Trajectory Tracking and Control of Car-like Robots

Suhan Shetty[1] and Ashitava Ghosal[1]

[1]Indian Institute of Science, Bangalore
Email: suhan.n.shetty@gmail.com, asitava@iisc.ac.in

Abstract

This paper deals with trajectory tracking control of a car-like robot. By exploiting the differential flatness property of the system based on the dynamics, a trajectory tracking controller using flatness-based control techniques is designed. A singularity in the system for the chosen control inputs, which does not allow direct application of feedback linearization control, is identified and this singularity is overcome by applying the dynamics-extension algorithm to obtain a dynamic feedback linearized controller. This controller results in asymptotic tracking convergence of the system's trajectory to the reference trajectory. Through numerical simulations, the control system is shown to track prescribed trajectories satisfactorily even in the presence of parametric uncertainties.

Keywords: Differential flatness, System dynamics, Car-like robot

1 Introduction

Dynamics of a wheeled mobile robot (WMR) is inherently nonlinear and it is subjected to non-holonomic constraints due to the no-slip condition at the wheel-ground contact. The developments in the control of non-holonomic systems have been a continuing topic of research and are explained in [1]. Most of the literature on trajectory tracking of car-like robots considers only the kinematics aspects of the system and a detailed presentation of kinematic models of non-holonomic car-like WMR can be found in [2]. However, system dynamics plays a major role in high-speed applications. The trajectory tracking of car-like robots based on system dynamics is a very challenging task due to multiple non-holonomic constraints which complicate the resulting equation of motion. In reference [3], researchers have attempted to use dynamics in the trajectory tracking control problem of a differential drive WMR. In reference [4], authors present trajectory tracking of car-like robots based on the dynamics of the system and they use Lyapunov stability theorem to derive control laws. However, reference [4] uses torque on the steering wheel as one of the control inputs and this is not very realistic.

In this work, we derive the dynamic equations of motion of a WMR using a version of the Lagrangian formulation applied to non-holonomic systems – known as Maggi's method [5]. We choose the driving force and rate of steering angle as control inputs.Then, we exploit the differential flatness property of the resulting system to design a flatness-based trajectory tracking controller [6, 7]. However, we identify that a direct application of flatness-based controller is not possible due to a singularity in the obtained equation of motion of the robot. We circumvent this by using the dynamic extension [8] of the system and then use flatness-based controller.

The paper is organized as follows: section 2 deals with the modeling of the wheeled mobile robot. In section 3 we identify the differential flatness of the system and design a flat controller. Section 4 presents the verification of the performance of the controller using numerical simulation and in section 5, we present the main conclusions.

