-out curve.
  - Torque-speed characteristic determined by the drive – Biploar chopper drives are best in terms of torque-speed performance.
  - Maximum slew-rate: maximum operating frequency with no load (related to maximum speed).

Actuators for Robots

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- Rotor is a permanent magnet and stator is a coil.
- Permanent magnets with rare earth materials (Samarium-cobalt, Neodymium) can provide large magnetic fields and hence high torques.
- Commutation done using brushes or in brushless motor using Hall-effect sensors and electronics.
- Widely available in large range of shape, sizes, power and torque range and low cost.
- Easy to control with optical encoder/tacho-generators mounted in-line with rotor.
- Brushless AC and DC servo-motors have low friction, low maintenance, low cost and are robust.

Actuators for Robots

Electric Actuators – DC/AC Servo-motors

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- Commutation done using brushes or in brushless motor using Hall-effect sensors and electronics.
- Widely available in large range of shape, sizes, power and torque range and low cost.
- Easy to control with optical encoder/tacho-generators mounted in-line with rotor.
- Brushless AC and DC servo-motors have low friction, low maintenance, low cost and are robust.

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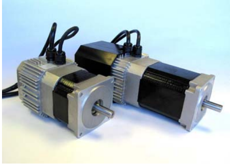
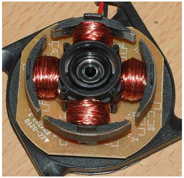
ACTUATORS FOR ROBOTS



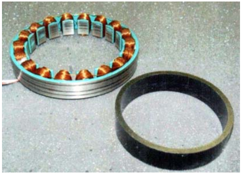
http://www.brushlessdcmotorparts.info



http://www.rc-book.com/wiki
/brushless-electric-motor

Small RC Servo motors
http://www.drivecontrol-details.info
/rc-servo-motor.html

Slotted-brushless DC Servo motors
http://www.motioncontroltips.com/



Direct drive motor, Applimotion, Inc
http://news.thomasnet.com/
company_detail.html?cid=20082162

Brushless Hub motor for E-bike
http://visforvoltage.org/
system-voltage/49-60-volts

Figure 23: Examples of DC servo motors

- See also Wikipedia entry on electric motors.

Actuators for Robots

Model of a DC permanent magnet motor

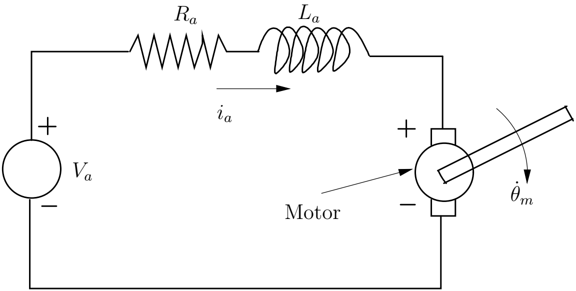


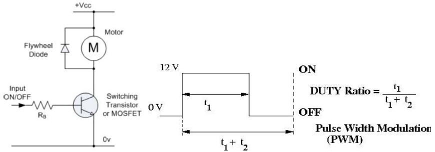
Figure 24: Model of a permanent magnet DC servo-motor

- Rotor – permanent magnet
- Stator – Armature coil with resistance R_a and inductance L_a.
- Applied voltage V_a, i_a current in coil.
- Rotation speed of motor $\dot{\theta}_m$ – mechanical part.

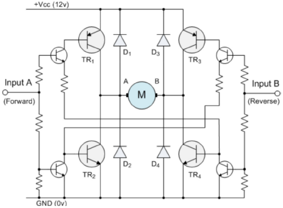
- Torque developed $T_m = K_t \; i_a$ – K_t is constant.
- Back-emf $V = K_g \; \dot{\theta}_m$ – K_g is constant.
- Motor dynamics can be modeled as first-order ODE

$$L_a \dot{i}_a + R_a i_a + K_g \dot{\theta}_m = V_a$$

- *Mechatronic* model – Mechanical + Electrical/Electronics



PWM Circuit for **One-direction – Control speed by varying duty ratio**



H – Bridge: Four switches using MOSFETS and diodes

Input **A** and **B** and the diagonal transistors can be used to change direction.

Input A	Input B	Motor Function
Transistor TR1 & TR4	Transistor TR2 & TR4	
0	**0**	Motor OFF
1	**0**	Forward direction
0	**1**	Reverse direction
1	**1**	Not Allowed

PWM and H-Bridge control both speed and direction of motor

Figure 25: PWM and H-bridge for DC servo motors

Transmissions used in Robots

- Purpose of transmission is to transfer power from source to load.
- The purpose of a transmission is *also* to transfer power at appropriate speed.
  - Typical rated speed of DC motors are about 1800-3600 rpm.
  - 3000 rpm $=$ 60 rps $\approx$ 360 rad/sec.
  - For a (typical) 1 m link $\rightarrow$ Tip speed is 36 m/sec – Greater than speed of sound!!
  - Need for large reduction in speed.
- Transmissions also convert rotary to linear motion and vice-versa.
- Transmissions also transfer motion to different places and to different direction.

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Transmissions used in Robots

- Transmissions in robots are decided based on motion, load and power requirements, and by the placement of motor relative to the joint.
- Transmissions for robots must be (a)stiff, (b) low weight, (c) backlash free, and (d) efficient.
- Direct drives with motor directly connected to joint – has advantages of low friction and low backlash but are expensive.
- Typical transmissions
  - Gear boxes of various kinds – spur, worm and worm wheel, planetary etc..
  - Belts and chain drives.
  - Harmonic drive for large reduction.
  - Ball screws and rack-pinion drives – to transform rotary to linear motions.
  - Kinematic linkages – 4-bar linkage.

