Robotics: Advanced Concepts & Analysis

Module 9 - Modeling and Analysis of Wheeled Mobile Robots

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Outline

1 Contents

2 Lecture 1

- Wheeled Mobile Robots on Flat Terrain

3 Lecture 2*

- Wheeled Mobile Robots on Uneven Terrain

4 Lecture 3*

- Kinematics and Dynamics of WMR on Uneven Terrain

5 Module 9 – Additional Material

- Problems, References and Suggested Reading

CONTENTS OF LECTURE

- Introduction to wheeled mobile robots.
- Different kinds of wheels and their modeling.
- Kinematics of wheeled mobile robots on flat surfaces.
- Modeling of wheel slip.
- Modeling of a WMR with omni-directional wheels.
- Simulation of a WMR with omni-wheels.
- Summary

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Introduction

- Last 8 modules, all robots considered were *stationary* with a *fixed* link or a *base*.
- Several robots are now designed to have *mobility* – they can move on a surface, in water or in air.
- Only robots with capability of mobility on a surface considered.
- Earliest examples are *automated guided vehicles* (AGV's) with wheels used on *flat* factory floors.
- More recently autonomous robots with *legs* and/or a combination or wheels and legs (*hybrid*) have been built.
- Vast majority are *wheeled mobile robots* or WMR's as they are *more efficient* and *faster* than legged or tracked vehicles.
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  - Industrial environments to move material (parts and finished goods) from one place to another.
  - Military and security applications.
  - Hazardous environments such as inside nuclear reactors or in deep sea bed.
  - Providing mobility to handicapped persons.
  - Planetary exploration.
- Analysis of WMR's involve kinematics, dynamics, control, sensing, motion planning, obstacle avoidance ...
- This Lecture – *deals* with kinematics and dynamics of WMR's moving on a *flat* surface or a plane.
- Next Lecture – WMR's moving on uneven or rough terrains.

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Examples of WMR's

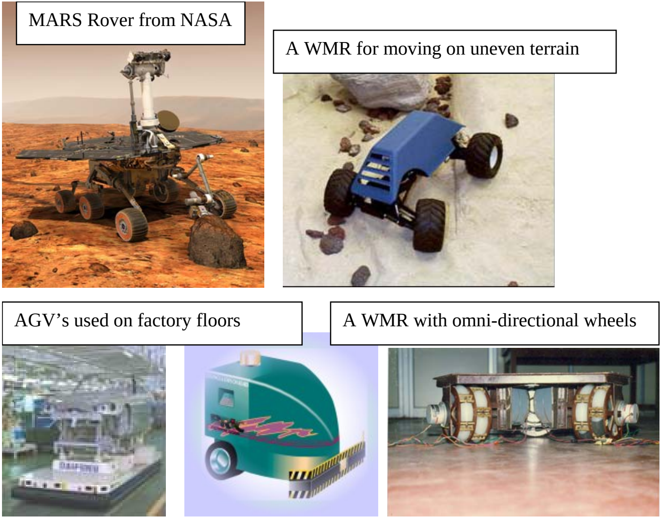

Figure 1: Some Wheeled Mobile Robots

Introduction

- Muir and Newman (1987)
  - A WMR is "a robot capable of locomotion on a surface solely through the wheel assemblies mounted on it and in contact with a surface.
  - A wheel assembly is a device which provides or allow relative motion between its mount and the surface on which it is intended to have single-point of rolling contact".
- In practical wheels, due to deformation, there is *area contact*.
- Different types of wheels used in WMR's – 1) Conventional, 2) Omni-directional and 3) Ball wheels

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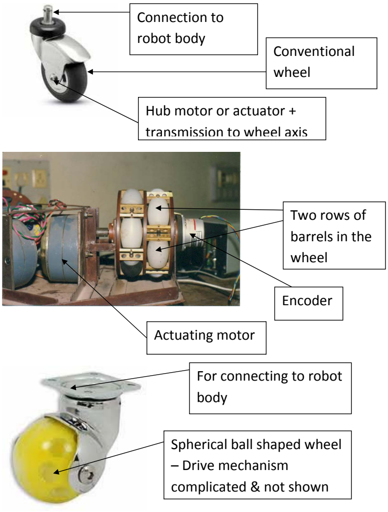

Figure 2: Three main types of wheels in WMR's

- Conventional wheel most commonly used $\rightarrow$ Rotation about axis of wheel & steering.
- Omni-directional wheels also called Swedish wheels:
  - Barrels on the periphery of a wheel.
  - Barrel can rotate about an axis at an angle to the wheel rotation axis – 90° in figure.
  - Barrel rotation is not actuated & barrel rotation leads to 'sliding' of wheel.
  - Two DOF in each wheel & steering
- Ball or spherical wheels
  - Essentially a sphere which can rotate about two axis.
  - Complicated drive and rotation measuring arrangement.

Single Wheel Kinematics

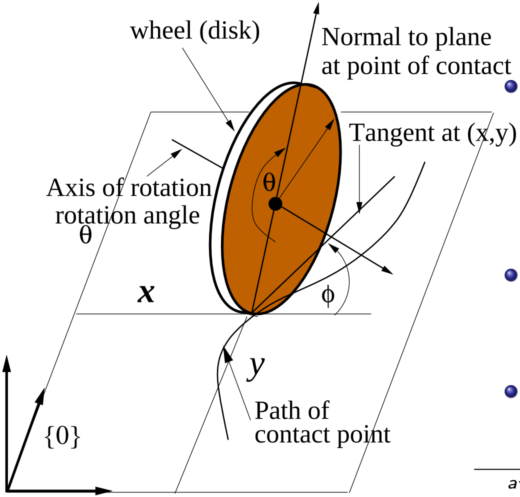

Figure 3: Wheel rolling on a plane

- Disk on a plane – 3 DOF configuration space (x, y, ϕ), ϕ is the steering angle[a], r is radius.
- Wheel rolling without slip – motion *only* along tangent to path $\dot{x}\sin\phi - \dot{y}\cos\phi = 0$. Velocity perpendicular to the tangent vector is zero.
- *Non-holonomic constraint* – Cannot be integrated to obtain $f(x, y, \phi) = 0$
- Constrains $\dot{\mathbf{q}} = (\dot{x}, \dot{y}, \dot{\phi})^T$, and *not* $\mathbf{q} = (x, y, \phi)^T$.

[a] Tilting of wheel from the normal not considered and rotation of wheel θ not required for analysis. If *thin* disk, tilt must be considered.

- Justification for *non-holonomic nature*
  - Assume $\dot{x}\sin\phi - \dot{y}\cos\phi = 0$ can be integrated.
  - This implies existence of $f(x,y,\phi) = 0 \Rightarrow$ Two DOF $\Rightarrow$ Choosing *any two* out of x, y, ϕ automatically determines the third!
  - For example, at $x = y = 0 \Rightarrow \phi = \phi^*$ is fixed[1] $\Rightarrow$ No other value(s) of ϕ is possible!
  - This is clearly false – Any ϕ is possible at $(0,0)$ by simply rotating disk about the normal.
  - Hence $f(x,y,\phi) = 0$ is *not integrable*.
- The kinematics of a single wheel is given by

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix} v + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \omega$$

where v and ω are the forward speed and steering rate.

[1] *At most finitely* many ϕ^* if $f(x,y,\phi) = 0$ is *nonlinear*.

Single Wheel Kinematics (Contd.)

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Two Wheel Robot – Bicycle

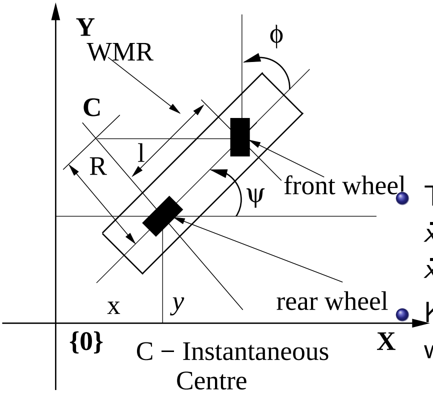

Figure 4: Two wheel robot (bicycle)

- Four generalised coordinates
  - (x, y) – point of contact of rear wheel.
  - ψ – orientation of body & rear wheel with X axis.
  - ϕ – steering angle of front wheel with respect to body
- Two rolling constraints in each wheel:
$\dot{x}\sin\psi - \dot{y}\cos\psi = 0$
$\dot{x}\sin(\psi+\phi) - \dot{y}\cos(\psi+\phi) - l\dot{\psi}\cos\phi = 0$
- Kinematics equations in terms of front wheel velocity v_f and steering rate ω

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} \cos\theta\cos\phi \\ \sin\theta\cos\phi \\ \sin\phi/l \\ 0 \end{pmatrix} v_f + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \omega$$

- Lines perpendicular to front and rear wheel velocity vector meet at C.
- Instantaneous centre: motion of the bicycle is rotation about C.

Three Wheel Robot – Tri-cycle

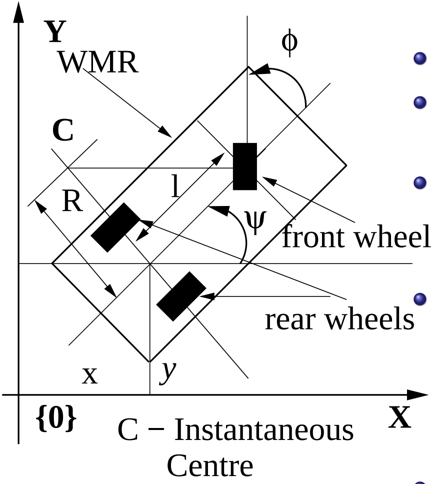

Figure 5: Three wheeled mobile robot or tri-cycle

- Front wheel is steered.
- Speeds of rear wheels are different when vehicle is making a turn.
- Speed of wheel i, $v_i = \dot{\theta}_i r_i$, $i = 1, 2, 3$ where $\dot{\theta}_i$ and r_i are rotational speed and radius of wheel i.
- One rear wheel is driven and the speed of the second rear wheel must adjust so that there is a single instantaneous centre C (the second rear wheel can be free or a differential is used).
- Kinematics of tri-cycle similar to bicycle.

- Instantaneous centre determine relation between R, $\dot{\theta}_i$, r_i, l, ψ and ϕ.
- If no single C $\rightarrow$ wheel (s) slip will occur.

‘Car like’ Mobile Robots & Others

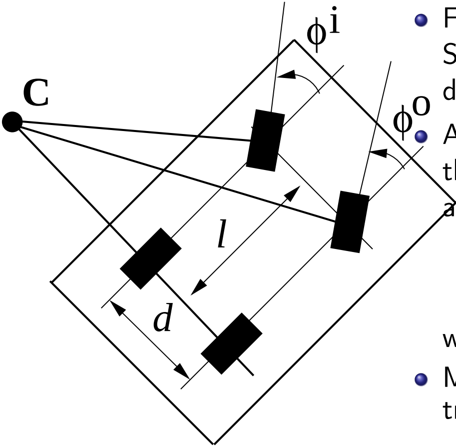

Figure 6: Four-wheeled mobile robot

- Four-wheeled (car like) mobile robot $\rightarrow$ Steering angle of front wheels is different in a turn.
- Ackerman steering to avoid slippage – the steering angle of ‘inside’ wheel ϕ^i and ‘outside’ wheel ϕ^o are related as

$$\cot(\phi^i) - \cot(\phi^o) = d/l$$

  where d and l as shown in figure.
- Multi-axle vehicles such as a truck with trailer
  - Steering front wheel to the ‘left’ results in ‘right’ motion of wheels on third axle – *non-minimum phase*.
  - More difficult to analyse and model.

Omni-directional wheels

- Six equally spaced 'free' barrels on the periphery.
- Each barrel can rotate about its axis as shown.
- Two rows of barrel → one barrel always in contact with ground.
- Distance of point of contact from the vehicle centre changes.

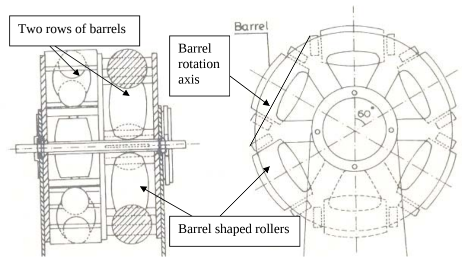

Figure 7: Omni-directional Wheel (Balakrishna & Ghosal, 1995)

- Rotation speed is $\dot{\theta}$ & Sliding speed is σ.
- Two components of velocity of wheel centre for a general case of barrel axis at an angle α to wheel axis – $(r\dot{\theta} + \sigma\cos\alpha, \sigma\sin\alpha)^T$.
- $\alpha = 90^\circ$ → components are $(r\dot{\theta}, \sigma)^T$, r is radius of wheel.

