Exercise Problems for Module 6

[**P6.1**] In addition, to the joint torques or forces, the end-effector of a serial manipulator is subjected to an external force ${}^0\mathbf{f}_{Tool}$ and a moment ${}^0\mathbf{n}_{Tool}$. What are the generalised forces $Q_i,\ i = 1, ..., n$ in this case?

[**P6.2**] Derive the expression for C_{ij} given in slides.

[**P6.3**] Show that the constraint force given by $[\mathbf{\Psi}(\mathbf{q})]^T\boldsymbol{\lambda}$ does not do any work.

[**P6.4**] If the planar four-bar mechanism is not broken at joint 3 (corresponding to ϕ_3) but at joint 4 (corresponding to ϕ_1), then we get a planar 3R manipulator with different constraint equations. Derive the symbolic equations of motion for this case and comment on the differences in the mass matrix, Coriolis/centripetal term and the gravity term obtained this way with the ones obtained in slides.

[**P6.5**] Obtain symbolic equations of motion of a planar five-bar mechanisms shown in exercise problem **P4.2** in Module 4.

[**P6.6**] Obtain the symbolic equations of motion of the spatial RRR manipulator shown in exercise problem **P3.5** in Module 3.

[**P6.7**] Obtain symbolic equations of motion of three-degree-of-freedom 3-RPS parallel manipulator discussed in Module 5.

[**P6.8**] For the planar 2R manipulator, assume that the trajectories of the joints are

$$\begin{aligned}\theta_1(t) &= \frac{\pi}{2}\sin(\frac{\pi}{20}t)\\ \theta_2(t) &= \frac{\pi}{4}\sin(\frac{\pi}{20}t)\end{aligned}$$

Plot $\tau_1(t)$ and $\tau_2(t)$. What and where are the highest τ_1 and τ_2?

[**P6.9**] The actuated joint of the four-bar mechanism oscillates between 0 and 90 degrees with θ_1 given as

$$\theta_1(t) = \frac{\pi}{2}\sin(\frac{\pi}{20}t)$$

What and where is the highest torque τ_1? Assume that the spring is not present for this problem and the joint is actuated by a motor.

[**P6.10**] For the planar 2R manipulator, assume that there is viscous damping at the joints given by $0.01\dot{\theta}_1$ and $0.01\dot{\theta}_2$. Simulate the equations of motion with the viscous damping included and plot θ_1 and θ_2 as functions of time.

[**P6.11**] The recursive Newton-Euler algorithm can be automated in Maple to yield the equations of motion for a serial manipulator. Write a Maple program to obtain the equations of motion for the planar 2R manipulator using the recursive Newton-Euler Algorithm.

[**P6.12**] The recursive Newton-Euler algorithm is known to be $\mathcal{O}(N)$. In addition to the order of an algorithm, it is also important to know what is the coefficient multiplying N. Obtain from the *recent* literature what is the constant term multiplying N in a recursive Newton-Euler algorithm for serial manipulators.