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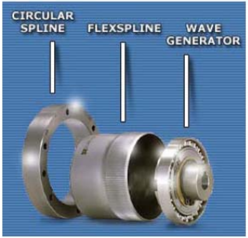
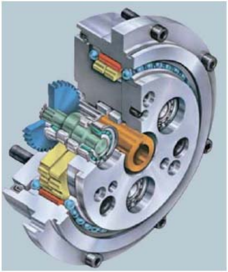
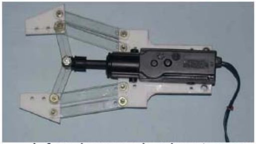


Parts of a harmonic drive
http://www.harmonicdrive.net/reference/applicationnotes/principles.php
See also http://en.wikipedia.org/wiki/Harmonic_drive for animation



RV series gear reduction mechanism
http://precision.nabtesco.com/en/

A ball screw
http://www.thk.com/in/products/class/ballscrew/



A four-bar mechanism to move gripper jaws
http://www.cs.cmu.edu/~amampett/pastProjects.html

Figure 26: Examples of transmissions used in robots

Outline

1. Contents
2. Lecture 1
   - Mathematical Preliminaries
   - Homogeneous Transformation
3. Lecture 2
   - Elements of a robot – Joints
   - Elements of a robot – Links
4. Lecture 3
   - Examples of D-H Parameters & Link Transformation Matrices
5. Lecture 4
   - Elements of a robot – Actuators & Transmission
6. **Lecture 5**
   - **Elements of a Robots – Sensors**
7. Module 2 – Additional Material
   - Problems, References, and Suggested Reading

INTRODUCTION

- A robot without sensors is like a human being without eyes, ears,touch,etc.
  - Sensor-less robots require costly /time consuming programming.
  - Can perform only in "playback" mode.
  - No change in their environment, tooling and work piece can be accounted for.
- Sensors constitute the perceptual system of a robot, designed:
  - To make inferences about the physical environment.
  - To navigate and localise itself, and
  - To respond more "flexibly" to the events occurring in its environment.
  - To enable learning, thereby endowing robots with "intelligence"
- Sensors allow less accurate modeling and control.
- Sensors enable robots to perform complex and increased variety of tasks reliably thereby reducing costs

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Importance of Sensors from Modeling and Control

- Dynamical system
  - System which changes with time – governed by differential equations.
  - For a given *input* there is a well defined *output*
  - Linear dynamical system – modeled by linear ordinary differential equations $\rightarrow$ Can be analysed using *Laplace transforms* and *transfer function*[2].
- Control system
  - Required to obtain a desired response from a dynamical system.
  - Stabilize an unstable dynamical system.
  - Improve performance
- Two kinds of system – Open Loop and Closed-loop or feedback control systems.
- Importance of sensors in modeling and control shown in next few slides.

[2] Other methods such as *frequency domain* can also be used.

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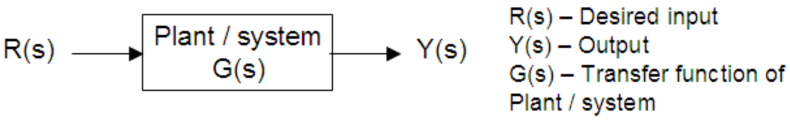

Figure 27: An open-loop control system

- Time domain linear function $f(t)$ converted to $s-$domain using Laplace transforms: $F(s) \triangleq \mathscr{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\, dt$
- Time derivatives become polynomials in s.
- Transfer function – Ratio of output to input in $s-$domain.
- Open-loop system – no feedback: Input $R(s) \rightarrow$ System $G(s) \rightarrow$ Output $Y(s)$ (see figure above)
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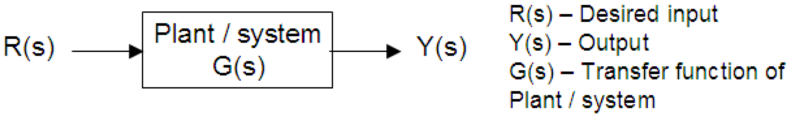

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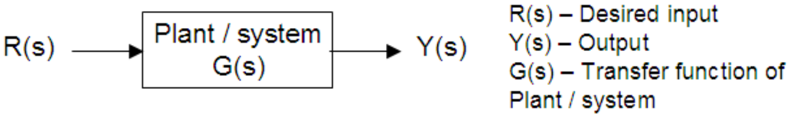
SENSORS IN MODELING AND CONTROL

OPEN-LOOP CONTROL SYSTEM



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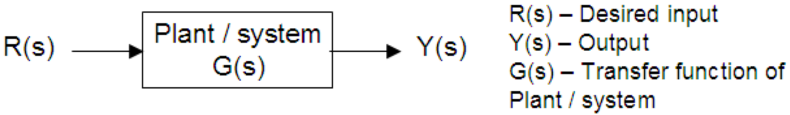

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Sensors in Modeling and Control

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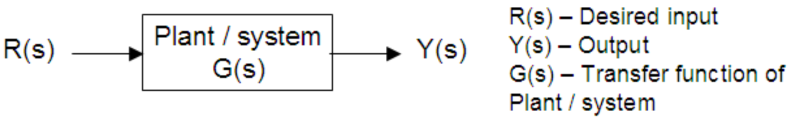

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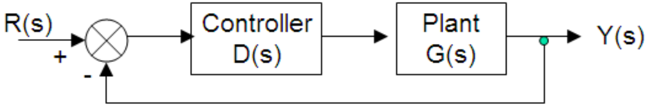

• Plant with sensors and feedback control
•Y(s) is sensed / measured and fed back

Figure 28: A closed-loop control system

$$Y(s) = \frac{D(s)G(s)}{1 + D(s)G(s)} R(s); \text{ Chosen } D(s)G(s) >> 1$$

- For error $\delta G(s)$ in the modeling of the plant

$$\frac{\delta Y(S)}{Y(s)} = \frac{\delta G(s)}{G(s)} \frac{1}{1 + D(s)G(s)}$$

- Since $1 + D(s)G(s) >> 1 \rightarrow x\%$ error in $G(s)$ results in much smaller error – $(\frac{1}{1+D(s)G(s)})$ $x\%$ in $Y(s)$

With sensors and feedback, *less* complex and expensive controllers and/or models of system can be used for *more* robust performance

Importance in Modeling and Control

Closed-loop Control System (Contd.)