WMR WITH THREE OMNI-DIRECTIONAL WHEELS

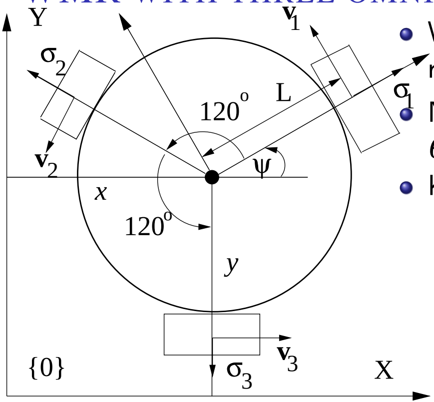

Figure 8: WMR with omni-directional wheels (Balakrishna & Ghosal, 1995)

- WMR with omni-directional wheels of radius r.
- No steering wheel, wheel rotation speed $\dot{\theta}_i,\ i = 1,2,3$.
- Kinematics – No slip condition at wheels

$$\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix} = \frac{1}{r}\begin{bmatrix} 0 & 1 & L_1 \\ -\sqrt{3}/2 & -1/2 & L_2 \\ \sqrt{3}/2 & -1/2 & L_3 \end{bmatrix}\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{pmatrix}$$

$$= [\ R\ ]\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{pmatrix}$$

$L_i,\ i = 1,2,3$ are the contact points.

- 3×3 matrix $[R]$ (**analogous to manipulator inverse Jacobian**) can be inverted $\rightarrow$ WMR controllable with $\dot{\theta}_i$.
- Can obtain sliding speeds $(\sigma_1, \sigma_2, \sigma_3)^T$ in terms of $(\dot{x}, \dot{y}, \dot{\psi})^T \rightarrow$ Not invertible $\Rightarrow$ WMR cannot be controlled by $\sigma_i,\ i = 1,2,3$ alone.

Modeling of Slip

- For rolling without slip in a conventional wheel of radius r, wheel centre velocity v and wheel angular velocity ω are related by $v = r\omega$.
- Either wheel and/or ground *must deform* for generating *tractive force* to drive a wheel.
- Deformable wheel and/or ground combination $\rightarrow$ Wheel slip will occur (Shekhar, 1997).
- Wheel slip is defined as

$$\lambda = (\dot{\theta} - \omega^*)/y$$

where $\omega^* \stackrel{\Delta}{=} v/r$ and $\dot{\theta}$ is the wheel angular velocity.

- $y = \omega^*$ when $\omega^* > \dot{\theta}$ and $y = \dot{\theta}$ when $\omega^* < \dot{\theta}*$
- Above implies $-1 \leq \lambda \leq 1$
  - Pure rolling $\rightarrow \lambda = 0$
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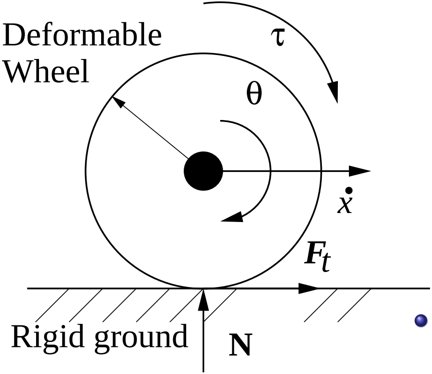

Figure 9: Single wheel dynamics

- Translational and rotation dynamics

$$
\begin{aligned}
F_t &= M_w \ddot{x} \\
\tau &= J_w \ddot{\theta} + F_t r
\end{aligned}
$$

  - F_t is the tractive force developed at wheel-ground interface
  - τ: torque applied at wheel axle.
  - M_w, J_w: mass and inertia of wheel.
  - $\ddot{x}$, $\ddot{\theta}$: linear and angular acceleration.

- In state-space form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}\tau$$

- Lie algebra approach $\rightarrow$ Not *locally* controllable if tractive force F_t is constant.
- F_t *must* be a function of linear and angular velocity of wheel.
- F_t is a function of normal reaction **N** and adhesion co-efficient μ_a.

Modeling of Slip (Contd.)

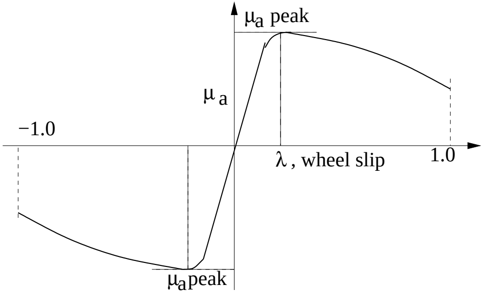

Figure 10: Typical adhesion coefficient Vs wheel slip

- Typically μ_a is a function of wheel slip λ and has a maximum μ_a peak.
- Typical plot shown in figure (Dugoff et al. 1970)
- Tractive force developed $\mathbf{F}_t = \mu_a(\lambda)\mathbf{N}$
- Actually $0 < |\mathbf{F}_t| \leq \mu_a \mathbf{N}$: Proper sign of $\mathbf{F}_t$ for acceleration or braking.
- Stable region – increasing μ_a with wheel slip.
- After μ_a peak, tractive force falls with increasing wheel slip $\rightarrow$ Unstable!

Modeling of WMR with Omni-directional Wheels

Equations of Motion

- Equations of motion using Lagrangian formulation (see Balakrishna & Ghosal, 1995)
  - Kinetic energy of platform of mass M_p and inertia I_p.
  - Kinetic energy of wheels of inertia $I_i,\ i = 1,2,3$.
  - No potential energy as motion on flat plane.
- Including tractive forces at wheels F_{t_i}, torque at each wheel τ_i and an approximate μ_a Vs λ curve.
- Equations of motion – 6 ODE's to take into account slip

$$\begin{bmatrix} M_p & 0 & 0 \\ 0 & M_p & 0 \\ 0 & 0 & I_p \end{bmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\psi} \end{pmatrix} + \dot{\Psi} \begin{bmatrix} 0 & -M_p & 0 \\ M_p & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{pmatrix} = r\,[\,R\,]^T \begin{pmatrix} F_{t_1} \\ F_{t_2} \\ F_{t_3} \end{pmatrix}$$

$$\begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{pmatrix} + r \begin{pmatrix} F_{t_1} \\ F_{t_2} \\ F_{t_3} \end{pmatrix} = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix}$$

Modeling of WMR with Omni-directional Wheels

Equations of Motion

- Equations of motion using Lagrangian formulation (see Balakrishna & Ghosal, 1995)
  - Kinetic energy of platform of mass M_p and inertia I_p.
  - Kinetic energy of wheels of inertia $I_i,\ i = 1,2,3$.
  - No potential energy as motion on flat plane.
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Modeling of WMR with Omni-directional Wheels

Control

- Desired Cartesian path $\mathbf{X}_d = (x_d(t), y_d(t), \psi_d(t))^T$ prescribed.
- PID control

$$\mathscr{F} = [K_v]\,(\dot{\mathbf{X}}_d - \dot{\mathbf{X}}) + [K_p]\,(\mathbf{X}_d - \mathbf{X}) + [K_i]\int(\mathbf{X}_d - \mathbf{X})\,dt$$

  $\mathbf{X} = (x(t),\, y(t),\, \psi(t))^T$.
- Cartesian forces $\mathscr{F}$ is related to (wheel) actuator torque by

$$\tau = [\,R\,]^{T^{-1}}\,\mathscr{F}$$

- Model based control scheme (see Module 7, Lecture 3) using 'ideal' rolling – $\tau = [\alpha]\,\tau' + \beta$ where

$$\begin{aligned}[\alpha] &= [\,R\,]^{-T}[M^*], \quad \beta = [\,R\,]^{-T}\{\dot{\Psi}\,[\,Q\,]\,\dot{\mathbf{X}}\} \\ \tau' &= \ddot{\mathbf{X}}_d + K_v(\dot{\mathbf{X}}_d - \dot{\mathbf{X}}) + K_p(\mathbf{X}_d - \mathbf{X})\end{aligned}$$

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Modeling of WMR with Omni-directional Wheels

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MODELING OF WMR WITH OMNI-DIRECTIONAL WHEELS

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Modeling of WMR with Omni-directional Wheels

Numerical simulation results

- Geometry of WMR: $L_1 = L_2 = L_3 = 0.5$ m, $r = 0.1$ m
- Inertia parameters: $M_p = 50.0$ Kg, $I_p = 5.0 \text{ kg m}^2$, $I_i = 2.0 \text{ kg m}^2$.
- Controller gains equal: $K_{p_i} = 15.0$, $K_{v_i} = 2\sqrt{K_{p_i}}$, and $K_{i_i} = 0.10$.

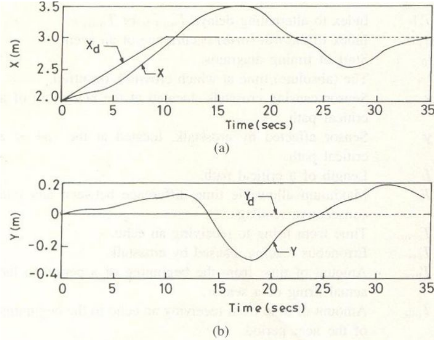

Figure 11: Straight line trajectory control using PID controller (Balakrishna & Ghosal, 1995)

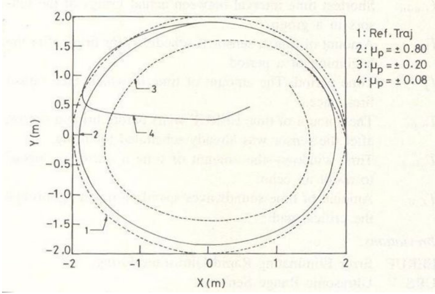

Figure 12: Circular trajectory control using model-based controller (Balakrishna & Ghosal, 1995)

Modeling of WMR with Omni-directional Wheels

Numerical simulation results (contd.)

- Performance of PID controller is quite poor.
- Maximum x error is approximately 0.5 m and maximum y error is approx 0.2 m.
- Model based controller using 'ideal' rolling as model performs slightly better.
- As the μ_a peak decreases from 0.8 to 0.08, performance of model based controller becomes poorer.
- Maximum radial error from desired circular trajectory of radius 2.0 m is approximately 1.7 m.
- It is important to model or take into account slip in WMR models – this is also borne out by experiments!!

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Summary of Modeling of WMR

- WMR modeling and analysis is very different from serial and parallel manipulators.
  - Base is not fixed.
  - Wheel-ground contact results in *non-holonomic* constraints – compare with *holonomic* constraints when two links are connected by a joint.
  - Non-holonomic constraints *does not* restrict the configuration space but *restrict* space of velocities!
- Various kinds of wheels in use – conventional, omni-directional and ball wheels.
  - Conventional wheels rotate about the wheel axis and can be steered about the normal.
  - Omni-directional wheel can rotate about its axis, slide along another direction and also steered.
  - Ball wheels can rotate about two different axis.
- WMR with three wheels simplest – four wheeled and multi-axle WMR's more difficult to model and analyse.
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Outline

1. Contents
2. Lecture 1
   - Wheeled Mobile Robots on Flat Terrain
3. Lecture 2*
   - Wheeled Mobile Robots on Uneven Terrain
4. Lecture 3*
   - Kinematics and Dynamics of WMR on Uneven Terrain
5. Module 9 – Additional Material
   - Problems, References and Suggested Reading

Contents of Lecture

- Introduction
- Modeling of torus-shaped wheel and uneven terrain
- Single wheel on uneven terrain – kinematic and dynamic modeling and simulation.
- A WMR for traversing uneven terrain without slip.
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INTRODUCTION

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- Recent interest in uneven and rough terrains & off-road environments.
  - Planetary exploration.
  - DARPA Grand Challenge – to develope a fully autonomous ground vehicle capable of completing a off-road course in limited time (see link for more details).
  - Luxury cars.
- Flat terrain – vehicle platform has 3 DOF consisting of position (x, y) and orientation ψ
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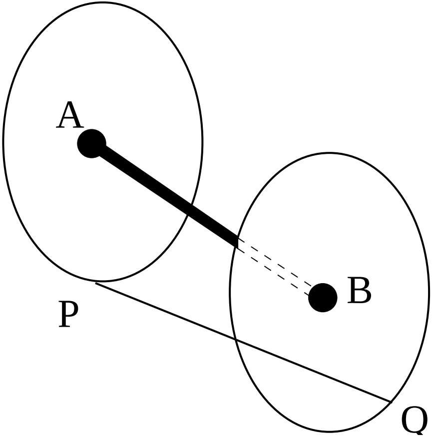

Figure 13: Wheel slip on uneven terrain

- Two wheels connected by a *fixed* length axle – distance AB is fixed.
- Points of contact with *uneven ground* P, Q can change $\rightarrow$ Length PQ is variable and *not equal* to AB.
- Variation of length PQ require a velocity component along axle AB and along the normal (Waldron, 1995).
- Also no instantaneous centre compatible with both wheels $\Rightarrow$ Wheel slip will occur.
- Wheel slip leads to a) localization errors, and b) wastage of fuel.