2 System Modeling

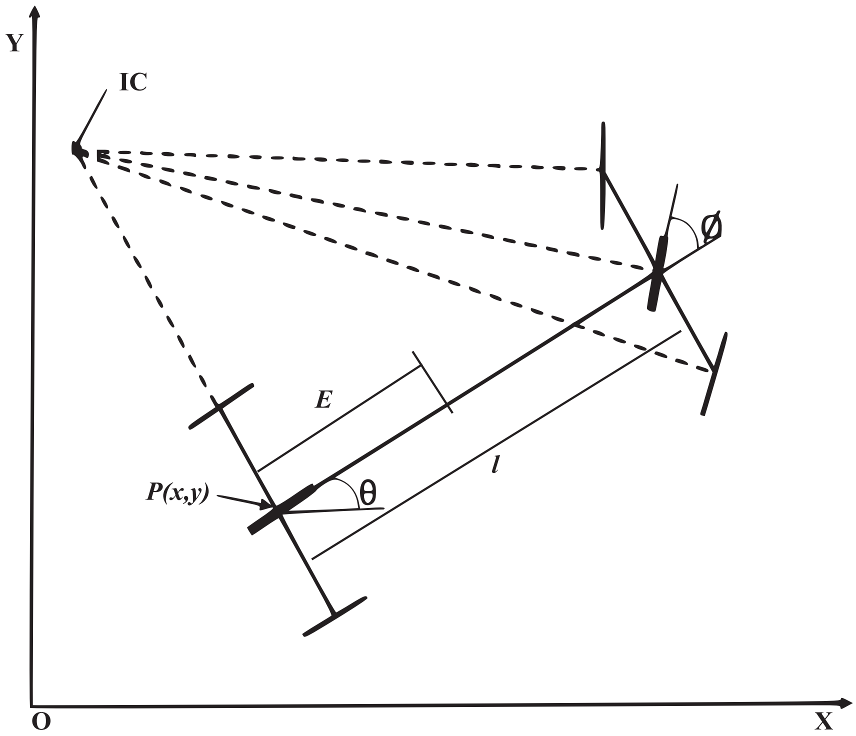


Figure 1: Schematic of a car-like robot and its equivalent bicycle model

We assume that the car-like robot uses Ackerman steering and is moving on a plane. We also assume that the car is driven using rear wheels and steered using front wheels. We model the car-like robot as a bicycle moving on a plane. The planar bicycle model is a widely used model for car-like robots as it captures essential kinematic and dynamic characteristics of the system under consideration.

The Fig. 1 represents the planar model of a car-like robot with its corresponding bicycle model. The bicycle model of the car is shown in thicker lines in the same figure. In the figure, $P(x, y)$ represents the midpoint of the rear wheel axle, ϕ represents equivalent steering angle, l represents length of the car, E represents the distance of the center of mass of the car from P, IC represents the instantaneous center of the car, and θ represents the orientation of the longitudinal axis of the car. All the angles are taken positive counter-clockwise.

2.1 Kinematic Model

We assume that the wheels are subjected to no-slip condition. This will result in the following two independent non-holonomic constraint equations given by

$$\dot{x}\sin(\theta) - \dot{y}\cos(\theta) = 0 \tag{1}$$

$$\dot{x}\sin(\theta+\phi) - \dot{y}\cos(\theta+\phi) - l\dot{\theta}\cos(\phi) = 0 \tag{2}$$

From the Eqn. (1) we can write the velocity v of the point P as

$$v = \dot{x}\cos(\theta) + \dot{y}\sin(\theta) \tag{3}$$

2.2 Dynamic Model

We use the Lagrangian approach to obtain the equation of motion of the wheeled mobile robot. The wheels are subjected to non-holonomic constraints and the traditional Lagrangian approach will involve Lagrangian multipliers. Since we are not interested in the solutions of Lagrange multipliers, we eliminate the Lagrange multipliers and then solve for the equations of motion – this is known as Maggi's method [5].

For a mechanical system described using n generalized coordinates (q), m non-holonomic constraints, we define n independent quasi-velocities ($v_i, i = 1$ to n), among which m of them are made equal to the m non-holonomic constraints. The other $(n-m)$ quasi-velocities are chosen appropriately. Let θ represent the corresponding n quasi-coordinates. We can show that [5] virtual displacements of the quasi-coordinates and the true coordinates are related by

$$\delta q_i = \sum_{j=1}^{n-m} \Phi_{ij}(q,t)\delta\theta_j, \qquad i = 1,\ldots,n$$

Let $Q_i(i = i,\ldots,n)$ be the generalized forces and L be the Lagrangian of the system. The Maggi's equation is then given as

$$\sum_{i=1}^{n}\left[\frac{\mathrm{d}}{\mathrm{d}t}(\frac{\partial L}{\partial \dot{q}_i}) - \frac{\partial L}{\partial q_i} - Q_i\right]\Phi_{ij} = 0 \quad j = 1,...,n-m \tag{4}$$

Denoting the generalized coordinates of the WMR by (x, y, θ), we have $n = 3$ and $m = 2$. We define the quasi-velocities v_i as follows:

$$v_1 = \dot{x}\cos(\theta) + \dot{y}\sin(\theta) = v$$
$$v_2 = \dot{x}\sin(\theta) - \dot{y}\cos(\theta) = 0$$
$$v_3 = \dot{x}\sin(\theta+\phi) - \dot{y}\cos(\theta+\phi) - l\dot{\theta}\cos(\phi) = 0$$

The above results in the following co-efficient matrix:

$$\Phi = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ \sin(\theta) & -\cos(\theta) & 0 \\ \tan(\phi)/l & 1/l & -1/l\cos(\phi) \end{bmatrix}$$

Let u denote the force generated by the drive wheel. The generalized forces can be written in terms of u as

$$Q_x = u\cos(\theta), \tag{5}$$
$$Q_y = u\sin(\theta), \tag{6}$$
$$Q_\theta = 0 \tag{7}$$

We assume that the WMR moves in a plane and hence the Lagrangian, L, is same as the total kinetic energy. The kinetic energy denoted by T is given by

$$L = T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}I_p\dot{\theta}^2 + m\left(-E\dot{x}\dot{\theta}\sin(\theta) + E\dot{y}\dot{\theta}\cos(\theta)\right) \tag{8}$$

where m is total mass of the WMR, I_p is mass moment of inertia of the WMR about vertical axis through the reference point P and E is the distance of the center of mass of the WMR from P.