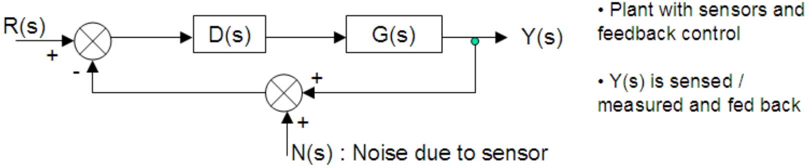

Figure 29: Closed-loop control with sensors

- Sensors must be *low noise* and 'good' for effectiveness.
- Consider a 'noisy' sensor (see figure)
- Output of system

$$Y(s) = \frac{D(s)G(s)}{1+D(s)G(s)}(R(s)-N(s));\ \text{Chosen} D(s)G(s) >> 1$$

- Note that output is proportional to *corrupted* $R(s)-N(s)$ input, hence output error can never be reduced!!

Importance in Modeling and Control

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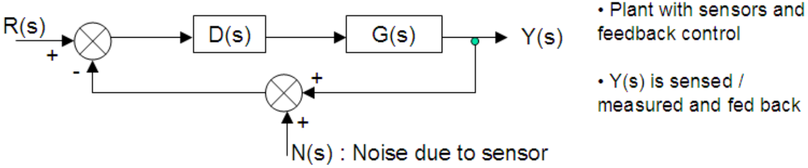

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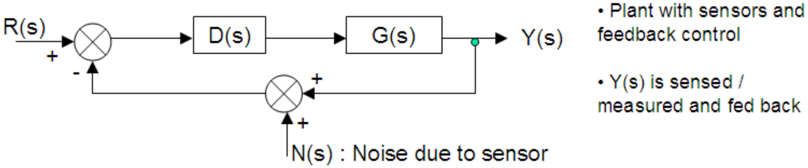

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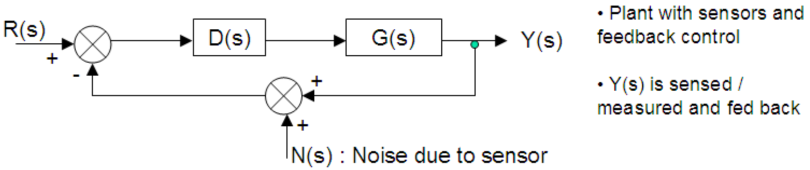

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Sensors in Robots

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- Desirable features in sensors are
  - High accuracy
  - High precision
  - Linear response
  - Large operating range
  - High speed of response
  - Easy to calibrate
  - Reliable and rugged
  - Low Cost
  - Ease of operation
- Broad classification of sensors in robots
  - Internal state sensors
  - External state sensors

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Sensors in Robots

Internal Sensors

- Internal sensors measure variables for control
  - Joint position
  - Joint velocity
  - Joint torque/force
- Joint position sensors (angular or linear)
  - Incremental & absolute encoders – optical, magnetic or capacitive
  - Potentiometers
  - Linear analog resistive or digital encoders
- Joint velocity sensors
  - DC tacho-generator & Resolvers
  - Optical encoders
- Force/torque sensors
  - At joint actuators for control
  - At wrist to measure components of force/moment being applied on environment
  - At end-effector to measure applied force on gripped object.

SENSORS IN ROBOTS

INTERNAL SENSORS

- Internal sensors measure variables for control
  - Joint position
  - Joint velocity
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Sensors in Robots

External Sensors

- Detection of environment variables for robot guidance, object identification and material handling.
- Two main types – contacting and non-contacting sensors
- Contacting sensors: Respond to a physical contact
  - Tactile/touch sensors – switches, Photo-diode/LED combination
  - Slip sensors
  - Tactile sensors – resistive/capacitive arrays
- Non-contacting sensors: Detect variations in optical, acoustic or electromagnetic radiations or change in position/orientation
  - Proximity sensors – Inductive, Capacitive, Optical and Ultrasonic
  - Range sensors – Capacitive and Magnetic, Camera, Sonar, Laser range finder, Structured light
  - Colour sensors
  - Speed/Motion sensors – Doppler radar, Doppler sound, Camera, Accelerometer, Gyroscope
  - Identification – Camera, RFID, Laser ranging, Ultrasound
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Sensors in Robots

- How to compute resolution of a sensor?
- Example: Optical encoder to measure joint rotation

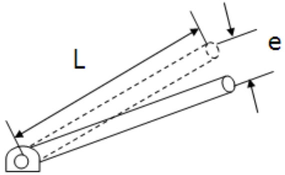


Figure 30: Resolution and required accuracy

- Optical encoders and resolvers – Desired accuracy $e \rightarrow$ One must be able to measure a minimum angle e/L.
- No. of bits/revolution required $= 2\pi/(\frac{e}{L})$
- Express as a binary number and rounded to next higher power of 2
- Example computation – $L = 1$m, $e \backsim 0.5\text{mm} \Longrightarrow \frac{e}{L} = 5x10^{-4}$
- $2\pi/(\frac{e}{L}) \backsim 10000 - 2^{13} = 8192, 2^{14} = 16384 \rightarrow 14$ bit encoder needed
- If L is small (last 3 joints in a robot), 12 bits are usually enough.