Introduction

- To overcome wheel slip $\rightarrow$ variable length axle (Choi & Sreenivasan, 1999).
- Add *passive* prismatic joint in axle – prismatic joint changes axle length by required amount to ensure compatible instantaneous centre for both wheels.
- At large inclination, gravity causes prismatic joint to change length in undesired way!
- Actuated (and controlled) prismatic joint – accurate sensing of slip is required.
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  - Use of torus shaped wheel – wheel has single point contact with uneven terrain.
  - Torus shaped wheels connected to WMR body with passive rotary joints.
  - Passive rotary joint allow lateral tilting and wheel-ground contact distance (PQ) to change.
  - Three actuated joints (for a 3DOF model) in WMR.
  - Rear wheels are driven and can tilt laterally.
  - Front wheel steered and can roll *freely*.
- Use of contact equations (Montana, 1988) to model wheel-ground contact.
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Modeling of Torus-shaped wheel and Uneven Terrain

Modeling of Surface

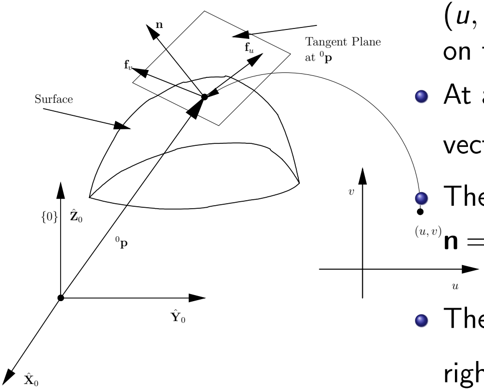

Figure 14: A surface in $\Re^3$

- A surface in $\Re^3$ can be represented in parametric form $(x, y, z)^T = \mathbf{f}(u, v)$, $(u, v) \in U \subseteq \Re^2$ maps to ${}^0\mathbf{p} = (x, y, z)^T$ on the surface.
- At a point, tangent plane defined by vectors $\mathbf{f}_u = \dfrac{\partial \mathbf{f}}{\partial u}$ and $\mathbf{f}_v = \dfrac{\partial \mathbf{f}}{\partial v}$.
- The normal to the surface is $\mathbf{n} = \dfrac{\mathbf{f}_u \times \mathbf{f}_v}{|\mathbf{f}_u \times \mathbf{f}_v|}$
- The vectors $\dfrac{\mathbf{f}_u}{|\mathbf{f}_u|}$, $\dfrac{\mathbf{f}_v}{|\mathbf{f}_v|}$, and $\mathbf{n}$ form a right-handed basis[a] at ${}^0\mathbf{p}$.

[a] If $\mathbf{f}_u$ and $\mathbf{f}_v$ are not orthogonal, then the orthogonal set is $\{\mathbf{f}_u/|\mathbf{f}_u|, \mathbf{n} \times \mathbf{f}_u/|\mathbf{f}_u|, \mathbf{n}\}$.

Modeling of Torus-shaped wheel and Uneven Terrain

Modeling of a Torus-shaped Wheel

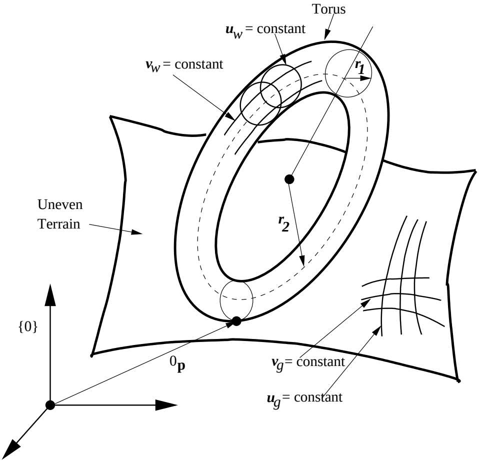

Figure 15: A torus-shaped wheel on uneven terrain

- Equation of a torus in parametric form

$$
\begin{aligned}
x &= r_1 \cos u_w \\
y &= \cos v_w (r_2 + r_1 \sin u_w) \\
z &= \sin v_w (r_2 + r_1 \sin u_w)
\end{aligned}
$$

- r_1 and r_2 are the two radii associated with a torus.
- Subscript w on the parameters u and v denote a wheel.
- Uneven terrain/ground $(x, y, z)^T = \mathbf{f}(u_g, v_g)$

Modeling of Torus-shaped wheel and Uneven Terrain

Metric, Curvature and Torsion of a Surface

- From the parametric equation, define
  - Second partials of $\mathbf{f}$, $\mathbf{f}_{(\cdot)(\cdot)} = \frac{\partial^2 \mathbf{f}}{\partial(\cdot)\partial(\cdot)}$, with respect to u and v,
  - Partial of $\mathbf{n}$, $\mathbf{n}_{(\cdot)}$, with respect to u and v.
  - Metric on a surface

$$[\mathbf{M}] = \begin{bmatrix} |\mathbf{f}_u| & 0 \\ 0 & |\mathbf{f}_v| \end{bmatrix}$$

  - Curvature form

$$[\mathbf{K}] = \begin{bmatrix} \frac{\mathbf{f}_u \cdot \mathbf{n}_u}{|\mathbf{f}_u|^2} & \frac{\mathbf{f}_u \cdot \mathbf{n}_v}{|\mathbf{f}_u||\mathbf{f}_v|} \\ \frac{\mathbf{f}_v \cdot \mathbf{n}_u}{|\mathbf{f}_u||\mathbf{f}_v|} & \frac{\mathbf{f}_v \cdot \mathbf{n}_v}{|\mathbf{f}_v|^2} \end{bmatrix}$$

  - Torsion form

$$[\mathbf{T}] = \begin{bmatrix} \frac{\mathbf{f}_v \cdot \mathbf{f}_{uu}}{|\mathbf{f}_u|^2|\mathbf{f}_v|} & \frac{\mathbf{f}_v \cdot \mathbf{f}_{uv}}{|\mathbf{f}_v|^2|\mathbf{f}_u|} \end{bmatrix}$$

- Metric 'defines' distance, Curvature determines 'in-plane' bending and Torsion determines 'out-of-plane' bending on a surface.

Modeling of Torus-shaped wheel and Uneven Terrain

Metric, Curvature and Torsion of a Surface

- From the parametric equation, define
  - Second partials of **f**, $\mathbf{f}_{(\cdot)(\cdot)} = \dfrac{\partial^2 \mathbf{f}}{\partial(\cdot)\partial(\cdot)}$, with respect to u and v,
  - Partial of **n**, $\mathbf{n}_{(\cdot)}$, with respect to u and v.
  - Metric on a surface
$$[\mathbf{M}] = \begin{bmatrix} |\mathbf{f}_u| & 0 \\ 0 & |\mathbf{f}_v| \end{bmatrix}$$
  - Curvature form
$$[\mathbf{K}] = \begin{bmatrix} \frac{\mathbf{f}_u \cdot \mathbf{n}_u}{|\mathbf{f}_u|^2} & \frac{\mathbf{f}_u \cdot \mathbf{n}_v}{|\mathbf{f}_u||\mathbf{f}_v|} \\ \\ \frac{\mathbf{f}_v \cdot \mathbf{n}_u}{|\mathbf{f}_u||\mathbf{f}_v|} & \frac{\mathbf{f}_v \cdot \mathbf{n}_v}{|\mathbf{f}_v|^2} \end{bmatrix}$$
  - Torsion form
$$[\mathbf{T}] = \begin{bmatrix} \dfrac{\mathbf{f}_v \cdot \mathbf{f}_{uu}}{|\mathbf{f}_u|^2|\mathbf{f}_v|} & \dfrac{\mathbf{f}_v \cdot \mathbf{f}_{uv}}{|\mathbf{f}_v|^2|\mathbf{f}_u|} \end{bmatrix}$$
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Modeling of Torus-shaped wheel and Uneven Terrain

Modeling of a Torus-shaped Wheel

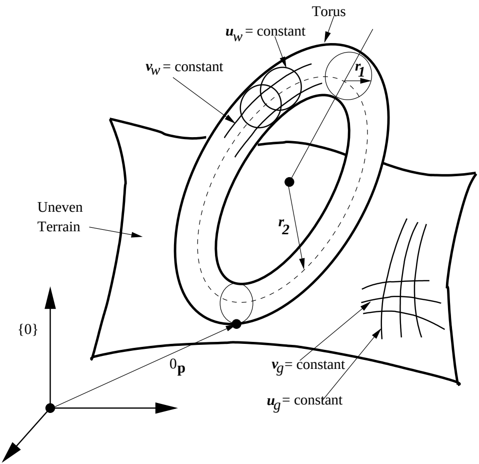

Figure 16: A torus-shaped wheel on uneven terrain

- For the torus-shaped wheel

$$[\mathbf{M}_w] = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 + r_1 \sin u_w \end{bmatrix}$$

$$[\mathbf{K}_w] = \begin{bmatrix} \frac{1}{r_1} & 0 \\ 0 & \frac{\sin u_w}{r_2 + r_1 \sin u_w} \end{bmatrix}$$

$$[\mathbf{T}_w] = [0 \quad \frac{\cos u}{r_2 + r_1 \sin u_w}]$$

Modeling of Torus-shaped wheel and Uneven Terrain

Modeling of Uneven Terrain

- Uneven terrain – smooth and hard, *not* terrains with sand, dirt or any discontinuities!
- Explicit or parametric equation of uneven terrain not available.
- Local elevation of a point is known from measurement (laser scanner).
- Ill-posed problem to obtain $\mathbf{f}(u, v)$ from measurements– *non-uniqueness*.
- Use bi-cubic and B-spline surfaces (see Mortenson, 1985)
  - Bi-cubic surface patch: $\mathbf{f}(u, v) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} u^i v^j, \quad (u, v) \in [0, 1]$
  - Can be determined from 4 corner points of the patch.
  - Patches can be smoothly connected to make up the whole surface.
  - Higher-order continuity can be obtained by using Non-uniform Rational B-Spline (NURBS)
- Matlab® spline toolbox – Partial derivatives of surface available to compute metric, curvature and torsion form.

Modeling of Torus-shaped wheel and Uneven Terrain

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Modeling of Torus-shaped wheel and Uneven Terrain

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Modeling of Torus-shaped wheel and Uneven Terrain

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Modeling of Torus-shaped wheel and Uneven Terrain

Examples of Uneven Terrains

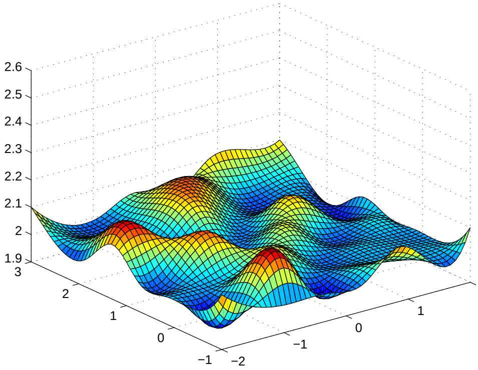

Figure 17: A bi-cubic uneven surface

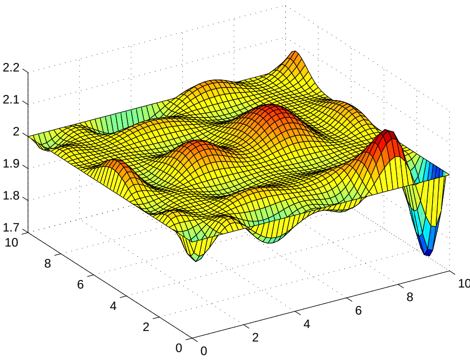

Figure 18: A B-spline uneven surface

Modeling of Torus-shaped wheel and Uneven Terrain

Kinematics of Contact

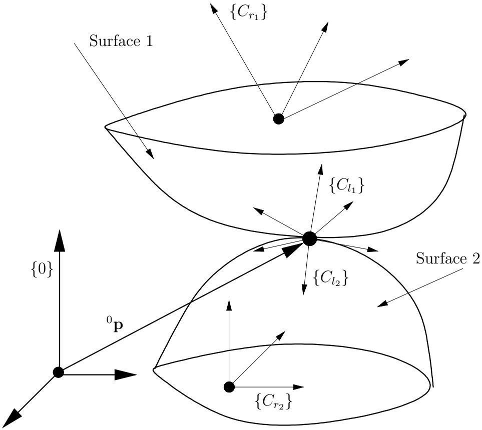

Figure 19: Two arbitrary surfaces in single-point contact

- Two surfaces 1 and 2 described with respect to $\{C_{r_1}\}$ and $\{C_{r_2}\}$.
- Parametric equations are $\mathbf{f}(u_1, v_1)$ and $\mathbf{f}(u_2, v_2)$.
- At point of contact ${}^0\mathbf{p}$, fix frames $\{C_{l_1}\}$ and $\{C_{l_2}\}$.
- ψ – angle between X axis of $\{C_{l_1}\}$ and $\{C_{l_2}\}$.
- (u_1, v_1), (u_2, v_2) and ψ define the 5 DOF for single-point contact.
- Define metric, $[M]$, Curvature $[K]$ and Torsion $[T]$ for the two surfaces at ${}^0\mathbf{p}$.

