Optimal synthesis of adjustable planar four-bar crank-rocker type mechanisms for approximate multi-path generation

Chanekar Prasad Vilas*, Ashitava Ghosal†
Dept. of Mechanical Engineering
Indian Institute of Science
Bangalore 560 012, INDIA

Abstract

This paper deals with an optimization based method for synthesis of adjustable planar four-bar, crank-rocker mechanisms. For multiple different and desired paths to be traced by a point on the coupler, a two stage method first determines the parameters of the possible driving dyads. Then the remaining mechanism parameters are determined in the second stage where a least-squares based circle-fitting procedure is used. Compared to existing formulations, the optimization method uses less number of design variables. Two numerical examples demonstrate the effectiveness of the proposed synthesis method.

Keywords: Adjustable four-bar mechanism, Approximate multi-path generation, Optimal synthesis, Least-squares based circle fitting.

1 Introduction

The classic path generation problem in four-bar mechanisms deals with obtaining linkage parameters such that a given point on the coupler of the four-bar mechanism follows a prescribed path [1]. There are two types of path generation problems namely, point-to-point path generation and continuous path generation. In the case of a planar four-bar mechanism, there are at most nine parameters and one Boolean value which defines the mode of the linkage assembly [2] and in point-to-point path generation the coupler point can be made to pass exactly through at most nine prescribed precision points [3]. In continuous path generation the path is specified by a large number of points (more than nine) and the coupler point may or may not pass through all of them exactly. The continuous path generation problem is solved as an optimization problem, and one can obtain the four-bar mechanism parameters [4, 5] which minimizes a desired objective function.

Adjustable mechanisms are capable of generating multiple paths with change in one or more mechanism parameters and with essentially the same hardware. The changeable parameter can either be length of one or more links or a change in the position of a fixed pivot [6]. As adjustments are incorporated in the simple four-bar mechanism, the number of design parameters become more than ten. The concept of adjustable four-bar mechanisms has a long history and one of the earliest mention of an adjustable four-bar mechanism appeared in the text book by Tao [7] which showed how an adjustable crank pivot in a four-bar mechanism can be used to generate variable straight line motion.

*Graduate Student. Email: prasadvc2007@gmail.com
†Corresponding author. Email: asitava@mecheng.iisc.ernet.in, Tel. +91 80 2293 2956

One of the first approaches towards synthesis of adjustable four-bar mechanisms was using the well-known complex number method [8]. An analytical method with closed-form solution and utilizing complex numbers to synthesize a four-bar path generator to generate two different paths was suggested by McGovern and Sandor [9]. The method used three precision points on each path with one point in common and the mechanism had an adjustable fixed pivot location. A graphical procedure using the geometrical properties of the four-bar mechanism to generate variable coupler curves, with cusps and double points, was suggested by Tao and Krishnamoorthy [10, 11]. Ahmad and Waldron [12] presented an analytical technique for the synthesis of four-bar linkages with adjustable driven crank pivot location to obtain variety of outputs. The two "phases" of motion were obtained by different combinations of the given five design positions. Four-bar linkages with fixed ground pivots and adjustable lengths for input and output links were synthesized by the use of Burmester curves [13]. A novel method using a seventh-order polynomials for the synthesis of four-bar adjustable slider-crank mechanism was suggested by Russell and Sodhi [14]. Their method used radial displacement, velocity, acceleration and jerk profiles with prescribed boundary conditions. A new procedure to synthesize variable coupler curve mechanism with one link replaced by an adjustable screw-nut link and driven by servomotor was presented by Soong and Wu [15]. Different coupler curves are obtained by controlling the angular displacement of the driving link and adjusting the length of adjustable links.

The continuous path design problem is non-linear in nature. With the advent of fast computers and efficient algorithms for optimization, these non-linear design problems can be easily solved using suitable numerical techniques. A genetic algorithm (GA) based optimization method was used by Zhou and Ting [16] to design adjustable four-bar slider-crank mechanisms capable of multiple path generation. The objective function for optimization was based on the position structural error of the slider guider. Zhou and co-workers have also used genetic algorithms on objective function based on driven link length structural error [17], optimal slider adjustment [18], and structural error in the orientation of the fixed link [19]. Adjustable four-bar linkages with continuous adjustment in one of the driven side links were also synthesized by Zhou et al. [20]. In a recent work on adjustable four-bar mechanisms [6], a two stage design method was proposed. In the first stage, the driving dyad is determined and in the second stage the driven dyad is obtained. Both design stages used sequential quadratic optimization (SQP) algorithm [21] to search for the optimal design variables.

