Exercise Problems for Module 3

[**P3.1**] Determine the expression for the area of the workspace of 2R manipulator with l_1 and l_2. Assume that l_1 and l_2 can be changed with $l_1 + l_2 = constant$. Show that $l_1 = l_2$ for maximum workspace.

[**P3.2**] Sketch the workspace of planar 2R manipulator with $-\pi/2 \leq \theta_i \leq \pi/2,\ i = 1, 2$.

[**P3.3**] For the planar 3R manipulator, discussed in Lecture 3, verify numerically that for a point (x, y) chosen in the dexterous region, i.e., between circles of radius $l_1 + l_2 - l_3$, and $l_1 - l_2 + l_3$, the inverse kinematics can be solved with arbitrary ϕ. Use $l_1 = 5$, $l_2 = 3$ and $l_3 = 1$.

[**P3.4**] Obtain the D-H parameters for the RRR manipulator shown in figure **??**. Derive expressions for the (x, y, z) coordinates of the point P on the manipulator, with respect to $\{Base\}$, as a function of $(\theta_1, \theta_2, \theta_3)$. Derive the expressions for θ_1, θ_2, and θ_3 for a given $(x, y, z)^T$.

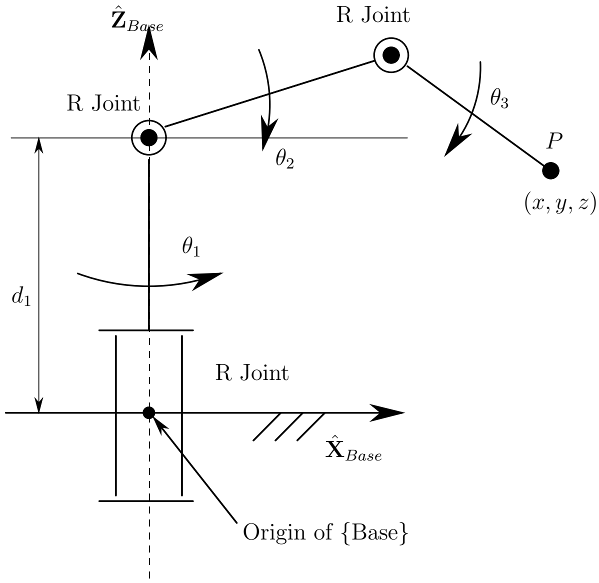

Figure 1: A RRR Manipulator

[**P3.5**] Assign coordinate systems, obtain D-H parameters and derive expressions for the (x, y, z) coordinates of the point P, on the manipulator, shown in figure **??**, as a function of $(\theta_1, \theta_2, \theta_3)$. Derive the expressions for θ_1, θ_2, and θ_3 for a given $(x, y, z)^T$.

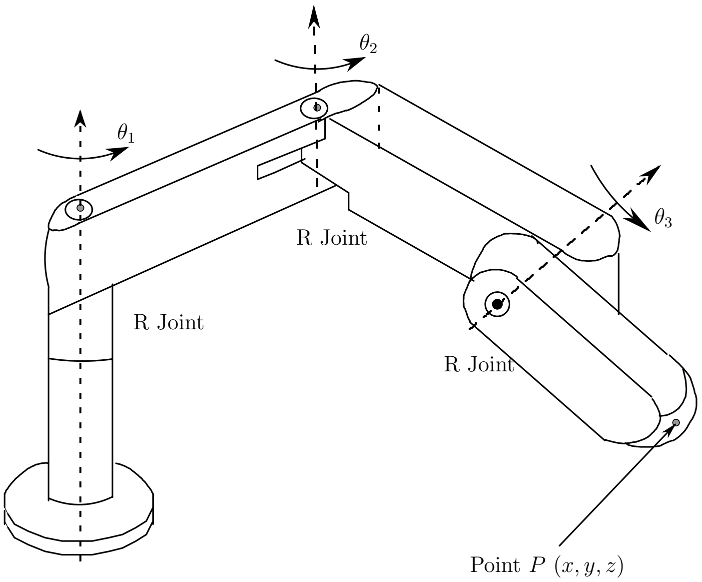

Figure 2: A different RRR Manipulator

[P3.6] Derive the direct and inverse kinematics for Stanford Arm shown in Exercise Problems in Module 2.

[P3.7] Show that the expressions for θ_1, θ_2 and θ_3 for a PUMA 560 robot can also be obtained in the form

$$\begin{aligned}
\theta_1 &= \text{Atan2}(O_{6y}, O_{6x}) - \text{Atan2}(d_3, \pm\sqrt{O_{6x}^2 + O_{6y}^2 - d_3^2}) \\
\theta_3 &= \text{Atan2}(a_3, d_4) - \text{Atan2}(K, \pm\sqrt{a_3^2 + d_4^2 - K^2}) \\
\theta_2 &= \text{Atan2}[(-a_3 - a_2c_3)O_{6z} - (c_1O_{6x} + s_1O_{6y})(d_4 - a_2s_3), \\
&\quad (a_2s_3 - d_4)O_{6z} + (c_1O_{6x} + s_1O_{6y})(a_3 + a_2c_3)] - \theta_3
\end{aligned}$$

where

$$K = (1/2a_2)(O_{6x}^2 + O_{6y}^2 + O_{6z}^2 - a_2^2 - a_3^2 - d_3^2 - d_4^2)$$

and $\text{Atan2}(y, x)$ is the four-quadrant $\tan^{-1}$ function.

[P3.8] Obtain other inverse kinematics solutions by using different initial guesses for the ${}^0_6[T]$ given for the non-intersecting wrist robot.

[P3.9] Derive the inverse kinematics equations of a SCARA manipulator shown Lecture 3 and sketch the workspace of the SCARA manipulator.

[P3.10] Take a straight line path in the workspace and plot θ_i when joint rotations are limited for the redundant planar 3R manipulator. Assume $-120^\circ \leq \theta_i \leq +120^\circ, i = 1, 2, 3$, and the link lengths are 5, 3, and 1 units, respectively.

[P3.11] Take a straight line path in the workspace and evaluate θ_i for a redundant planar 4R manipulator. Assume $-120^\circ \leq \theta_i \leq +120^\circ, i = 1, 2, 3, 4$, and the link lengths are 5, 2.5, 1, and 0.5 units, respectively. Plot the θ_i's and compare with problem **P3.10**.

[P3.12] The equations of two circles with centre $(a_i, b_i),\ i = 1, 2$ and radii $r_i,\ i = 1, 2$ are

$$(x - a_i)^2 + (y - b_i)^2 = r_i^2, \quad i = 1, 2$$

Transform (x, y) to homogeneous coordinates $x = x/w$ and $y = y/w$. Obtain expressions for two regular points of intersection with $w = 1$. With $w = 0$, obtain the expressions for the line at ∞ and the two more solutions at infinity. Show that they are independent of the centre and radius of the circles.

[P3.13] Obtain the Bezout matrix for two polynomials, $P(x) = \sum_{i=0}^{4} a_i x^i$ and $Q(x) = \sum_{i=0}^{2} b_i x^i$.

[P3.14] Determine the inverse kinematics equations for six degree of freedom RRPRRR manipulator analogous to the 14 equations for a 6R manipulator discussed in the text.