Exercise Problems for Module 7

[P7.1] Obtain the coefficients of a cubic polynomial

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

if $\theta(0)$, $\dot{\theta}(0)$, $\ddot{\theta}(0)$ and $\theta(t_f)$ are specified.

[P7.2] Obtain expressions for the six coefficients of a quintic polynomial

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

when the position, velocity and acceleration are specified at $t = 0$ and $t = t_f$.

[P7.3] Using the numerical values of Case 2 Example (Figure 4), assume that $\ddot{\theta}_i$ are specified at the initial, final and via points as

$$\ddot{\theta}_1(0) = -2.0834, \quad \ddot{\theta}_1(2) = 11.6662, \quad \ddot{\theta}_1(3) = -110.8336$$

The velocity at the via point is also specified as

$$\dot{\theta}_1(2) = 19.5833,$$

Plan a trajectory using quintic polynomials between $t = 0$ and $t = 3$. What is the difference between this trajectory and the one obtained in Case 2 Example (Figure 4)?

[P7.4] Assume that the accelerations at the initial and final times are

$$\ddot{\theta}_1(0) = -5.0, \quad \ddot{\theta}_1(3) = -100.0$$

with all other specifications the same as in **P7.3** and case 2 Example (Figure 4). Plan a trajectory using quintic polynomials between $t = 0$ and $t = 3$. What is the difference between this trajectory and the one obtained in **P7.3**.

[P7.5] The tip of a planar 2R manipulator is to trace a straight line in its workspace. Write a Matlab program to plan a smooth cubic trajectory for known link lengths and given initial and final (x, y) and $(\dot{x}, \dot{y})$ such that the tip of the planar 2R manipulator *exactly* traces a straight line. Plot x, y, θ_1, θ_2 and their derivatives as a function of time. Use $l_1 = l_2 = 1.0$ m.

[**P7.6**] For the θ_1, θ_2 and their derivatives obtained in **P7.5**, using the dynamic equations of motion obtained in Module 6, obtain the torques τ_1 and τ_2 as a function of time. Where is the torque largest? Of $[\mathbf{M}]\ddot{\mathbf{\Theta}}$, the Coriolis/centripetal term and the gravity term, whose contribution is the largest?

[**P7.7**] The tip of a planar 2R manipulator is to trace a full circle in its workspace with centre at (a, b) and radius r in 1.0 second. Assume the full circle is inside the workspace and at the start and final time, the Cartesian velocities are zero. Write a Matlab program to plan a smooth cubic trajectory for known link lengths. Plot x, y, θ_1, θ_2 and their derivatives as a function of time. Use $l_1 = l_2 = 1.0$ m and $(a, b) = (0.5, 0.5)$ and $r = 0.5$.

[**P7.8**] As in **P7.7**, the tip of the 2R manipulator is to trace a circle in its workspace. Instead of a cubic trajectory, we can also assume the parameter $\theta(t)$ used to describe the circle in the $X-Y$ plane as $A\sin(\omega t)$, where A and ω are chosen to satisfy initial and final conditions. Plot $\theta_1(t)$, $\theta_2(t)$ and their derivatives as a function of time for this case. Comment on the difference between the joint trajectories obtained this way and in **P7.7**.

[**P7.9**] For the θ_1, θ_2 obtained from **P7.8**, using the dynamic equations of motion obtained in Module 6, obtain the torques τ_1 and τ_2 as a function of time. Where is the torque largest for the trajectory obtained using $A\sin(\omega t)$? Of $[\mathbf{M}]\ddot{\mathbf{\Theta}}$, the Coriolis/centripetal term and the gravity term, whose contribution is the largest?

[**P7.10**] Write a Matlab program to generate smooth cubic trajectories in the Cartesian space for the end-effector of the PUMA 560 robot discussed in Modules 2 and 3. Use the dimensions given in Module 3 and test your program for simple linear and circular trajectories.