Optical Encoder

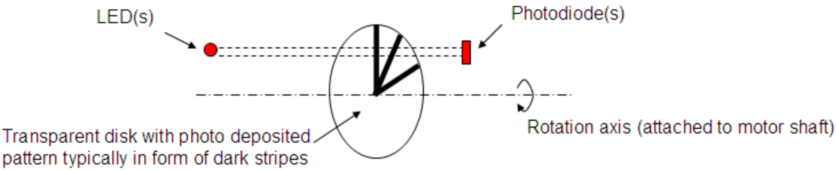
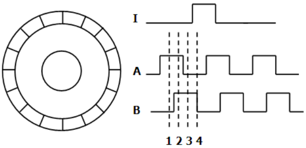


Figure 31: Optical encoder

- One of the most important and widely used internal sensor.
- Consists of an etched encoding disk with photo-diodes and LEDS – disk made from
  - Glass, for high-resolution applications (11 to >16 bits)
  - Plastic (Mylar) or metal, for applications requiring more rugged construction (resolution of 8 to 10 bits)
- As disk rotates, light is alternately allowed to reach photo-diode, resulting in digital output similar to a square wave

Internal Sensors in Robots

Optical Encoder (Contd.)



State	Ch A	Ch B
S_1	High	Low
S_2	High	High
S_3	Low	High
S_4	Low	Low

Figure 32: Optical encoder outputs

- Typically 3 signals available – Channel A, B and I; A and B are phase shifted by 90 degrees and I is called as the *index* pulse obtained every full rotation of disk.
- Signals read by a microprocessor/counter
- Output of counter includes rotation and direction.
- Output can be absolute or relative joint rotation.
- Can be used for estimating velocity.

Internal Sensors in Robots

Other Joint Rotation Sensors

- Potentiometers – voltage $\propto$ resistance and resistance $\propto$ rotation at joint
  - Not very accurate but very inexpensive.
  - More suitable for slow rotations
  - Adds to joint friction
- Resolvers – rotary electrical transformer to measure joint rotation
  - Analog output, need ADC for digital control
  - Electromagnetic device – Stator + Rotor (connected to motor shaft)
  - Voltage (at stator) $\propto \sin(\theta)$ – θ = rotor angle
- Tachometers measures joint velocity
  - Similar to resolver
  - Voltage (at stator) $\propto \dot{\theta}$ – $\dot{\theta}$ = angular velocity of the rotor
  - Analog output $\rightarrow$ required ADC for digital control.

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- Employed for force/torque sensing – Can be achieved by *joint* and *wrist* sensing
- Force/Torque joint sensors
  - Direct sensing of force/torque in a compliant shaft attached to motor by means of strain gages
  - A model of the motor and the shaft required.
  - Introduced compliance at joint not desirable – system dynamics altered!
  - For DC motor – joint torque $\propto$ armature current.
  - Requires model of motor & accuracy not very good.
- Force/torque wrist sensors
  - Mounted between end of robot arm and end-effector.
  - Can measure all six components of force/torque using strain gages.
  - Extensively used in *force* control.

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Force/Torque Wrist Sensor (contd.)

- Performance specifications to ensure that the wrist motions generated by the force/torque sensors do not affect the position accuracy of the manipulator:
- High stiffness to ensure quick dampening of the disturbing forces which permits accurate readings during short time intervals
- Compact design to ensure easy movement of the manipulator
- Need to be placed close to end-effector/tool
- Linear relation between applied force/torque and strain gauge readings.
- Made from single block of metal $\rightarrow$ no hysteresis

Internal Sensors in Robots

Force/Torque Wrist Sensor (contd.)

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FORCE/TORQUE WRIST SENSOR (CONTD.)

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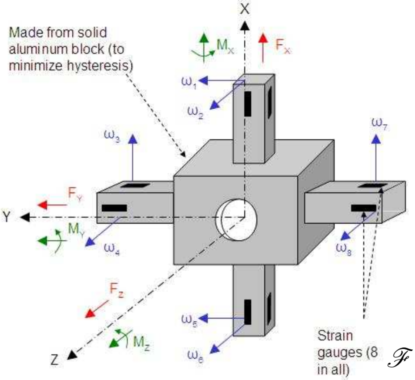

Figure 33: Six axis force/torque sensors at wrist

$$\mathscr{F} = [R_{\mathscr{F}}]W$$

$$R_{\mathscr{F}} =$$

$$\begin{pmatrix} 0 & 0 & r_{13} & 0 & 0 & 0 & r_{17} & 0 \\ r_{12} & 0 & 0 & 0 & r_{25} & 0 & 0 & 0 \\ 0 & r_{32} & 0 & r_{34} & 0 & r_{36} & 0 & r_{38} \\ 0 & 0 & 0 & r_{44} & 0 & 0 & 0 & r_{48} \\ 0 & r_{52} & 0 & 0 & 0 & r_{56} & 0 & 0 \\ r_{61} & 0 & r_{63} & 0 & r_{65} & 0 & r_{67} & 0 \end{pmatrix}$$

$\mathscr{F} = (F_x, F_y, F_z, M_x, M_y, M_z)^T$
$W = (w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8)$
w_i 8 strain gauge readings

R_{ij} are found during calibration process using least-square techniques.

External Sensors in Robots

Touch Sensor

- Allows a robot or manipulator to interact with its environment – to "touch and feel", "see" and "locate".
- Two classes of external sensors – contact and non-contact

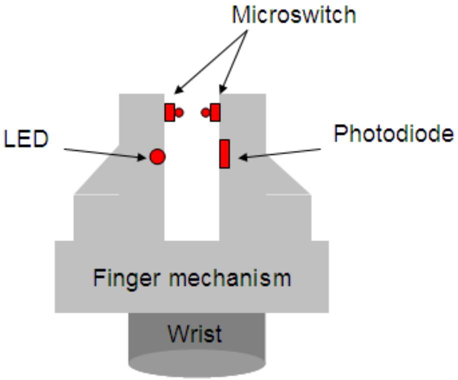

Figure 34: Touch sensor

- Simple – LED-Photo-diode pair used to detect presence/absence of object to be grasped
- Micro-switch to detect touch.

