Exercise Problems for Module 2

[P2.1] The position vectors of three points, not lying on a line, on a rigid body $\{B\}$, are given by ${}^A\mathbf{p}_1$, ${}^A\mathbf{p}_2$, and ${}^A\mathbf{p}_3$ with respect to a fixed coordinate system $\{A\}$. Obtain the rotation matrix ${}^A_B[R]$ in terms of the three position vectors.

[P2.2] Figure 1 shows two orientations of a commonly used dice. The frames $\{A\}$ and $\{B\}$ are as shown and for clarity the origins are *not* shown coincident. Estimate a) the rotation matrix ${}^A_B[R]$, b) the equivalent $(\hat{\mathbf{k}}, \phi)$, and c) the $X-Y-Z$ Euler angles which can take $\{A\}$ to $\{B\}$. For part a), use of a 3D CAD software to draw the dice in the two orientation may be helpful.

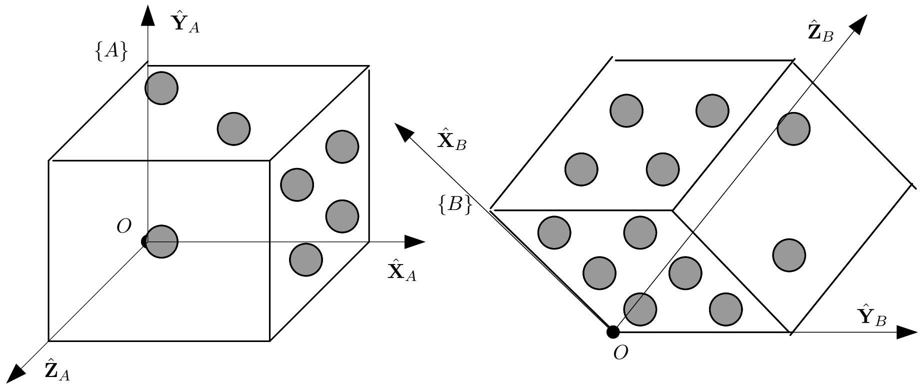

Figure 1: Two orientations of a dice

[P2.3] Consider a vector ${}^A\mathbf{Q}$ being rotated about the vector $\hat{\mathbf{k}}_A$ by an angle θ to form the vector ${}^A\mathbf{Q}'$. Show

$${}^A\mathbf{Q}' ={}^A \mathbf{Q}\cos\theta + \sin\theta({}^A\hat{\mathbf{k}} \times{}^A \mathbf{Q}) + (1-\cos\theta)({}^A\hat{\mathbf{k}} \cdot{}^A \mathbf{Q}){}^A\hat{\mathbf{k}}$$

This is known as *Rodrigues' formula.*

[P2.4] Use the Rodrigues' formula to derive the expressions given in equation (10) in Lecture 1.

[P2.5] Consider three successive rotations of $[{}^A\hat{\mathbf{X}}, \theta_1]$, $[{}^A\hat{\mathbf{Y}}, \theta_2]$, and $[{}^A\hat{\mathbf{Z}}, \theta_3]$ – note the rotations are about the axis of $\{A\}$. Derive the rotation matrix ${}^A_B[R]$. (Hint: Transform the second and third rotation axes, $\hat{\mathbf{Y}}_A$ and $\hat{\mathbf{Z}}_A$ to the moving $\{B\}$ and use equation (10) in Lecture 1).

[**P2.6**] Show that the rotation matrix obtained in [**P2.5**] above is the same as 3-2-1 (or Z-Y-X) Euler angles about the moving axis. Explain why this is so.

[**P2.7**] If the rotation angle is small, i.e., $\sin\phi = \phi$ and $\cos\phi = 1$ holds, then obtain ${}^{A}_{B}[R]$ from equation (10). Using this result, show two infinitesimally small rotations commute.

[**P2.8**] Obtain expressions for $r_{ij},\ i = 1, 2, 3$, in terms of the four Euler parameters, $\epsilon_1, \epsilon_2, \epsilon_3$ and ϵ_4.

[**P2.9**] Figure 2 shows a commonly used dice in two locations. The origin of $\{B\}$ with respect to O_A is as shown in figure 2. The frames $\{A\}$ and $\{B\}$ are as shown. Estimate ${}^{A}_{B}[T]$. Use of a 3D CAD software may be helpful in visualization.

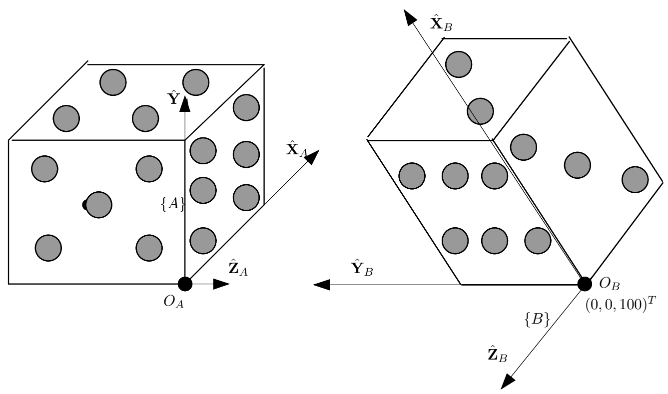

Figure 2: Two locations of a dice

[**P2.10**] Assign coordinate systems and obtain the D-H parameters for the robot shown in figure 3. The arrangement of the non-intersecting joints at the wrist is shown on the right-hand side of the figure. The arrangement of the first three joints is similar to the PUMA 560 manipulator shown in figure **16** in Lecture 3.

[**P2.11**] Obtain the D-H parameters for the Stanford Arm shown in figure 4 below.

[**P2.12**] Obtain the transformation matrices for the robots given in Problems **P2.10** and **P2.11**.

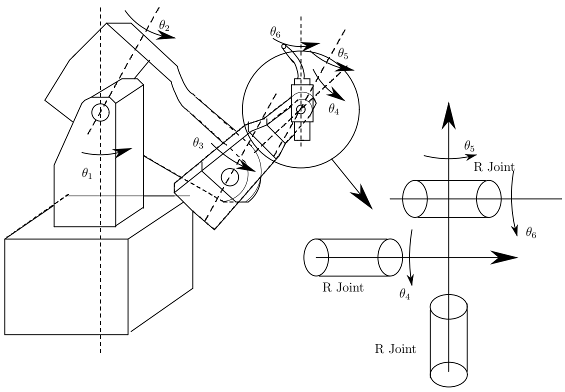

Figure 3: The IGM Robot with non-intersecting wrist

[**P2.13**] In the example of 3- degree-of-freedom manipulator, assume that the top and bottom platforms are both equilateral triangles of sides a and b, respectively. Consider $\{Top\}$ fixed at the centroid of the top (moving) platform and the $\{Base\}$ fixed at the centroid of the bottom (fixed) platform as discussed in the example. Obtain ${}^{Base}_{Top}[T]$ in terms of a, b and the D-H parameters of each leg as given in the example. (Hint: The centroid is given by $(1/3)({}^{Base}\mathbf{S}_1 +{}^{Base} \mathbf{S}_2 +{}^{Base} \mathbf{S}_3)$)

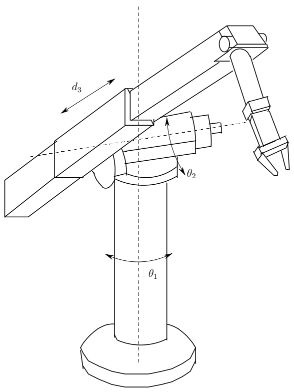

Figure 4: The Stanford arm