The most important step in the use of optimization based methods for continuous paths is the formulation of an appropriate objective function and a choice of an efficient optimization method to solve the synthesis problem. From the review of literature above, genetic algorithm based optimization has been proposed by Zhou and co-workers. Genetic algorithms based optimization is known to be slow and the objective functions used in GA based approaches uses large number of variables. The work reported in this paper is closest to the work of Peng and Sodhi [6] and we also use a two stage approach with sequential quadratic optimization (SQP) algorithm. In the first stage, possible driving dyads are obtained which are then passed on to the second stage where all other four-bar mechanism parameters are obtained. The objective functions of both stages are based on the geometry of the four-bar mechanism. The main difference between this work and reference [6] is that we use a least-squares based circle fitting procedure in the second stage which results in less number of search variables which in turn results in more efficient optimization. In this work, we propose a novel way to choose an initial guess making the optimization independent of the initial guess. The method presented in this work can also be used for adjustment of all possible four-bar parameters except the location of the fixed pivot – objective functions are provided for adjustment of crank length, coupler length and angle and rocker length, and are directly formulated in terms of linkage parameters. Finally, to the best of our knowledge this work presents the first attempt in optimal design of adjustable four-bar mechanisms with adjustments in the driving side. The proposed approach is illustrated with the help of two examples.

The paper is organized as follows: In section 2, for the sake of completeness, we define all the variables associated with a planar four-bar mechanism and present well-known kinematic formulas dealing with position analysis. In section 3, we present the formulation of optimization schemes and in section 4, we present a discussion on the choice of driving or driven side adjustments. In section 5, we present numerical examples to illustrate our approach and in section 6, we present the conclusions.

2 A Planar Four-Bar Mechanism

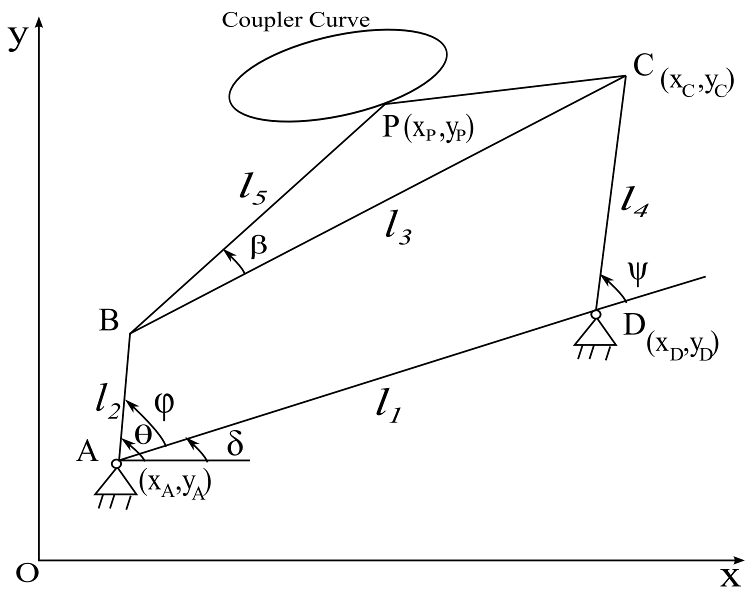


Figure 1: Schematic of a four-bar mechanism

The four-bar mechanism **ABCDP** with its parameters is shown in figure 1. The angles θ and δ are measured relative to the X-axis and the angles ϕ and ψ are measured relative to the base AD. The coupler angle β is measured relative to BC. The parameters l_2 and l_5 are referred to as the *driving* side parameters, and the parameters β, l_3, l_4 and $D(x_D, y_D)$ are referred to as the *driven* side parameters throughout the paper. The location of the driving crank pivot $A(x_A, y_A)$ remains unchanged in our approach. Figure 1 also shows the sketch of a typical coupler curve traced by $P(x_P, y_P)$. We assume that 50 to 100[1] points are prescribed for each desired coupler path and we use a superscript on the mechanism parameters to indicate the particular path. If lesser number of points are prescribed, we can use spline interpolation to generate additional points in the desired path.