External Sensors in Robots – Contact

Slip Sensor

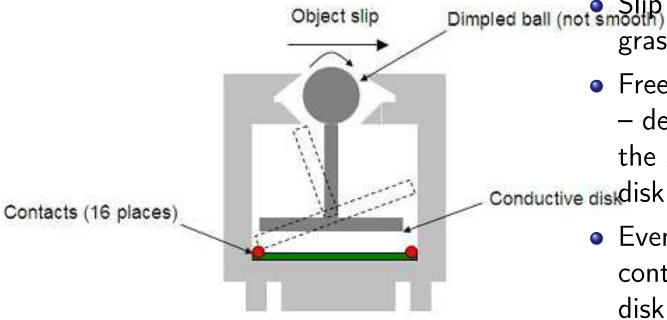

Figure 35: Slip sensor

- Slip sensor – to detect if grasped object is slipping
- Free moving dimpled ball – deflects a thin rod on the axis of a conductive disk
- Evenly spaced electrical contacts placed under disk

- Object slips past the ball – moving rod and disk – electrical signal from contact to detect slip.
- Direction of slip determined from sequence of contacts.

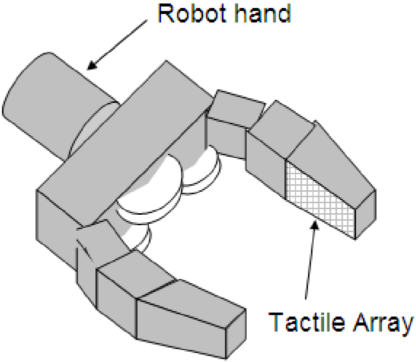

Figure 36: Robot hand with tactile array

- "Skin" like membrane to "feel" the shape of the grasped object
- Also used to measure force/torque required to grasp object
- Change in resistance/capacitance due to local deformation from applied force

External Sensors in Robots – Contact

Tactile sensors (contd.)

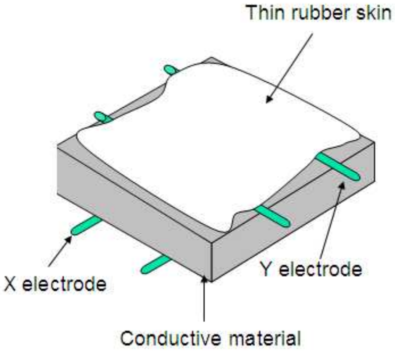

Figure 37: Artificial Skin

- Send current in one set, measure current in other set
- Magnitude of current $\propto$ change in resistance due to deformation
- Magnitude of current $\propto$ change in capacitance

- Fluid filled membrane
- Array of Hall-effect sensors
- MEMS – silicon micro-machined with doped strain-gauge flexure

- Detect presence of an object near a robot or manipulator
- Works at very short ranges (<15-20 mm)
- Frequently used in stationary and mobile robots to avoid obstacles and for safety during operation
- Four main types of proximity sensors
  - Inductive proximity sensors
  - Capacitive proximity sensor
  - Ultrasonic proximity sensor
  - Optical proximity sensors

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External Sensors in Robots – Non-contact

Inductive proximity sensors

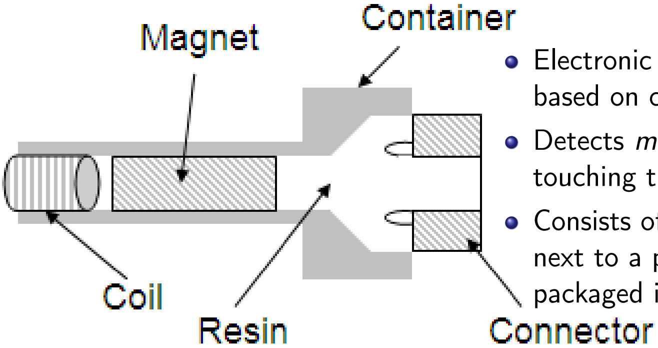

Figure 38: A magnetic proximity sensor

- Electronic proximity sensor based on change of inductance
- Detects *metallic* objects without touching them
- Consists of a wound coil, located next to a permanent magnet, packaged in a simple housing

Inductive proximity sensors – magnetic

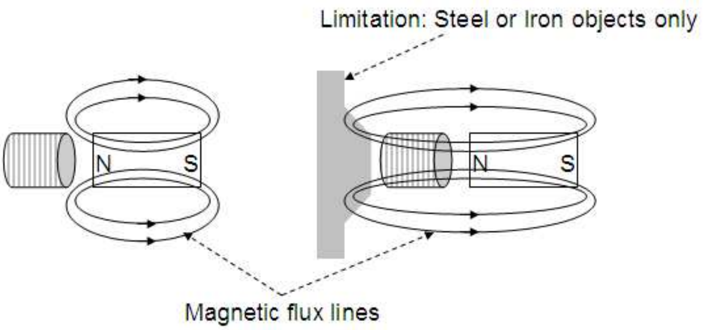

Figure 39: Flux lines in a magnetic sensor

- Ferromagnetic material enters or leaves the magnetic field → Flux lines of the permanent magnet change their position
- Change in flux → Induces a current pulse with amplitude and shape proportional to rate of change in flux

External Sensors in Robots – Non-contact

Inductive proximity sensors – magnetic (Contd.)