Modeling of Torus-shaped wheel and Uneven Terrain

Kinematics of Contact

- Contact equations: Relationship between $(\dot{u}_1, \dot{v}_1, \dot{u}_2, \dot{v}_2, \dot{\psi})$ and the linear and angular velocity components v_x, v_y, v_z and $\omega_x, \omega_y, \omega_z$

$$
\begin{aligned}
(\dot{u}_1, \dot{v}_1)^T &= [\mathbf{M}_1]^{-1}([\mathbf{K}_1]+[\mathbf{K}^*])^{-1}[(-\omega_y, \omega_x)^T - [\mathbf{K}^*](v_x, v_y)^T] \\
(\dot{u}_2, \dot{v}_2)^T &= [\mathbf{M}_2]^{-1}[R_\psi]([\mathbf{K}_1]+[\mathbf{K}^*])^{-1}[(-\omega_y, \omega_x)^T + [\mathbf{K}_1](v_x, v_y)^T] \\
\dot{\psi} &= \omega_z + [\mathbf{T}_1][\mathbf{M}_1](\dot{u}_1, \dot{v}_1)^T + [\mathbf{T}_2][\mathbf{M}_2](\dot{u}_2, \dot{v}_2)^T \\
0 &= v_z
\end{aligned}
$$

where the *relative* curvature of surface 2 with respect to 1 is $[\mathbf{K}^*] = [R_\psi][\mathbf{K}_2][R_\psi]^T$ and the rotation matrix $[R]$ is

$$
[R_\psi] = \begin{pmatrix} \cos\psi & -\sin\psi \\ -\sin\psi & -\cos\psi \end{pmatrix}
$$

- Can also invert the equations

$$
\begin{aligned}
(v_x, v_y)^T &= -[\mathbf{M}_1](\dot{u}_1, \dot{v}_1)^T + [R_\psi][\mathbf{M}_2](\dot{u}_2, \dot{v}_2)^T \\
(\omega_y, -\omega_x)^T &= -[\mathbf{K}_1][\mathbf{M}_1](\dot{u}_1, \dot{v}_1)^T - [R_\psi][\mathbf{K}_2][\mathbf{M}_2](\dot{u}_2, \dot{v}_2)^T \\
\omega_z &= \dot{\psi} - [\mathbf{T}_1][\mathbf{M}_1](\dot{u}_1, \dot{v}_1)^T - [\mathbf{T}_2][\mathbf{M}_2](\dot{u}_2, \dot{v}_2)^T \\
v_z &= 0
\end{aligned}
$$

- $v_z = 0$ *holonomic* constraint $\Rightarrow$ Ensures surfaces stay in contact!
- Five other equations need to be *numerically* integrated with initial conditions for solution.

- Can also invert the equations

$$
\begin{aligned}
(v_x, v_y)^T &= -[\mathbf{M}_1](\dot{u}_1, \dot{v}_1)^T + [R_\psi][\mathbf{M}_2](\dot{u}_2, \dot{v}_2)^T \\
(\omega_y, -\omega_x)^T &= -[\mathbf{K}_1][\mathbf{M}_1](\dot{u}_1, \dot{v}_1)^T - [R_\psi][\mathbf{K}_2][\mathbf{M}_2](\dot{u}_2, \dot{v}_2)^T \\
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Modeling of Torus-shaped wheel and Uneven Terrain

Kinematics of Contact

- Contact equations similar to constraint equations for joints (see Module 2, Lecture 2)
- *Main difference*: equations contain derivatives with respect to time!
- *Two main types of contacts* :
  - *Pure rolling* – $v_x = v_y = 0 \rightarrow$ important for WMR's
  - *Pure sliding* – $\omega_x = \omega_y = 0$
- Pure rolling $v_x = v_y = v_z = 0 \rightarrow$ Three DOF *in velocities*.
- Very much unlike a 3 DOF spherical joint $\rightarrow$ S joint $x = y = z$ are also same for both links!
- Pure rolling – *non-holonomic* $\rightarrow$ Only $v_x = v_y = 0$ (also $v_z = 0$) and x, y, z coordinates of the contact point can change as the rolling proceeds!

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Single Wheel on Uneven Terrain

Kinematic Analysis

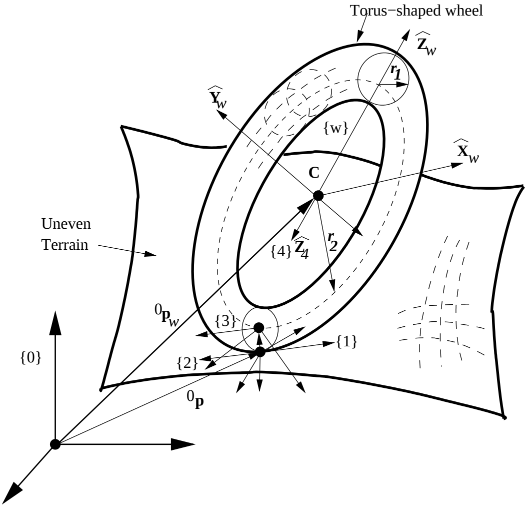

Figure 20: Single wheel on uneven ground

- $\{C_{r_2}\}$ is same as $\{0\}$.
- $\{C_{r_1}\}$ is fixed at the wheel centre C, same as $\{w\}$.
- $\{C_{l_1}\}$ and $\{C_{l_2}\}$ at the contact point – denoted by $\{2\}$ and $\{1\}$.
- $\{3\}$ and $\{4\}$ as shown in figure.

Single Wheel on Uneven Terrain

Kinematic Analysis (Contd.)

- 4×4 transformation matrices from assigned frames:

$$
{}^0_1[T] = \begin{pmatrix} l_1 & m_1 & n_1 & u_g \\ l_2 & m_2 & n_2 & v_g \\ l_3 & m_3 & n_3 & f(u_g, v_g) \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad {}^1_2[T] = \begin{pmatrix} \cos\psi & -\sin\psi & 0 & 0 \\ -\sin\psi & -\cos\psi & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$l_i,\ m_i,\ n_i,\ i = 1,2,3$ are the components of $\{\mathbf{f}_u/|\mathbf{f}_u|, \mathbf{n} \times \mathbf{f}_u/|\mathbf{f}_u|, \mathbf{n}\}$.

- Transformation matrices from $\{2\}$ to $\{4\}$ are

$$
{}^2_3[T] = \begin{pmatrix} \sin u_w & 0 & \cos u_w & 0 \\ 0 & 1 & 0 & 0 \\ -\cos u_w & 0 & \sin u_w & -r_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad {}^3_4[T] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\sin v_w & \cos v_w & 0 \\ 0 & -\cos v_w & -\sin v_w & -r_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

- Transformation matrix from $\{4\}$ to $\{w\}$ is

$$
{}^4_w[T] = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

- The transformation matrix from $\{w\}$ to $\{0\}$ is

$$ {}^{0}_{w}[T] = {}^{0}_{1}[T]^{1}_{2}[T]^{2}_{3}[T]^{3}_{4}[T]^{4}_{w}[T] $$

- The contact equation for a single-wheel *rolling without slip*

$$
\begin{aligned}
(\dot{u}_w, \dot{v}_w)^T &= [\mathrm{M}_w]^{-1}([\mathrm{K}_w] + [\mathrm{K}^*])^{-1}[(-\omega_y, \omega_x)^T] \\
(\dot{u}_g, \dot{v}_g)^T &= [\mathrm{M}_g]^{-1}[\mathrm{R}_\psi]([\mathrm{K}_g] + [\mathrm{K}^*])^{-1}[(-\omega_y, \omega_x)^T] \\
\dot{\psi} &= \omega_z + [\mathrm{T}_w][\mathrm{M}_w](\dot{u}_w, \dot{v}_w)^T + [\mathrm{T}_g][\mathrm{M}_g](\dot{u}_g, \dot{v}_g)^T \\
0 &= v_z
\end{aligned}
$$

w denotes wheel and g denotes ground.

- Inputs: ω_x, ω_y, ω_z.
- Integrate contact equations to obtain evolution of point of contact on wheel *and* ground in time.

- The transformation matrix from $\{w\}$ to $\{0\}$ is

$$ {}^0_w[T] = {}^0_1[T]\,{}^1_2[T]\,{}^2_3[T]\,{}^3_4[T]\,{}^4_w[T] $$

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Single Wheel on Uneven Terrain

Kinematic Analysis (contd.)

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Single Wheel on Uneven Terrain

Kinematic Analysis (Contd.)

- The transformation matrix from $\{w\}$ to $\{0\}$ is

$$ {}^0_w[T] = {}^0_1[T]^1_2[T]^2_3[T]^3_4[T]^4_w[T] $$

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$$
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Single Wheel on Uneven Terrain

Kinematic Analysis (contd.)

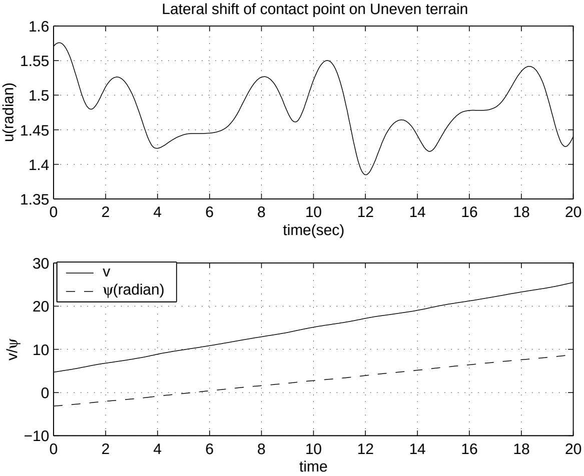

Figure 21: Variation of u_w, v_w and ψ with t

- Simulation for **bi-cubic surface** shown earlier.
- $r_1 = 0.05$ m, $r_2 = 0.25$ m.
- Wheel tilts as it rolls.

Single Wheel on Uneven Terrain

Kinematic Analysis (Contd.)

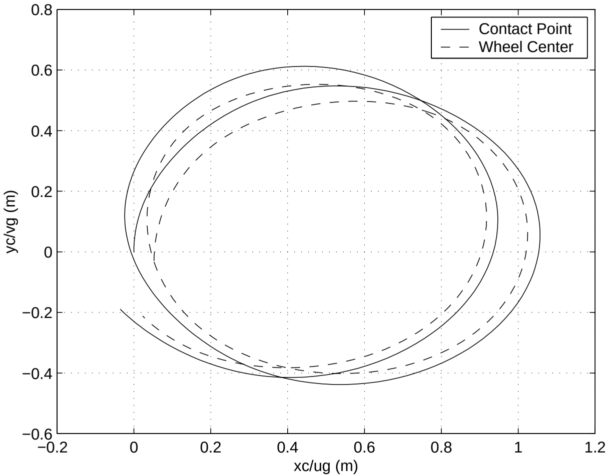

Figure 22: Plot of wheel centre and ground contact point

- Trace of wheel centre and ground contact point is different.
- Unlike a disk rolling on flat surface.

- Equations of motion of a torus-shaped wheel on uneven terrain using Lagrangian formulation.
- Non-holonomic constraints

$$(v_x, v_y)^T = -[\mathbf{M}_w](\dot{u}_w, \dot{v}_w)^T + [R_\psi][\mathbf{M}_g](\dot{u}_g, \dot{v}_g)^T = (0,0)^T$$

after rearranging $[\Psi(\mathbf{q})]\dot{\mathbf{q}} = 0$

- Kinetic energy of wheel
  - Angular velocity ${}^0_w[\Omega]$ from rotation matrix as ${}^0_w[\dot{R}]\ {}^0_w[R]^T$.
  - Linear velocity by differentiating position of wheel centre – ${}^0\mathbf{V}_w = {}^0\dot{\mathbf{p}}_w$
  - Kinetic energy: $KE = \frac{1}{2}\Omega^T[I_w]\Omega + \frac{1}{2}m_w {}^0\mathbf{V}_w{}^2$
- Potential energy: $PE = m_w g z_{wc}$
- Equations of motion (see Module 6, Lecture 1)

$$[\mathbf{M}(\mathbf{q})]\ddot{\mathbf{q}} + [\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})]\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \tau + [\Psi(\mathbf{q})]^T\lambda$$

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Single Wheel on Uneven Terrain

Dynamic Analysis

- Equations of motion of a torus-shaped wheel on uneven terrain using Lagrangian formulation.
- Non-holonomic constraints

$$(v_x, v_y)^T = -[\mathbf{M}_w](\dot{u}_w, \dot{v}_w)^T + [R_\psi][\mathbf{M}_g](\dot{u}_g, \dot{v}_g)^T = (0,0)^T$$

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Single Wheel on Uneven Terrain

Dynamic Analysis (contd.)