[**P7.11**] For the single-link manipulator discussed in Lecture 2, choose $J = K = F = 1$, $K_p = 1$, and $\theta_d(t)$ as a step input. Vary K_v between 1 and 3 and numerically obtain the plots of $\theta(t)$. What happens at $K_v = 2$? Note: This problem can be done by numerically solving the ordinary differential equation or by using tool boxes in Matlab (see Control Tool Box or Simulink).

[**P7.12**] For the single link manipulator in Lecture 2, choose $J = K = F = 1$, $K_p = 1$ and $K_v = 2$. Assume a step input for θ_d and add a constant disturbance $T_d = 0.1$. What is the plot of $\theta(t)$? Next consider a PID controller with everything else remaining the same. Vary the integral gain K_i and plot $\theta(t)$ for different K_i's. For what value of K_i is the system unstable? Note: As in problem **P7.11**, one can use Matlab and its tool boxes for numerical simulation.

[**P7.13**] Consider the non-linear dynamical system

$$\ddot{x} + 7\dot{x}^2 + x\dot{x} + x^3 = u(t)$$

where $u(t)$ is the control input. Design a control system using the concepts given in Lecture 3 such that the error response is critically damped and the natural frequency ω_n is 1 rad/sec. Draw a block diagram of the system.

[**P7.14**] A researcher has proposed the following model-based control scheme for a serial manipulator:

$$\begin{aligned}\boldsymbol{\tau} &= [\mathbf{M}(\mathbf{q})]\ddot{\mathbf{q}}_d + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) \\ &\quad + [K_p](\mathbf{q}_d - \mathbf{q}) + [K_v](\dot{\mathbf{q}}_d - \dot{\mathbf{q}})\end{aligned}$$

where $[K_p]$ and $[K_v]$ are positive-definite gain matrices and the other symbols have the same meaning as in Lecture 3. Draw a block diagram of the proposed controller along the lines of Figure 13 (Lecture 3). What is the error equation? What is the possible advantage of this scheme? What are the possible disadvantages?

[**P7.15**] Choose a circular trajectory for the planar 2R manipulator as discussed in exercise problem **P7.9**. Using the numerical data given in Lecture 3 for the 2R manipulator, simulate its motion for a PD and model-based (with estimates) controller. Use the symbolic equations of motion derived for a planar 2R manipulator in Module 6.

[**P7.16**] A planar 2R manipulator is to trace an arc of a circle whose parametric equation is given by $x = l_0 + l_3 \cos\phi$ and $y = l_3 \sin\phi$, where l_0 and l_3 are constants, and $\pi/2 \leq \phi \leq \pi$. Following the developments in Lecture 4, determine the following:

a) the symbolic expressions of the terms in the equation of motion as a function of ϕ

b) for an arbitrarily chosen $\boldsymbol{\tau}$, verify numerically that the tip actually traces a circle. The numerical data pertaining to the planar 2R manipulator given in Lecture 3 (simulations) can be used. Choose an appropriate l_0 and l_3 such that the arc of the circle can be traced by the tip of the 2R manipulator (Hint: See Module 4 for conditions on link lengths of a planar four-bar mechanism).

[**P7.17**] The tip of a planar 2R manipulator is to move along a slot as shown in Figure 1. Following the developments in Lecture 4 determine
a) the symbolic expressions for the Jacobian $[J_{\mathbf{h}}]$ and
b) the symbolic expressions for $\boldsymbol{\tau}_{\mathbf{n}}$ and $\boldsymbol{\tau}_{\phi}$.