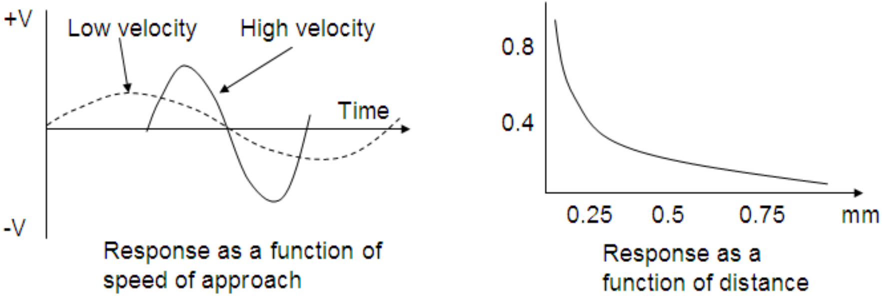

Figure 40: Flux response in magnetic proximity sensor

- Coils output voltage waveform → for proximity sensing
- No physical contact required – useful where access is a challenge
- Applications – Metal detectors, Traffic light changing, Automated industrial processes
- *Limited to ferromagnetic materials*

- Hall effect relates the voltage between two points in a conducting or semiconducting material subjected to a strong magnetic field across the material.
- When a semi-conductor magnet device is brought in close proximity of a ferromagnetic material
  - the magnetic field at the sensor weakens due to bending of the field lines through the material,
  - the Lorentz forces are reduced, and
  - the voltage across the semiconductor is reduced.
- The drop in the voltage is used to sense the proximity
- Applications: Ignition timings in IC engines, tachometers and anti-lock braking systems, and brushless DC electric motors.

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External Sensors in Robots – Non-contact

Inductive proximity sensors – Hall effect sensors

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CAPACITIVE PROXIMITY SENSORS

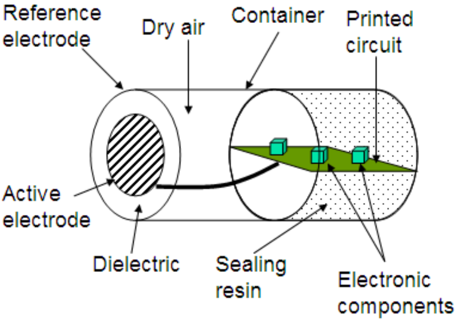

Figure 41: Capacitive sensor

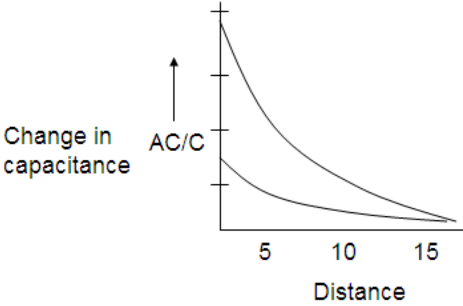

Figure 42: Capacitive response

- Similar to inductive, but uses *electrostatic* field
- *Can sense metallic as well as non-metallic materials*
- Sensing element is a capacitor composed of a sensitive electrode and a reference electrode

- Object's entry in electrostatic field of electrodes changes capacitance
- Oscillations start once capacitance exceeds a predefined threshold
- Triggers output circuit to change between on and off
- When object moves away, oscillator's amplitude decreases, changing output back to original state
- Larger size and dielectric constant of target, means larger capacitance and easier detection
- Useful in level detection through a barrier

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External Sensors in Robots – Non-contact

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ULTRASONIC PROXIMITY SENSORS

- Electro-acoustic transducer to send and receive high frequency sound waves.
- Emitted sonic waves are reflected by an object back to the transducer which switches to receiver mode
- Same transducer is used for both receiving and emitting the signals – fast damping of acoustic energy is essential to detect close proximity objects
- Achieved by using acoustic absorbers and by decoupling the transducer from its housing
- Important specifications – 1) Maximum operating distance, 2) Repeatability, 3) Sonic cone angle, 4) Impulse frequency, and 5) Transmitter frequency
- Some well-known applications – 1) Useful in difficult environments, 2) Liquid level detection and 3) Car parking sensors

External Sensors in Robots – Non-contact

Ultrasonic proximity sensors

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- Emitted sonic waves are reflected by an object back to the transducer which switches to receiver mode
- Same transducer is used for both receiving and emitting the signals – fast damping of acoustic energy is essential to detect close proximity objects
- Achieved by using acoustic absorbers and by decoupling the transducer from its housing
- Important specifications – 1) Maximum operating distance, 2) Repeatability, 3) Sonic cone angle, 4) Impulse frequency, and 5) Transmitter frequency
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External Sensors in Robots – Non-contact

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- Also known as light beam sensors – solid state LED acting as a transmitter by generating a light beam.
- A solid-state photo-diode acts as a receiver
- Field of operation of the sensor – long pencil like volume, formed due to intersection of cones of light from source and detector
- Any reflective surface that intersects the volume gets illuminated by the source and is seen by the receiver
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- Measure distance of objects at larger distances
- Uses electromagnetic or electrostatic or acoustic radiation – looks for changes in the field or return signal
- Highly reliable with long functional life and no mechanical parts.
- Four main kinds of range sensing techniques in robots
  - Triangulation
  - Structured lighting approach
  - Time of flight range finders
  - Vision
- Applications: 1) Navigation in mobile robots, 2) Obstacle avoidance, 3) Locating parts.

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Range sensors -triangulation

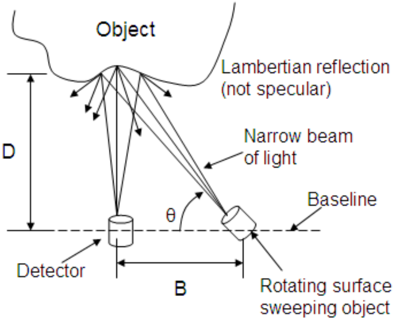

Figure 43: Triangulation

- A narrow beam of light sweeps the plane defined by the detector, the object and the source, illuminating the target
- Detector output is peak when illuminated patch is in front
- With B and θ known $\rightarrow$ obtain $D = B \tan\theta$
- Changing B and θ, one can get D for all visible portions of object

- Very little computation required
- Very slow as one point is done at a time

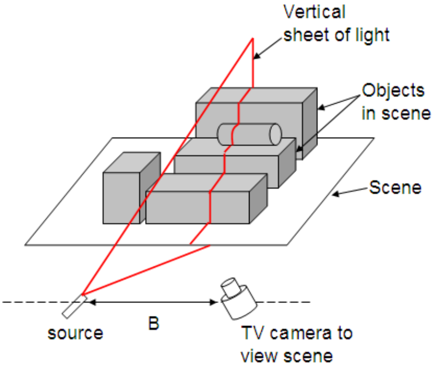

Figure 44: Structured lighting

- A "'sheet" of light generated through a cylindrical lens or narrow slit, is projected on a target
- Intersection of the sheet with target yields a light stripe
- A camera offset slightly from the projector, views and analyze the shape of the line

RANGE SENSORS - STRUCTURED LIGHTING (CONTD.)