- Wheel dimensions: $r_1 = 0.05$ m, $r_2 = 0.25$ m.
- Mass of wheel $m_w = 1.0$ kg.
- Inertia components of a torus-shaped wheel

$$[I_w] = \begin{pmatrix} \frac{1}{4}m_w({3r_1}^2 + {4r_2}^2) & 0 & 0 \\ 0 & \frac{1}{8}m_w({5r_1}^2 + {4r_2}^2) & 0 \\ 0 & 0 & \frac{1}{8}m_w({5r_1}^2 + {4r_2}^2) \end{pmatrix}$$

- Initial conditions *satisfy* non-holonomic constrains.
- No external force $\tau = \mathbf{0}$
- Torus-shaped wheels rolls down under gravity on surface shown next.

Single Wheel on Uneven Terrain

Dynamic Analysis (Contd.)

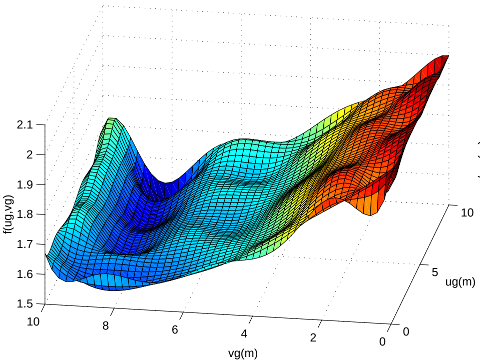

Figure 23: B-spline surface used for single wheel dynamics

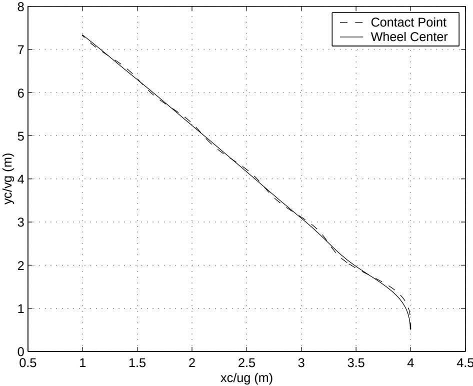

Figure 24: Trace of wheel centre and ground contact points

Single Wheel on Uneven Terrain

Dynamic Analysis (Contd.)

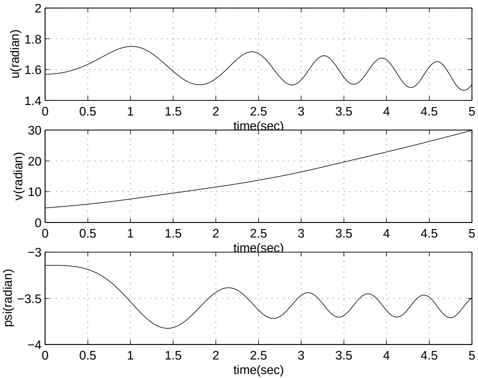

Figure 25: Variation of u_w, v_w and ψ with t

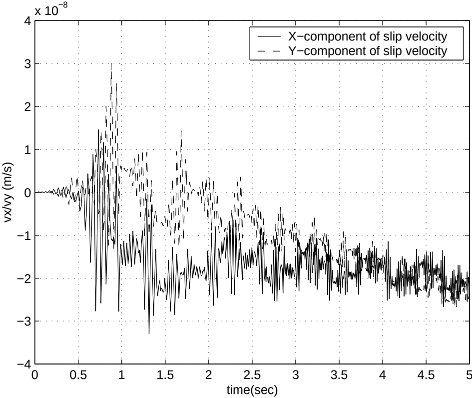

Figure 26: Components of slip velocity at wheel-ground contact point

- Slip components are very small (10^{-8}m/sec).
- Simulation checked for conservation of energy.

- Three torus-shaped wheels connected to rigid platform with rotary (R) joints.
- Two possible configurations – platform with 3 DOF
  - Each wheel attached to platform with two R joints.
  - For rear wheels — one R joint is actuated by a motor making the wheel roll.
  - For front wheel – one R joint represents steering.
  - For rear wheels – one R joint is passive allowing lateral tilting about axis perpendicular to wheel rotation axis.
  - For front wheel – one R joint represents free rolling of the wheel.
- WMR platform with 6 DOF
  - Each wheel attached to platform by three R joints.
  - 2 R joints in rear and front wheel function as above.
  - Additional R joint in rear wheel allow for steering.
  - Additional R joint in front wheel allow lateral tilt.

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A WMR for Traversing Uneven Terrain

Configurations

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A WMR for Traversing Uneven Terrain

3 DOF Configuration

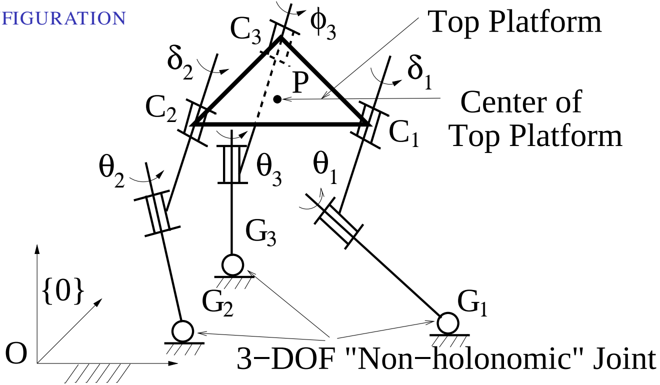

Figure 27: Schematic of a three-wheeled mobile robot with 3 DOF

- Wheel-ground contact has 3 DOF instantaneously – only velocities are restricted $\rightarrow$ *non-holonomic joint*
- Grübler criterion – DOF $= 6(N - J - 1) + \sum_{i=1}^{J} F_i$ – $N = 8$, $J = 9$, and $\sum_{i=1}^{J} F_i = 15 \rightarrow$ DOF $= 3$.

A WMR for Traversing Uneven Terrain

6 DOF Configuration

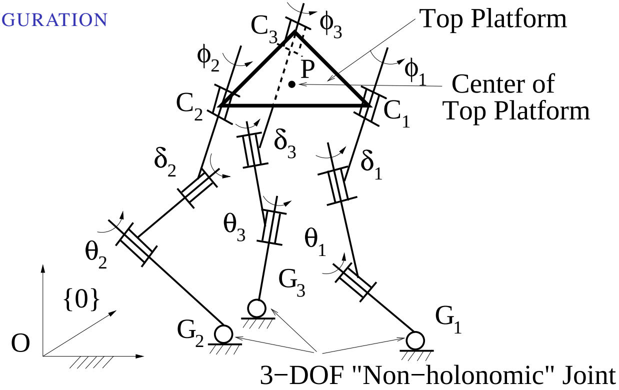

Figure 28: Schematic of three wheeled mobile robot with 6 DOF

- Wheel-ground contact – 3 DOF *non-holonomic joint*.
- Grübler criterion – $\text{DOF} = 6(N - J - 1) + \sum_{i=1}^{J} F_i$, $N = 11$, $J = 12$ and $\sum_{i=1}^{J} F_i = 18 \rightarrow$ DOF $= 6$.
- Study kinematics and dynamics of 3 DOF configuration.

SUMMARY

- WMR with fixed length axle, moving on uneven terrain, can slip.
- Variable length axle concepts and concept of passive tilting.
- Geometric modeling of torus-shaped wheel and uneven terrain.
- Contact equations representing 5 DOF between two surfaces in single point contact.
- Kinematic and dynamic analysis and simulation of single wheel on uneven terrain.
- Configuration of a three-wheeled mobile robot for traversing uneven terrain without slip.
- Kinematic, dynamic and stability analysis in next lecture.

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Outline

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- Kinematic analysis a three-wheeled mobile robot
  - Solution of the direct kinematics problem.
  - Solution of the inverse kinematics problem.
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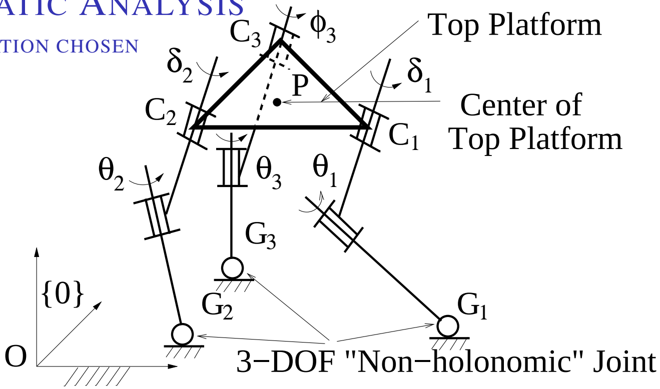

Figure 29: Equivalent instantaneous parallel manipulator

- Instantaneous parallel manipulator with a platform "connected" to ground by three serial chains.
- Actuated: Rear wheel rotations, θ_1 and θ_2, front wheel steering ϕ_3.
- Passive: Rear wheel tilt δ_1, δ_2 and front wheel rotation θ_3.

Kinematic Analysis

- Analyse the WMR as a parallel manipulator at *every* instant.
- *Instantaneously* – Wheels are not fixed as in a parallel manipulator!
- Non-holonomic (no slip) constraints and hence kinematics in terms of joint rates!
- Direct Kinematic: Given actuation rates $\dot{\theta}_1$, $\dot{\theta}_2$ and $\dot{\phi}_3$, the terrain and WMR geometry, find orientation of top platform ${}^0_p[R]$ and the position vector of the centre of the platform.
- Inverse kinematics: Given geometry of WMR and terrain and given any three of $V_{p_x}, V_{p_y}, V_{p_z}$, $\Omega_{p_x}, \Omega_{p_y}, \Omega_{p_z}$, find rear wheel actuator inputs $\dot{\theta}_1$ and $\dot{\theta}_2$ and the steering input to the front wheel $\dot{\phi}_3$.

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- *Step 1: Generate the uneven terrain surface:*
  - Use bi-cubic patches or B-splines to reconstruct the surface from elevation data.
  - Find [**M**], [**K**] and [**T**] for the ground and wheels at the three wheel-ground contact points.
- *Step 2: Form contact equations:*
  - For *each* wheel, obtain 5 ODE's in u_i, v_i, u_{g_i}, v_{g_i}, and ψ_i $i = 1, 2, 3$.
  - For no-slip motion, set $v_x = v_y = 0$ for each of the three wheels.
  - ω_x, ω_y, and ω_z are related to Ω_{p_x}, Ω_{p_y}, Ω_{p_z}, and the input and passive joint rates

$$^0(\omega_x, \omega_y, \omega_z)^T = ^0 (\Omega_{p_x}, \Omega_{p_y}, \Omega_{p_z})^T + ^0 \omega_{\text{input}}$$

  - Above equation *couples* all five sets of ODE's resulting in a set of 15 ODE's in 21 variables – 15 contact variables, $\theta_1, \theta_2, \theta_3$, δ_1, δ_2, and ϕ_3.
  - Out of 21 variables, θ_1, θ_2, and ϕ_3 are actuated and known.

Kinematic Analysis

Algorithm

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- *Step 3: Obtain the angular and linear velocities of centre of the platform:*
  - If γ, β, α be a Z-Y-X Euler angle parametrization (see Module 2, Lecture 1) representing the orientation of the platform, then

$$
\begin{aligned}
{}^0\Omega_{p_x} &= \dot{\alpha}\cos\beta\cos\gamma - \dot{\beta}\sin\gamma = f_1(u_i, v_i, u_{g_i}, v_{g_i}, \psi_i, \dot{u}_i, \dot{v}_i, \dot{u}_{g_i}, \dot{v}_{g_i}, \dot{\psi}_i)\\
{}^0\Omega_{p_y} &= \dot{\alpha}\cos\beta\sin\gamma + \dot{\beta}\cos\gamma = f_2(u_i, v_i, u_{g_i}, v_{g_i}, \psi_i, \dot{u}_i, \dot{v}_i, \dot{u}_{g_i}, \dot{v}_{g_i}, \dot{\psi}_i)\\
{}^0\Omega_{p_z} &= \dot{\gamma} - \dot{\alpha}\sin\beta = f_3(u_i, v_i, u_{g_i}, v_{g_i}, \psi_i, \dot{u}_i, \dot{v}_i, \dot{u}_{g_i}, \dot{v}_{g_i}, \dot{\psi}_i), \quad i = 1,2,3
\end{aligned}
$$

  - If x_c, y_c, and z_c denote the coordinates of the centre of the platform in $\{0\}$, the linear velocity of the centre of the platform is

$$
{}^0(V_{p_x}, V_{p_y}, V_{p_z})^T \stackrel{\Delta}{=} {}^0(\dot{x}_c, \dot{y}_c, \dot{z}_c)^T = {}^0\mathbf{V}_{w_i} + {}^0(\Omega_{p_x}, \Omega_{p_y}, \Omega_{p_z})^T \times {}^0\mathbf{p}_{c_i}
$$

    $i = 1,2,3$ denote three wheels, ${}^0\mathbf{p}_{c_i}$ locates the point of attachment of the wheel to the platform from the centre of the platform, and ${}^0\mathbf{V}_{w_i}$ is the velocity of the centre of the wheel.