Using Eqn. (4) we have

$$\left[\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} - Q_x\right]\Phi_{11} + \left[\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} - Q_y\right]\Phi_{21} + \left[\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} - Q_\theta\right]\Phi_{31} = 0 \tag{9}$$

Using the first column of Φ and substituting Eqn. (8) and Eqn. (5) into Eqn. (9), we get one of the differential equations of motion. The other two equations of motion can be determined by differentiating the constraint equations Eqn. (1) and Eqn. (2). The equations of motion for the WMR are given as

$$\ddot{\theta} = \frac{1}{l}[\ddot{x}(\sin(\theta) + \cos(\theta)\tan(\phi)) + \ddot{y}(-\cos(\theta) + \sin(\theta)\tan(\phi)) + \dot{\theta}v + v\frac{d}{dt}(\tan(\phi))] \tag{10}$$

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = (\dot{\theta}v)\begin{pmatrix} -\sin(\theta) \\ \cos(\theta) \end{pmatrix} + (C_1 u - C_2\dot{\theta}\frac{\mathrm{d}(\tan(\phi))}{\mathrm{dt}})\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} \tag{11}$$

where, $C_1(\phi) = \frac{l^2}{I_\phi}$, $C_2(\phi) = \frac{lI_p}{I_\phi}$ and $I_\phi = ml^2 + I_p\tan^2(\phi)$.

3 Controller Design

We choose the control inputs (u_1, u_2) to be the external force u generated by the drive wheel and the rate of steering angle $\frac{d}{dt}(\tan(\phi))$, respectively. Using Eqn. (1), Eqn. (2), Eqn. (3) and Eqn. (11) it can be shown that (x, y) is a flat output, i.e., the states of the system $(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})$ and the control inputs (u_1, u_2) can be expressed in terms of (x, y) and its derivatives [6]. The Eqn. (11) can be written in the affine form as

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = (\dot{\theta}v) \begin{pmatrix} -\sin(\theta) \\ \cos(\theta) \end{pmatrix} + \begin{bmatrix} C_1\cos(\theta) & -C_2\dot{\theta}\cos(\theta) \\ C_1\sin(\theta) & -C_2\dot{\theta}\sin(\theta) \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{12}$$

It can be observed that the characteristic matrix is singular in the above equation. To overcome this difficulty, we apply the technique of dynamic extension to the above dynamical system and choose a new variable w_1 as

$$w_1 = C_1 u_1 - C_2\dot{\theta}u_2 \tag{13}$$

Using w_1, Eqn. (11) can be re-written as

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = (\dot{\theta}v) \begin{pmatrix} -\sin(\theta) \\ \cos(\theta) \end{pmatrix} + w_1 \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} \tag{14}$$

Differentiating the above equation with respect to time we can extend the above system to the following form:

$$\dddot{X} = F + G.W \tag{15}$$

where

$$W = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = \begin{pmatrix} \dot{w_1} \\ u_2 \end{pmatrix}$$

$$\beta = -C_2\dot{\theta}^2 + \frac{v^2}{l}$$

$$X = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$F = (-\dot{\theta}^2 v) \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} + (2\dot{v}\dot{\theta} + C_1\dot{\theta}u_1) \begin{pmatrix} -\sin(\theta) \\ \cos(\theta) \end{pmatrix}$$

and

$$G = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \beta \end{bmatrix}$$

In Eqn. (15), G is non-singular except at $\beta = 0$, i.e., at $v = 0$. Now we transform the control input by choosing a new control input as

$$V = F + G.W \tag{16}$$

Substituting the above in Eqn. (15) we have

$$\dddot{X} = V \tag{17}$$

This transformed system is in the linear form and we can use the linear control techniques for designing a trajectory tracking control law for V to track a desired trajectory $(X_d(t))$. Using pole-placement technique we design V as

$$V = \dddot{X}_d + K_2(\ddot{X}_d - \ddot{X}) + K_1(\dot{X}_d - \dot{X}) + K_0(X_d - X) \tag{18}$$

where K_2, K_1 and K_0 are control gain matrices.