- Distortion of the line is related to distance and can be calculated
- Horizontal displacement (in image) proportional to depth gradient
- Integration gives absolute range.
- Calibration is required
- Advantages:
  - Fast, very little computation is required
  - Can scan multiple points or entire view at once
- Structured lighting should be permitted

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- Utilizes pulsed lasers and ultrasonics to measure time taken by the pulse to coaxially return from a surface
- If D is the target's distance, c is the speed of radiation, and t is elapsed time taken for the pulse to return

$$D = \frac{ct}{2}$$

- Light or electromagnetic radiation more useful for large (kilometers) distances.
- Light not very suitable in robotic applications as c is large
  - To measure range with ± 0.25 inch accuracy, one needs to measure *very small* time intervals – $\backsim$ 50 ps
- Suitable for acoustic (ultrasonic) radiation, since $c \backsim 330$ m/s
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External Sensors in Robots – Non-contact

Range sensors - time-of-flight (contd.)

Phase shift and continuous laser light for measuring distance

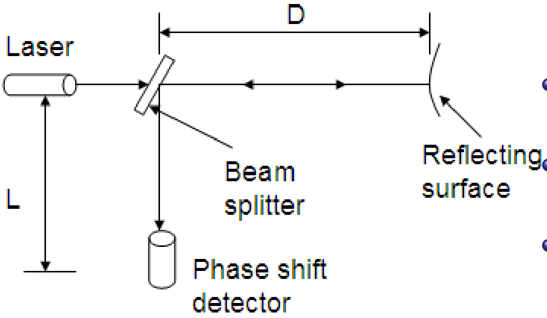

Figure 45: Range measurement using phase shift

- Laser beam of wavelength λ is split into two beams and travel to a detector through *two different paths*
- The phase delay between the two beams is measured
- Distance traveled by first beam L
- Total distance traveled by the second beam $D' = L + 2D \rightarrow$ $D' = L + \frac{\theta}{360}\lambda$, θ is the phase shift.

- For $\theta = 360^\circ$, two waveforms are aligned and $D' = L$ and $D' = L + n\lambda$ $\rightarrow$ waveforms cannot be differentiated on phase shift alone.
- Restrict $\theta < 360^\circ$ and $2D < \lambda$

$$D = \frac{\theta}{360}\left(\frac{\lambda}{2}\right)$$

- λ typically small $\rightarrow$ impractical for robot application $\rightarrow$ Modulate laser light with a waveform of much higher wavelength
- Example: modulating frequency=10 MHz $\rightarrow \lambda = \frac{c}{f} = 30m$ and D up to 15m can be measured
- Advantages of continuous light technique
  - Yields intensity as well range information,
  - Requires very little computation,
  - Lasers do not suffer from specular reflection, and
  - Expensive, not so robust and require higher power.

Range sensors - time-of-flight (Contd.)

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External Sensors in Robots – Non-contact

Range sensors - ultrasonic

- Similar to the pulsed laser technique
- An ultrasonic chirp is transmitted over a short time period
- From the time difference between the transmitted and reflected wave, D can be obtained
- Generally used for navigation and obstacle avoidance in robots
- Much cheaper than laser range finder
- Shorter range as waves disperse
- Wavelength of ultrasonic radiation much larger $\rightarrow$ not reflected very well from small objects and corners.
- Ultrasonic waves not reflected very well from plastics and some other material.

- Most powerful and complex form of sensing, analogous to human eyes
- Comprising of one or more video cameras with integrated signal processing and imaging electronics
- Includes interfaces for programming and data output, and a variety of measurement and inspection functions.
- Also referred to as machine or computer vision
- Computations required are very large compared to any other form of sensing
- Computer vision can be sub-divided into six main areas: 1) Sensing, 2) Pre-processing, 3) Segmentation, 4) Description, 5) Recognition and 6) Interpretation

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External Sensors in Robots – Non-contact
Vision Sensors (contd.)

- Three levels of processing
- Low level vision
  - Primitive in nature, requires no intelligence on the part of the vision functions
  - Sensing and pre-processing can be considered as low level vision functions
- Medium level vision
  - Processes that extract, characterize and label components in an image resulting from a low level vision
  - Segmentation, description and recognition of the individual objects refer to the medium level function
- High level vision: Processes that attempt to emulate cognition

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VISION SENSORS (CONTD.)

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Vision Sensors (Contd.)

- Smaller number of robotic applications – primarily due to computational complexity and low speed.
- Vision system can
  - Determine distances of objects.
  - Determine geometrical shape and size of objects.
  - Determine optical (color, brightness) properties of objects in an environment.
  - Can be used for navigation (map making), obstacle avoidance, Cartesian position and velocity feedback, locating parts, and many other uses.
  - Can learn about environment.
  - Acquire knowledge and intelligence.
- Vision systems extensively used in autonomous navigation in mobile robots (Mars rovers)
- Use of vision systems increasing rapidly as technology improves!