- *Step 4: Form holonomic constraint equations:*
  - Distance between the three points C_1, C_2, and C_3 must remain constant for the moving platform to be *rigid*.
  - *Holonomic* constraint are

$$\begin{aligned} \|{}^0\mathbf{p}_{C_1} - {}^0\mathbf{p}_{C_2}\|^2 &= {l_{12}}^2 \\ \|{}^0\mathbf{p}_{C_2} - {}^0\mathbf{p}_{C_3}\|^2 &= {l_{23}}^2 \\ \|{}^0\mathbf{p}_{C_3} - {}^0\mathbf{p}_{C_1}\|^2 &= {l_{31}}^2 \end{aligned}$$

  ${}^0\mathbf{p}_{C_i}$, $i = 1,2,3$ locate C_1, C_2, C_3, from the origin of $\{0\}$ and l_{ij} is the distance between centres of wheels i and j, respectively.
  - Holonomic constraints are same as spherical-spherical pair (S-S) joint constraint of Module 2, Lecture 2.

Kinematic Analysis

Algorithm (contd.)

- *Solution of Direct Kinematics Problem:*
  - Steps 1, 2 & 4 $\rightarrow$ 15 ODE's and 3 algebraic equations in 21 variables.
  - 18 Unknown variables – $\dot{\theta}_1$, $\dot{\theta}_2$ and $\dot{\phi}_3$ are given!
  - Differentiate holonomic constraints $\rightarrow$ Convert to a system of 18 ODE's.
  - Integrate using ODE solver with initial conditions.
  - Obtain position vector of centre and orientation of platform from 21 variables at each t
- *Solution of Inverse Kinematics Problem:*
  - Steps 1 to 4 $\rightarrow$ 21 first order ODE's and 3 algebraic equations.
  - Assuming linear velocity of the platform $\dot{x}_c$, $\dot{y}_c$ and the angular velocity about the vertical, $\dot{\gamma}$, are given $\rightarrow$ 24 unknowns.
  - Convert DAE's into ODE's by differentiating holonomic constraints.
  - Integrate set of 24 ODE's and obtain required θ_1, θ_2 and ϕ_3 as function of t.
- *Initial conditions* for the direct and inverse kinematics problems *must* satisfy the *holonomic and non-holonomic* constraints.

KINEMATIC ANALYSIS

ALGORITHM (CONTD.)

- *Solution of Direct Kinematics Problem:*
  - Steps 1, 2 & 4 $\rightarrow$ 15 ODE's and 3 algebraic equations in 21 variables.
  - 18 Unknown variables – $\dot{\theta}_1$, $\dot{\theta}_2$ and $\dot{\phi}_3$ are given!
  - Differentiate holonomic constraints $\rightarrow$ Convert to a system of 18 ODE's.
  - Integrate using ODE solver with initial conditions.
  - Obtain position vector of centre and orientation of platform from 21 variables at each t
- *Solution of Inverse Kinematics Problem:*
  - Steps 1 to 4 $\rightarrow$ 21 first order ODE's and 3 algebraic equations.
  - Assuming linear velocity of the platform $\dot{x_c}$, $\dot{y_c}$ and the angular velocity about the vertical, $\dot{\gamma}$, are given $\rightarrow$ 24 unknowns.
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- Geometry of the WMR
  - Length of the rear axle 1 m.
  - Distance of the centre of front wheel from middle of the rear axle 1 m.
  - Torus-shaped wheel: $r_1 = 0.05$ m, $r_2 = 0.25$ m.
  - WMR centre is at the centroid of the triangular platform.
- Initial conditions: $u_1 = 1.5816$, $v_1 = \frac{3\pi}{2}$, $u_{g_1} = 4.089$ m, $v_{g_1} = 0.3917$ m, $\psi_1 = -3.1414$, $u_2 = 1.5560$, $v_2 = \frac{3\pi}{2}$, $u_{g_2} = 3.1$ m, $v_{g_2} = 0.4$ m, $\psi_2 = -3.1413$, $u_3 = 1.5735$, $v_3 = \frac{3\pi}{2}$, $u_{g_3} = 3.5977$ m, $v_{g_3} = 1.4097$ m, $\psi_3 = -3.1404$, $\theta_3 = 0$, $\delta_1 = 0$, and $\delta_2 = 0$ – All angles in radians.
- Actuator inputs: $\dot{\theta}_1 = -1$ rad/sec, $\dot{\theta}_2 = -0.9$ rad/sec, $\dot{\phi}_3 = 0.005t$ rad/sec.
- Uneven surface same as used for single wheel dynamic simulation.

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  - Length of the rear axle 1 m.
  - Distance of the centre of front wheel from middle of the rear axle 1 m.
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Kinematic Analysis

Numerical simulations results for direct kinematics

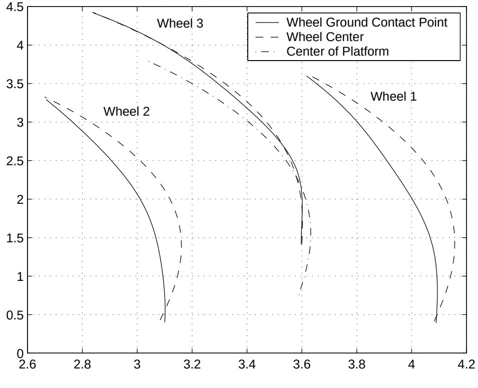

Figure 30: Locus of wheel-ground contact point, wheel centre and platform centre

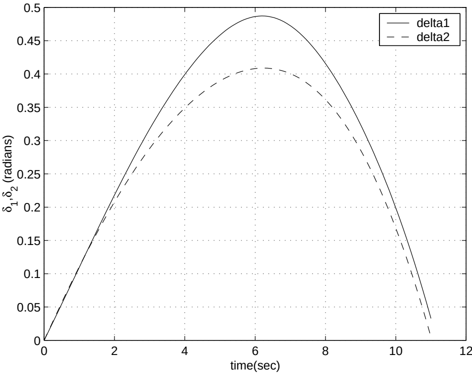

Figure 31: Variation of lateral tilt δ_1 and δ_2

- The locus of wheel centres are *not* the same as the wheel-ground contact point – due to uneven terrain and lateral tilt!!
- Lateral tilt changes at different points of the uneven terrain.

Kinematic Analysis

Numerical simulations results for direct kinematics

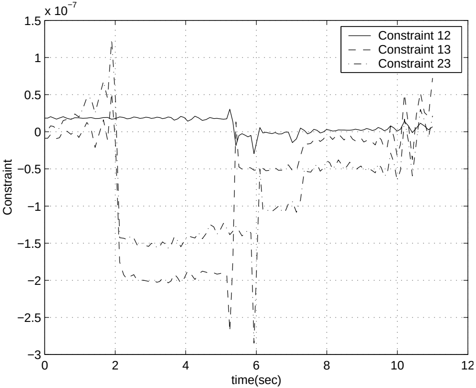

Figure 32: Satisfaction of holonomic constraints

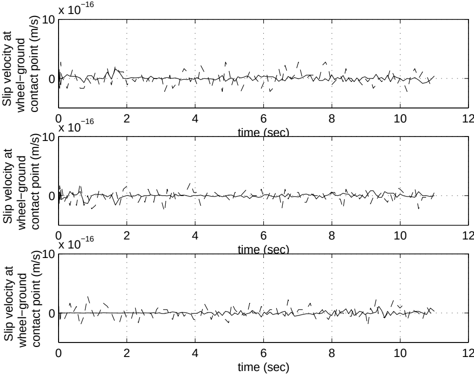

Figure 33: Slip velocities at wheel-ground contact points for three wheels

- The holonomic constraints are satisfied up to 10^{-7} m
- There is virtually no slip – WMR traverses uneven terrain without slip!!

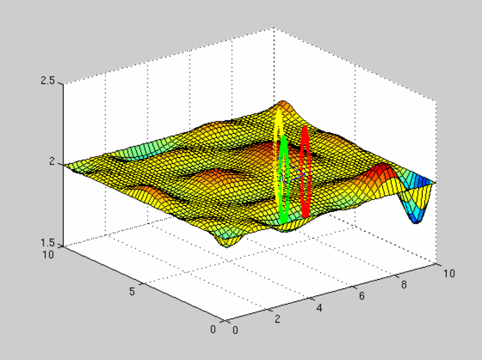

Figure 34: Video of direct kinematics of WMR

See video for a simulation of the three wheeled mobile robot on uneven terrain.

- Geometry of the WMR – same as used in direct kinematics
  - Length of the rear axle 1 m.
  - Distance of the centre of front wheel from middle of the rear axle 1 m.
  - Torus-shaped wheel: $r_1 = 0.05$ m, $r_2 = 0.25$ m.
  - WMR centre is at the centroid of the triangular platform.
- Initial conditions: $u_1 = 1.5801$, $v_1 = \frac{3\pi}{2}$, $u_{g_1} = 3.9904$ m, $v_{g_1} = 0.4895$ m, $\psi_1 = -3.1415$, $u_2 = 1.5535$, $v_2 = \frac{3\pi}{2}$, $u_{g_2} = 2$ m, $v_{g_2} = 0.5$ m, $\psi_2 = -3.1411$, $u_3 = 1.5808$, $v_3 = \frac{3\pi}{2}$, $u_{g_3} = 3.4898$ m, $v_{g_3} = 1.5702$ m, $\psi_3 = -3.1414$, $z_c = 2.22$ m, $\alpha = -0.0448$, and $\beta = -0.0498$ – All angles in radians.
- Given inputs: $\dot{x}_c = 0.03$ m/sec, $\dot{y}_c = 0.15$ m/sec, and $\dot{\gamma} = -0.005t$ rad/sec.
- Uneven surface same as used for direct kinematics.

Kinematic Analysis

Numerical simulations results for inverse kinematics

- Geometry of the WMR – same as used in direct kinematics
  - Length of the rear axle 1 m.
  - Distance of the centre of front wheel from middle of the rear axle 1 m.
  - Torus-shaped wheel: $r_1 = 0.05$ m, $r_2 = 0.25$ m.
  - WMR centre is at the centroid of the triangular platform.
- Initial conditions: $u_1 = 1.5801$, $v_1 = \frac{3\pi}{2}$, $u_{g_1} = 3.9904$ m, $v_{g_1} = 0.4895$ m, $\psi_1 = -3.1415$, $u_2 = 1.5535$, $v_2 = \frac{3\pi}{2}$, $u_{g_2} = 2$ m, $v_{g_2} = 0.5$ m, $\psi_2 = -3.1411$, $u_3 = 1.5808$, $v_3 = \frac{3\pi}{2}$, $u_{g_3} = 3.4898$ m, $v_{g_3} = 1.5702$ m, $\psi_3 = -3.1414$, $z_c = 2.22$ m, $\alpha = -0.0448$, and $\beta = -0.0498$ – All angles in radians.
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- Uneven surface same as used for direct kinematics.

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- Given inputs: $\dot{x}_c = 0.03$ m/sec, $\dot{y}_c = 0.15$ m/sec, and $\dot{\gamma} = -0.005t$ rad/sec.
- Uneven surface same as used for direct kinematics.

KINEMATIC ANALYSIS

NUMERICAL SIMULATIONS RESULTS FOR INVERSE KINEMATICS

- Geometry of the WMR – same as used in direct kinematics
  - Length of the rear axle 1 m.
  - Distance of the centre of front wheel from middle of the rear axle 1 m.
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- Uneven surface same as used for direct kinematics.