Once we have V, we can get the corresponding W from Eqn. (16) as

$$W = G^{-1}(V - F) \tag{19}$$

and once W is obtained, we can get the original control inputs (u_1, u_2) as follows:

$$u_2 = W_2, \quad w_1 = \int_0^t \dot{w_1} dt = \int_0^t W_1 dt$$

and from Eqn. (13), we have

$$u_1 = \frac{w_1 + C_2 \dot{\theta} u_2}{C_1}$$

To address the problem of singularity of matrix G, that occurs when $v = 0$, we judiciously select inverse as follows :

If $v \neq 0$, then G is invertible, we use

$$G^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\beta} \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

else if $v = 0$, we choose G^{-1} as

$$G^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

and keep u_2 unchanged.

The reasoning behind the above choice of the inverse is that when $v = 0$, it can be observed from Eqn. (15) that control input $W_2 = u_2$ has no influence on the system dynamics.

4 Numerical Simulation

The controller is validated for commonly used trajectories with car-like robots using MATLAB. For numerical simulations the nominal system parameters are taken from a typical car as $l = 2$ m, $m = 200$ kg, and $I_p = 100$ kgm^2. An initial offset is added to the desired trajectory so as to test disturbance handling. To make the simulation more realistic and test the controller in the presence of uncertainties in the system parameter values, in the simulation we use the system parameters as $l = 2.1$ m, $m = 210$ kg, and $I_p = 110$ kgm^2.

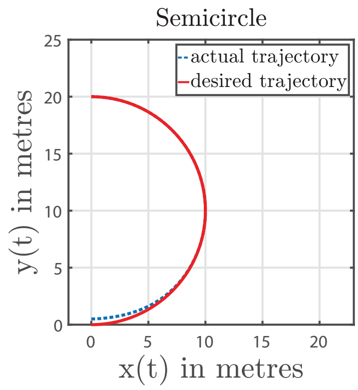
The controller gain matrices are chosen as follows for all the simulations :

$$K_2 = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}, \quad K_1 = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.75 \end{bmatrix}$$
$$K_0 = \begin{bmatrix} 0.125 & 0 \\ 0 & 0.125 \end{bmatrix}$$

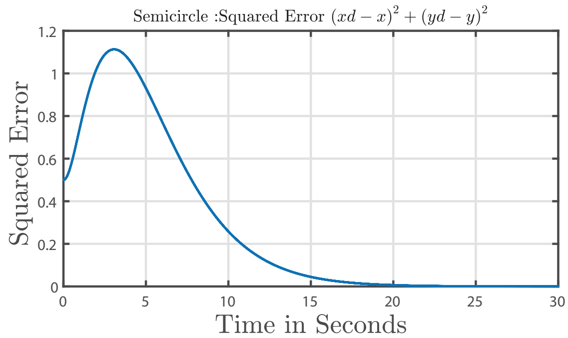
The trajectory tracking performance for two chosen representative trajectories, a semicircle of radius 10m and lane-change curve, are shown in Fig. 2 and Fig. 3. We observe that the error in trajectory tracking converges to zero, and the rate of convergence can be increased by increasing the controller gains. Simulations are performed for different parametric uncertainties and disturbances in the form of initial offsets of the desired trajectory. The controller is observed to be robust against these uncertainties.