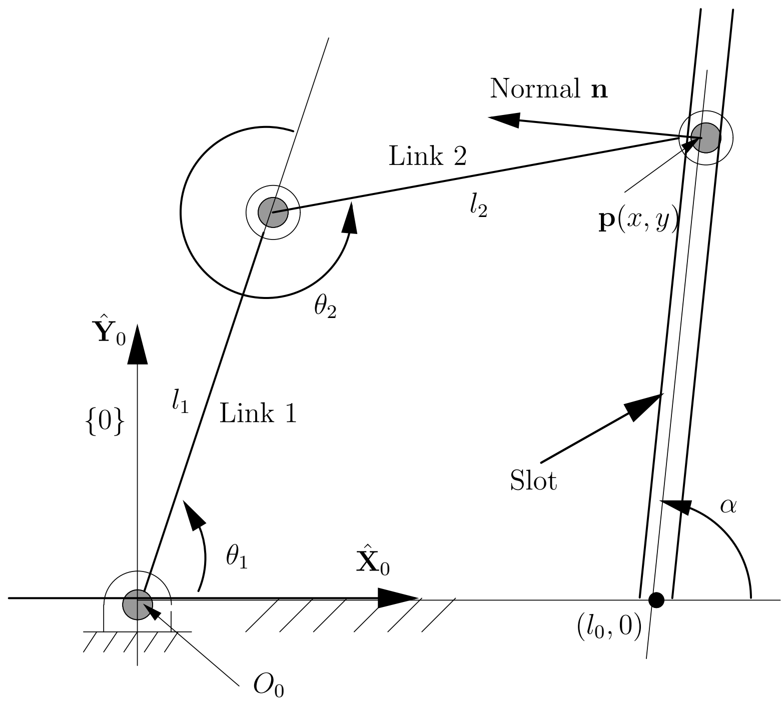

Figure 1: A planar 2R manipulator

[**P7.18**] For the four-bar mechanism simulated in Module 6, we wish to rotate θ_1 from 0 to 150 degrees in 15 seconds with $\dot{\theta}_1$ being zero at $t = 0$ seconds and $t = 15$ seconds. Instead of the spring, assume that the actuation is by a DC permanent magnet motor and all other mass and geometrical data remain the same as in four-bar simulation example

in Module 6. Design a model-based controller such that the motion is critically damped along the trajectory.
From the simulation, what is the maximum torque during the motion? If a simple PD controller is used with the same gains as in the model-based controller, what is the maximum torque during the simulation?

[**P7.19**] For the planar 2R manipulator, obtain the symbolic expressions for the Cartesian mass matrix, the Cartesian Coriolis/centripetal term, and the Cartesian gravity term.

[**P7.20**] Figure 2 shows a manipulator tightening a screw. What are the natural and artificial constraints?

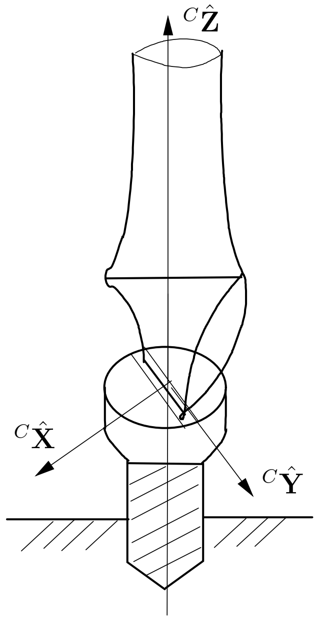

Figure 2: A manipulator tightening a screw

[**P7.21**] Show that

$$[\dot{\mathbf{M}}(\mathbf{q})] - 2[\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})]$$

is skew-symmetric by considering expression of C_{ij} derived in Module 6.

[**P7.22**] Show that the control scheme

$$\boldsymbol{\tau} = -[\widehat{\mathbf{M}(\mathbf{q})}][\ [K_p]\mathbf{q} + [K_v]\dot{\mathbf{q}}] + \mathbf{G}(\mathbf{q})$$

for a multi-degree-of-freedom manipulator gives asymptotical stability. The matrix $[\widehat{\mathbf{M}(\mathbf{q})}]$ is positive definite and is an estimate of the mass matrix for the multi-degree-of-freedom manipulator.