Kinematic Analysis

Numerical simulations results for inverse kinematics

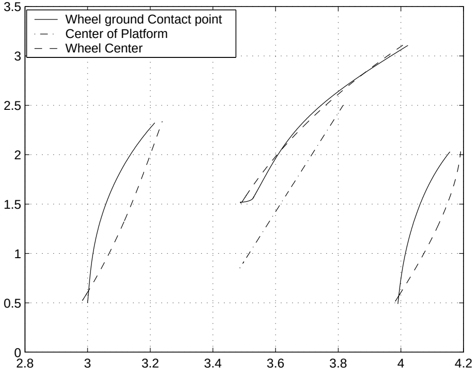

Figure 35: Locus of wheel-ground contact point, wheel centre and platform centre

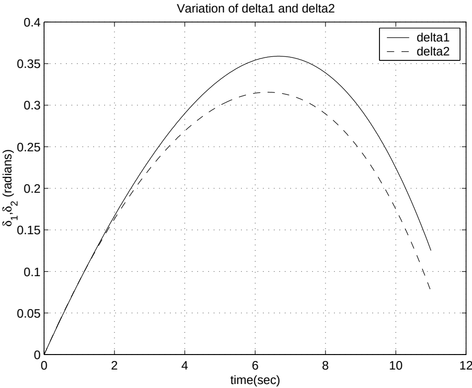

Figure 36: Variation of lateral tilt δ_1 and δ_2

- Due to uneven terrain, the locus of wheel centres are *not* the same as the wheel-ground contact point.
- Lateral tilt changes at different points of the uneven terrain.

Kinematic Analysis

Numerical simulations results for direct kinematics

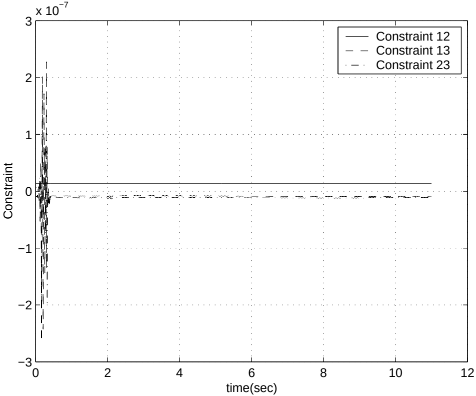

Figure 37: Satisfaction of holonomic constraints

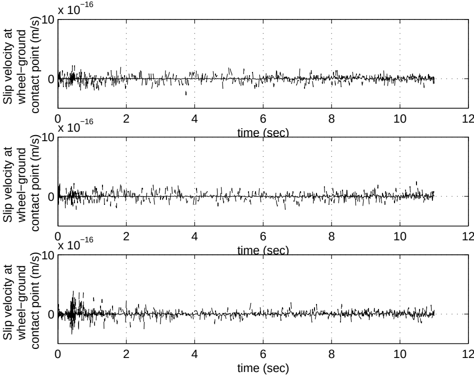

Figure 38: Slip velocities at wheel-ground contact points for three wheels

- The holonomic constraints are satisfied up to 10^{-7} m
- There is virtually no slip – WMR traverses uneven terrain without slip!!

Dynamic Analysis

Overview

- Formulation of equation of motion for WMR using Lagrangian formulation.
  - Kinetic energy of wheels, platform and ‘links’ connecting actuated and passive joints to platform.
  - Potential energy due to gravity.
- 15 contact variables at three wheel-ground contact points, 3 passive, 3 actuated and 6 variables for position and orientation of WMR platform $\rightarrow$ Total 27 generalised coordinates.
- Three actuating torques – two in rear wheel and for front wheel steering.
- Need 24 independent constraint equations for the system to be well-posed – inverse kinematics equations!!
- Derive equations of motion subjected to holonomic and non-holonomic constraints (see Module 6, Lecture 1).

DYNAMIC ANALYSIS

OVERVIEW

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DYNAMIC ANALYSIS

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Dynamic Analysis

Lagrangian formulation

- Total kinetic energy

$$KE = (KE)_{w_1} + (KE)_{w_2} + (KE)_{w_3} + (KE)_{\text{platform}} + (KE)_{\text{actuators}} + (KE)_{\text{links}}$$

- Total potential energy

$$PE = (PE)_{w_1} + (PE)_{w_2} + (PE)_{w_3} + (PE)_{\text{platform}} + (PE)_{\text{actuators}} + (PE)_{\text{links}}$$

- All kinetic energy and potential energy components can be found (see Chakraborty & Ghosal, 2005).
- Constraints equation from inverse kinematics: $[\Psi]\dot{\mathbf{q}} = 0$ – $[\Psi]$ is a 24×27 matrix.
- Equations of motion from Lagrangian formulation

$$[\mathbf{M}(\mathbf{q})]\ddot{\mathbf{q}} + [\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})]\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \tau + [\Psi(\mathbf{q})]^T \lambda$$

$[\mathbf{M}(\mathbf{q})]$ is a 27×27 mass matrix, $[\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})]\dot{\mathbf{q}}$, $\mathbf{G}(\mathbf{q})$ and τ are 27×1 vectors.

- *Only* three elements of τ, corresponding to θ_1, θ_2, ϕ_3, are non-zero.

Dynamic Analysis

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Dynamic Analysis

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Dynamic Analysis

Lagrangian Formulation (Contd.)

- λ is 24×1 vector of the Lagrange multipliers.
- Solve for λ as shown in Moudule 6.
- Finally obtain a set of 27 second-order ODE's – Obtained in symbolic form by using Mathematica®extensively!
- With actuators locked and wheels tilted, WMR falls under own weight! – contrary to a normal parallel manipulator !!
- Wheel-ground contact modeled as *instantaneous* 3 DOF joint – *not* like a spherical (S) joint.
- *Form closure* not present $\rightarrow$ Additional terms modeling a torsion springs and a damper (preventing falling under own weight with actuators locked) is added in τ corresponding to lateral tilts

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k_{s_i} and k_{d_i} are spring constant and damping.

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- *Step 1: Generate the uneven terrain surface*
  - Reconstruct the surface from elevation data.
  - Derivative of constraints are required $\rightarrow \mathscr{C}^3$ continuity required.
  - Fourth degree B-spline surface using Matlab®Spline Tool Box.
- *Step 2: Form equations of motion* – 27 second-order ODEs.
- *Step 3: Obtain initial conditions*
  - Initial conditions must satisfy no-slip and holonomic constraints.
  - 3 actuated variables can be chosen arbitrarily.
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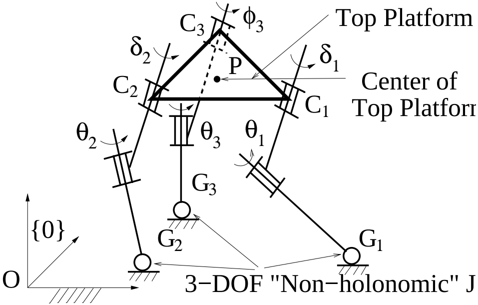

Figure 39: Schematic of the 3 DOF WMR

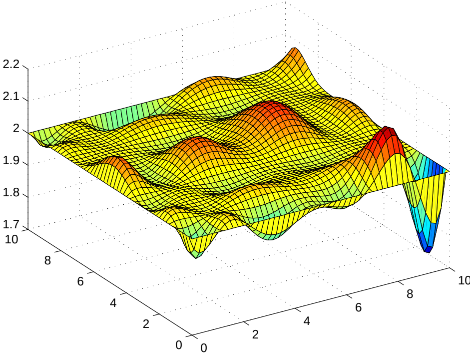

Figure 40: B-spline surface with $\mathscr{C}^3$ continuity

- Synthetic elevation data
- Uneven terrain generated using Matlab®Spline Tool Box

- Mass of platform = 10 kg, Mass of each wheel = 1 Kg.
- Maximum allowable deflection of δ_i is $\pi/4$ under self-weight $\Rightarrow$ $k_{s_i} = 16.24$ N-m/rad and $k_{d_i} = 0.57$ N-m-s/rad.
- Initial conditions: $u_1 = 1.57$, $v_1 = \frac{3\pi}{2}$, $u_{g_1} = 5.008$ m, $v_{g_1} = 1.5067$ m, $\psi_1 = -3.142$, $u_2 = 1.5648$, $v_2 = \frac{3\pi}{2}$, $u_{g_2} = 4$ m, $v_{g_2} = 1.5$ m, $\psi_2 = -3.1418$, $u_3 = 1.5818$, $v_3 = \frac{3\pi}{2}$, $u_{g_3} = 4.4897$ m, $v_{g_3} = 2.483$ m, $\psi_3 = -3.1419$, $\theta_1 = 0$, $\theta_2 = 0$, $\theta_3 = 0$, $\delta_1 = 0$, $\delta_2 = 0$, $\phi_3 = 0$, $z_c = 2.3088$ m, $\alpha = -0.0127$, and $\beta = 0.0133$ – all angles in radians.
- All the initial values of the first derivatives are chosen to be 0.
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Dynamic Analysis

Numerical simulations results

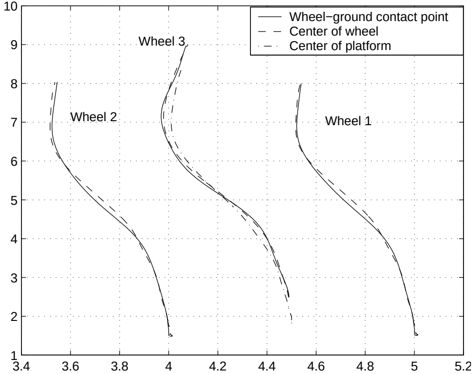

Figure 41: Locus of wheel-ground contact point, wheel centre and platform centre

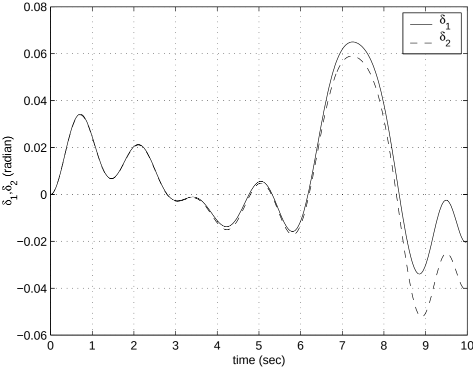

Figure 42: Variation of δ_1 and δ_2

- Uneven terrain – δ_1 and δ_2 varies automatically to avoid slip.

Dynamic Analysis

Numerical simulations results

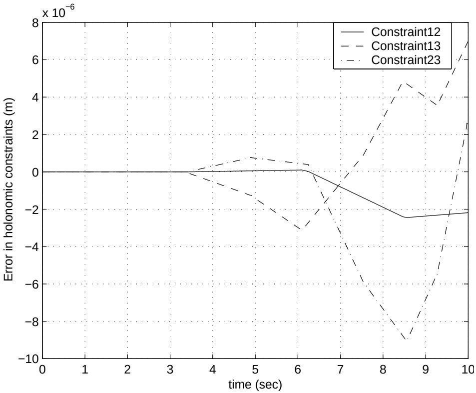

Figure 43: Satisfaction of holonomic constraints in dynamics

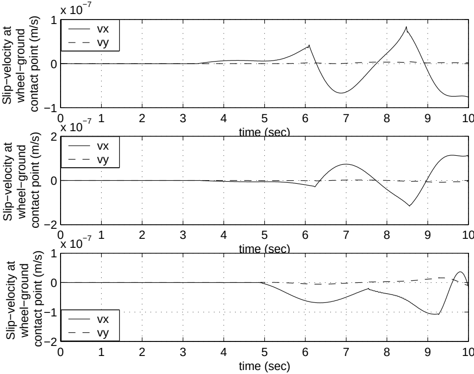

Figure 44: Slip velocities at wheel-ground contact points for three wheels

- All constraints are satisfied at least up to 10^{-7} m.
- Three-wheeled mobile robot can traverses uneven terrain without slip!!

Stability Analysis
Overview

- Uneven terrain – loss of vehicle stability due to tip-over or roll over.
- Tip-over – Vehicle undergoes rotation resulting in reduction in number of vehicle-ground contact points.
- Mobility is lost and, if rotational motion is not arrested, vehicle overturns.
- Need a 'measure' of stability to warn operator.
- Placement of centre of mass, speed, acceleration, external forces/moments and nature of terrain determine tip-over or stability.
- Various measures of stability – force-angle stability measure (Papadopoulos and Rey, 1996) used.
- Investigate stability of the earlier studied three-wheeled mobile robots with torus-shaped wheels for different conditions.