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- Provide an interface to sense, process and/or control the surrounding environment
- Generally silicon based, allowing integration with microelectronics
- Sensing mechanisms employed are a) Capacitive, b) Piezoelectric, c) Piezoresistive, d) Ferroelectric, e) Electromagnetic and f) Optical.
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- Consist of very small electrical, electronics and mechanical components integrated on a single chip
- Provide an interface to sense, process and/or control the surrounding environment
- Generally silicon based, allowing integration with microelectronics
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New Developements in Sensor Technology

MEMS Sensors (Contd.)

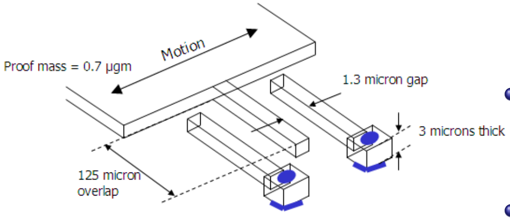

Figure 46: A typical MEMS capacitive sensor

- Polysilicon springs suspend MEMS structure above the substrate – proof mass is free to move in X and Y direction.
- Radial fingers fixed to proof mass are positioned between plates fixed to the substrate
- Each finger and pair of fixed plates make up a differential capacitor.

- In-plane acceleration causes the proof mass to move relative to the plates changing the capacitance
- Change in capacitance converted to a voltage signal
- Can sense both dynamic acceleration and static acceleration

- Applications of MEMS Sensors:
  - Automotive electronics and crash systems
  - High resolution seismic sensing
  - Medical equipments
  - Hard disk drives
  - Computer peripherals
  - Wireless devices
  - Smart portable devices such as phones, PDAs etc.
- MEMS sensors are finding their way into mobile robots.

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New Developements in Sensor Technology

MOEMS(Micro-opto-electromechanical systems) sensors

- Silica based MEMS sensors, though very effective, can't be used in high temperature and pressure conditions, due to the limiting mechanical and electrical properties of silica.
- For harsh environmental conditions, MEMS sensors are built out of robust materials such as (Si, SiC) and are integrated with optical signal detection technique forming MOEMS
- MOMES sensors are highly resistant to electromagnetic interference (EMI) and radio frequency interference (RFI)
- Eliminate the need for on-board electronics

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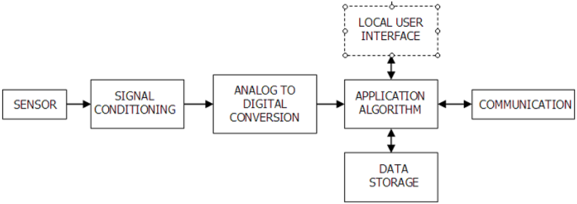

Figure 47: Components of a smart sensor

- "Smart Sensor" – a single device combining data collection and information output
- As per IEEE 1451.2, a smart sensor is:
  "*a transducer that provides functions beyond those necessary for generating a correct representation of a sensed or control quantity*".
- Integration of transducer into applications in a networked environment.
- Evolving concept – Often sensors not complying with IEEE definition are also called smart sensors!

- Three primary kinds of smart sensors:
  - Transmitter plus micro-controller capable of a certain amount of internal signal processing, such as linearisation, conversion to engineering units, diagnostics etc.
  - A sensor that can communicate over a digital communications network, either wired or wireless
  - A smart plug-and-play transducer electronic data sheet (TEDS) sensor with applications in aerospace and defense, transportation and research labs
- Smart sensors are capable of a) self identification and diagnosis, b) Time and location awareness for time and position stamping, c) Conform to standard data communications and control protocols for interoperability
- Other features of smart sensors – a)remote diagnostics, b) improved reliability, and c) better signal to noise ratio.

New Developements in Sensor Technology

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- *Intelligent sensors* take some load off the controller by
  - Taking some predefined action based on the input received
  - Efficiently and precisely measuring the parameters, enhancing or interrupting them
- Intelligent sensors: An adaptation of smart sensors with embedded algorithms for detection of noise, instrumentation anomalies and sensor anomalies
- Enabling technology for ISHM (Integrated Systems Health Management) used in spaceships by NASA

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A MILESTONE FOR SENSOR-BASED ROBOTICS

ASIMO: ADVANCED STEP IN INNOVATIVE MOBILITY

- A humanoid robot developed by HONDA car company of Japan
- Large number of sensors and actuators
  - Sensors for vision, speed, balance, force, angle, and foot area
  - 34 degrees of freedom controlled by servo motors
- Capable of advanced movement
  - Walking and running
  - Maintaining posture and balance
  - Climbing stairs
  - Avoiding obstacle
- Intelligence
  - Charting a shortest route
  - Recognizing moving objects
  - Distinguish sounds and recognize faces and gestures

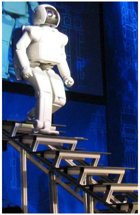

Figure 48: ASIMO climbing stairs

Figure 49: ASIMO playing soccer

Sensors in Robots – Summary

- Sensors enhance performance of robots/manipulators, and makes them more flexible for different application.
- Two main types of sensors
  - Internal sensors for measuring variables such as joint rotation, velocity, force, and torque
  - External sensors for measuring variables such as proximity, touch, range, and geometry.
- Large varieties of sensors exist for large variety of measurement tasks.
- More than one sensor is used in a robot.
- Vision system is the most powerful and complex sensor.
- New and modern sensor technologies are becoming mature and are finding their way into robots.

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OUTLINE

1. CONTENTS
2. LECTURE 1
   - Mathematical Preliminaries
   - Homogeneous Transformation
3. LECTURE 2
   - Elements of a robot – Joints
   - Elements of a robot – Links
4. LECTURE 3
   - Examples of D-H Parameters & Link Transformation Matrices
5. LECTURE 4
   - Elements of a robot – Actuators & Transmission
6. LECTURE 5
   - Elements of a Robots – Sensors
7. MODULE 2 – ADDITIONAL MATERIAL
   - Problems, References, and Suggested Reading

- Exercise Problems
- References & Suggested Reading
- Homogeneous coordinates, lines & screws