Figure 2: Semicircle Trajectory Tracking Performance

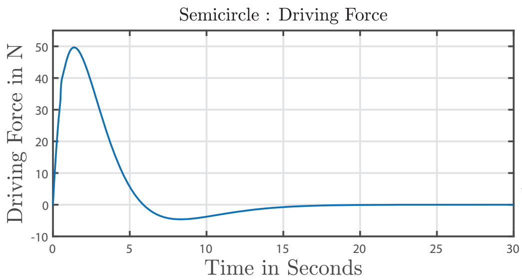
(a) Semicircle Trajectory



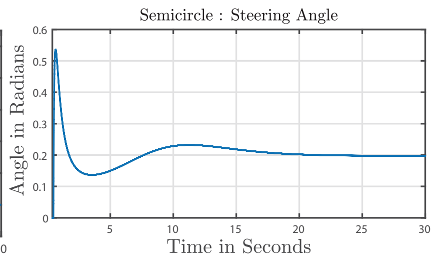
(b) Squared Error



(c) Control Input : Driving Force



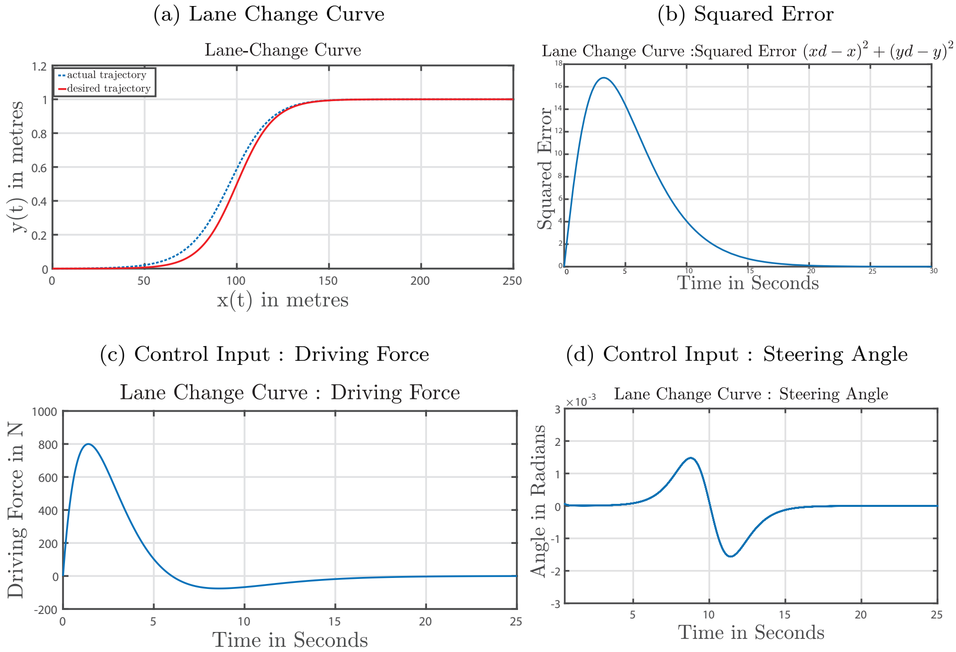
(d) Control Input : Steering Angle



5 Conclusion

In this work, we have proposed a novel trajectory control technique for car-like robots based on the system dynamics. The differential-flatness property of the system is exploited to obtain a simple and robust controller. The singularity in the dynamics is identified and we have used dynamic-extension technique to overcome the singularity issues.The designed controller results in asymptotic convergence of the robot trajectory to a desired trajectory. The controller is validated using numerical simulations for commonly used trajectories.

Figure 3: Lane Change Curve Tracking Performance



(a) Lane Change Curve (b) Squared Error (c) Control Input : Driving Force (d) Control Input : Steering Angle

References

[1] I. Kolmanovsky and N. H. McClamroch, "Developments in nonholonomic control systems," *IEEE Control Systems Magazine*, vol. 15(6), p. 2036, 1995.

[2] A. De Luca, G. Oriolo and C. Samson, "Feedback control of a nonholonomic car-like robot," *Lecture Notes in Control and Information Sciences, Springer*, vol. 229, pp. 171–253, 2005.

[3] K. Shojaei, A. M. Shahri and A. Tarakemeh, "Adaptive feedback linearizing control of nonholonomic wheeled mobile robots in presence of parametric and nonparametric uncertainties," *Robotics and Computer-Integrated Manufacturing*, vol. 27, 2011.

[4] H.-H. Lee, "A new trajectory control of a car-like wheeled robots," in *Proceedings of the ASME 2012 International Mechanical Engineering Congress and Expositions*, 2012.

[5] D. T. Greenwood, *Advanced Dynamics.* Cambridge University Press, 2006.

[6] G. G. Rigatos, *Nonlinear Control and Filtering Using Differential Flatness Approaches : Applications to Electromechanical Systems.* Springer, 2011.

[7] J. Levine, *Analysis and Control of Nonlinear Systems: A Flatness-based Approach.* Springer-Verlag Berlin Heidelberg, 2009.

[8] J-J. E. Slotine and W. Li, *Applied Nonlinear Control.* Pearson, 1991.