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Stability Analysis

Force-angle Stability Measure

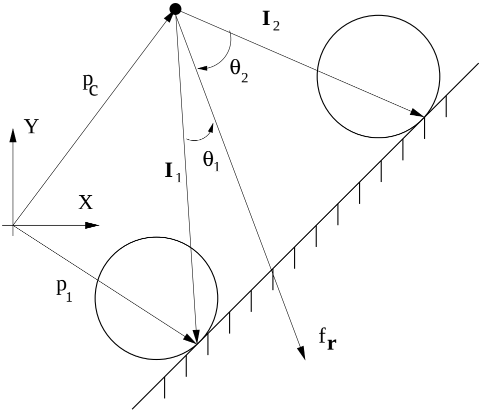

Figure 45: Planar force-angle stability measure

- Centre of mass subjected to a net force $\mathbf{f}_r$.
- $\mathbf{f}_r$ makes angle θ_1 and θ_2 with tip-over axis normals $\mathbf{l}_1$ and $\mathbf{l}_2$.
- Force-angle stability measure $\xi = \min\{\theta_1, \theta_2\}||\mathbf{f}_r||$

Stability Analysis

Force-angle Stability Measure for WMR

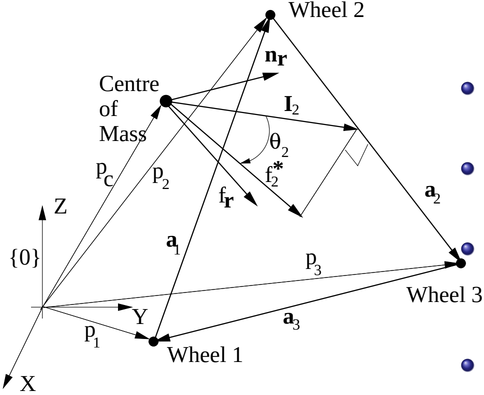

Figure 46: Force-angle stability measure for WMR

- Wheel-ground contact points $\mathbf{p}_i, = (u_{g_i}, v_{g_i}, z_i)^T, \ i = 1,2,3$ known.
- Location of centre of mass $\mathbf{p}_c = (x_c, y_c, z_c)^T$ known in $\{0\}$.
- Line joining wheel-ground contact points $\mathbf{a}_i, \ i = 1,2,3$ are tip-over axis.
- Component of net resultant force is $\mathbf{f}_2^*$ for tip-over axis $\mathbf{a}_2$ (see next slide).
- Angle θ_2 for tip-over axis $\mathbf{a}_2$.
- Likewise find θ_1 for $\mathbf{a}_1$ and θ_3 for $\mathbf{a}_3$.
- If any $\theta_i = 0$, WMR can tip-over $\mathbf{a}_i$.

Force-angle Stability Measure

- Net resultant force at centre of mass: $\mathbf{f}_r = \mathbf{f}_{\text{gravity}} + \mathbf{f}_{\text{dist}} - \mathbf{f}_{\text{inertial}}$
  - Force due to $\mathbf{f}_{\text{gravity}}$ and inertial $\mathbf{f}_{\text{inertial}}$ obtained from dynamic simulation.
  - $\mathbf{f}_{rmdist}$ – External disturbance.
- Net resultant moment at centre of mass: $\mathbf{n}_r = \mathbf{n}_{\text{gravity}} + \mathbf{n}_{\text{dist}} - \mathbf{n}_{\text{inertial}}$
- Interested in component $\mathbf{f}_i$ and $\mathbf{n}_i$ about tip-over axis $\mathbf{a}_i$.
- Combine $\mathbf{f}_i$ and $\mathbf{n}_i$ to get a net resultant force

$$\mathbf{f}_i^* = \mathbf{f}_i + \frac{\mathbf{l}_i \times \mathbf{n}_i}{|\mathbf{l}_i|}$$

  $\mathbf{l}_i$ is the tip-over axis normal.
- Angle for stability measure, θ_i, angle between $\mathbf{f}_i^*$ and unit vector along $\mathbf{l}_i$. Sign of θ_i determines if the net resultant force in *inside* the support polygon or not.
- Overall force-angle stability measure

$$\xi_i = \min\{\theta_1, \theta_2, \theta_3\} \, ||\mathbf{f}_r||$$

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$$\xi_i = \min\{\theta_1, \theta_2, \theta_3\} \, ||\mathbf{f}_r||$$

Stability Analysis

Force-angle Stability Measure

- Net resultant force at centre of mass: $\mathbf{f}_r = \mathbf{f}_{\text{gravity}} + \mathbf{f}_{\text{dist}} - \mathbf{f}_{\text{inertial}}$
  - Force due to $\mathbf{f}_{\text{gravity}}$ and inertial $\mathbf{f}_{\text{inertial}}$ obtained from dynamic simulation.
  - $\mathbf{f}_{rmdist}$ – External disturbance.
- Net resultant moment at centre of mass: $\mathbf{n}_r = \mathbf{n}_{\text{gravity}} + \mathbf{n}_{\text{dist}} - \mathbf{n}_{\text{inertial}}$
- Interested in component $\mathbf{f}_i$ and $\mathbf{n}_i$ about tip-over axis $\mathbf{a}_i$.
- Combine $\mathbf{f}_i$ and $\mathbf{n}_i$ to get a net resultant force

$$\mathbf{f}_i^* = \mathbf{f}_i + \frac{\mathbf{l}_i \times \mathbf{n}_i}{|\mathbf{l}_i|}$$

  $\mathbf{l}_i$ is the tip-over axis normal.
- Angle for stability measure, θ_i, angle between $\mathbf{f}_i^*$ and unit vector along $\mathbf{l}_i$. Sign of θ_i determines if the net resultant force in *inside* the support polygon or not.
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- Geometry, mass and inertial parameters same as in dynamic analysis
  - Mass of platform = 10 kg and Mass of each wheel = 1 kg.
  - Spring constant = 16.24 Nm/rad and damping = 0.57 Nms/rad.
- Various terrains: Curved path on flat terrain, two rear wheels on two different planes and uneven terrains.
- For each chosen paths and/or input torques, compute at every instant of time t
  - Net resultant force $\mathbf{f}_r$ at centre of mass.
  - Net resultant moment $\mathbf{n}_r$ at centre of mass.
  - Compute tip-over axis $\mathbf{a}_i$ and normal $\mathbf{l}_i$ for $i = 1, 2, 3$
  - Compute $\mathbf{f}_i^*$
  - Compute force-angle stability measure ξ_i.

Stability Analysis

Numerical Simulation Results

- Geometry, mass and inertial parameters same as in dynamic analysis
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Stability Analysis

Numerical Simulation Results – Flat plane

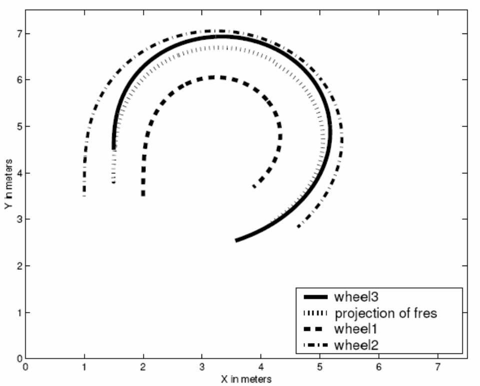

Figure 47: Curve path on a plane

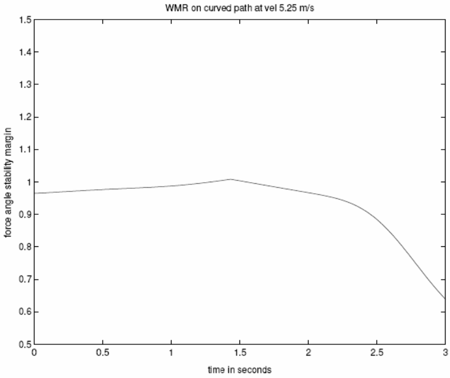

Figure 48: Stability margin for curve path on a plane

- Actuator torques: $\tau_1 = -0.5$ N-m, $\tau_2 = -0.75$ N-m, $\tau_3 = -0.004t$ N-m.
- As the WMR turns, stability margin reduces.

Stability Analysis

Numerical Simulation Results – Inclined Plane

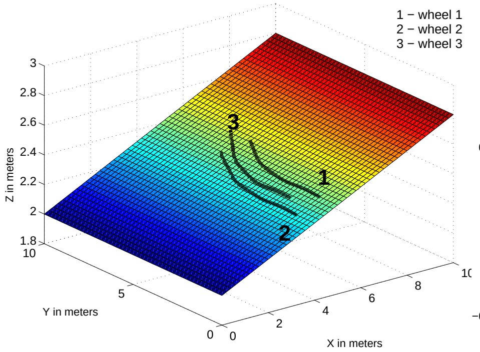

Figure 49: Path traced by WMR on incline plane

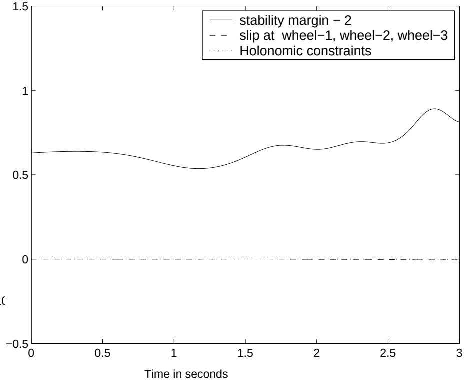

Figure 50: Stability margin for path on an inclined plane

- Input torques: $\tau_1 = -2.4$ N-m, $\tau_2 = -4$ N-m and $\tau_3 = -0.08t$ N-m.
- Initially least stability about axis 2 – As the WMR turns, tip-over starts shifting from axis 2 to axis 1 $\rightarrow$ stability increases.

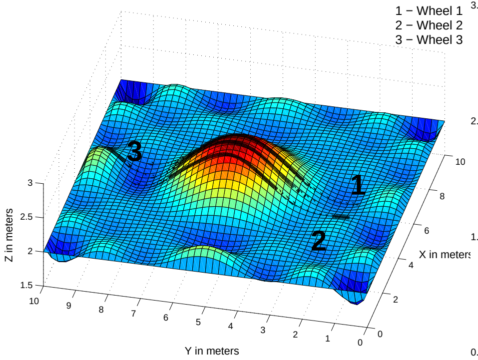

Figure 51: WMR climbing obstacle on a straight path

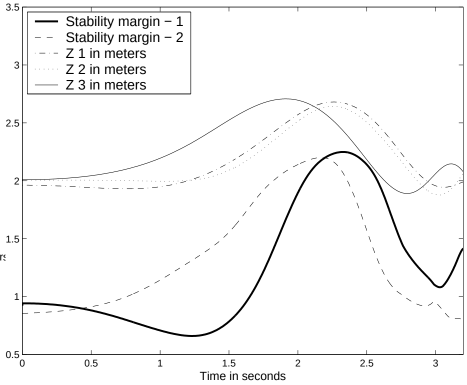

Figure 52: Stability margin along straight path

- Input torques: $\tau_1 = -4$ N-m, $\tau_2 = -4.0$ N-m and $\tau_3 = -0.0$ N-m.
- WMR is able to negotiate obstacle on uneven terrain without tip-over.

Stability Analysis

Numerical Simulation Results – Uneven Terrain

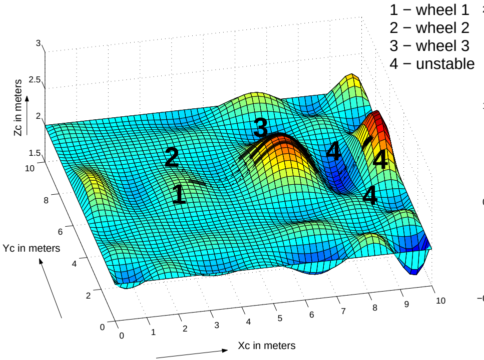

Figure 53: Path traced by WMR on an uneven terrain

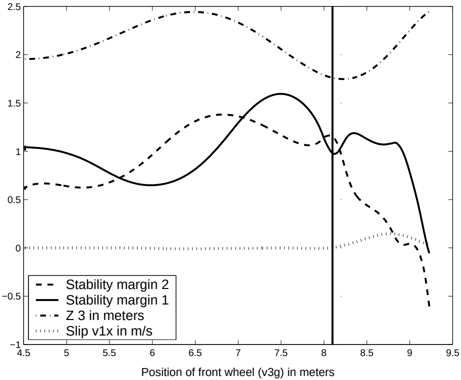

Figure 54: Stability margin on uneven terrain

- Input torques: $\tau_1 = -4$ N-m, $\tau_2 = -4.0$ N-m and $\tau_3 = -0.0$ N-m.
- ξ reduces while climbing second peak, tip-over occurs about axis 1.
- Simulation *strictly* not valid after the vertical line – wheel slip increases to more than 10^{-3} m!

- A three-wheeled mobile robot with torus-shaped wheels.
- Rear wheels with passive lateral tilting capability.
- Modeled as an instantaneous parallel robot with 3 DOF.
- Solution of direct and inverse kinematics by integration.
- Dynamic modeling and simulation.
- Simulation show the capability of traversing uneven terrain without slip.
- Use of force-angle stability measure for tip-over stability.
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Kinematics and Dynamics of WMR on Uneven Terrain

Summary

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Outline

1. Contents

2. Lecture 1
   - Wheeled Mobile Robots on Flat Terrain

3. Lecture 2*
   - Wheeled Mobile Robots on Uneven Terrain

4. Lecture 3*
   - Kinematics and Dynamics of WMR on Uneven Terrain

5. Module 9 – Additional Material
   - Problems, References and Suggested Reading

- Simulation movies in ADAMS for a three-wheeled mobile robot – Movie clip 1 and Movie clip 2.
- Exercise Problems
- References & Sugested Reading