For any dyad (or a planar 2R manipulator) shown in figure 2, the workspace of the end-point lies in between two concentric circles [22]. The point A is the optimized crank pivot location and A lies inside the desired coupler path if $l_2 > l_5$ and outside the path if $l_2 < l_5$ [6, 19]. This fact helps in choosing fixed pivot A. For N given points, $P_i(x_{P_i}, y_{P_i}), i = 1, 2, ..., N$, on each coupler curve,

[1]The proposed methods also work for less number of points but larger number of points help in increasing the accuracy of circle fitting used in Stage II of the design process. After extensive simulations we have found that 50-100 points give sufficient accuracy.

we define

$$\begin{aligned} l_{max} &= \max\{l_{P_1}, l_{P_2}, \ldots, l_{P_N}\} \\ l_{min} &= \min\{l_{P_1}, l_{P_2}, \ldots, l_{P_N}\} \\ \text{where,} \quad l_{P_i} &= \sqrt{(x_A - x_{P_i})^2 + (y_A - y_{P_i})^2} \quad \text{for} \quad i = 1, 2, \ldots, N \end{aligned} \tag{1}$$

Since the coupler point P is on the driving dyad, the curve traced by P must lie in the area between two concentric circles with radii $l_2 + l_5$ and $|l_5 - l_2|$ centered at the crank pivot A. The lengths l_2 and l_5 are computed such that the above two concentric circles are tangential to the coupler curve. Hence, we must have,

$$\begin{aligned} l_{max} &= l_2 + l_5 \\ l_{min} &= |l_5 - l_2| \end{aligned} \tag{2}$$

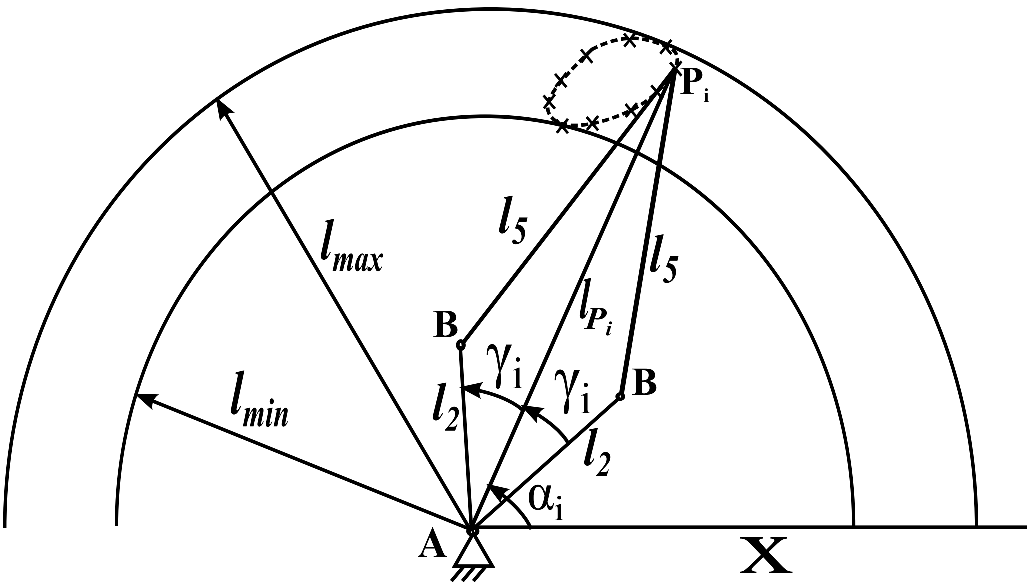


Figure 2: Driving dyad, workspace and crank angle

As the coupler point P moves along the path, the points B and C also change their positions accordingly. For each P_i, θ_i, B_i and C_i, can be computed as follows:

The crank angle θ_i ($\angle BAX$) is given as

$$\theta_i = \alpha_i \pm \gamma_i \tag{3}$$

where α_i is the orientation of $\overrightarrow{AP_i}$ in XY-plane relative to the positive fixed X-axis and γ_i is given by

$$\gamma_i = \cos^{-1}\left(\frac{l_2^2 + l_{P_i}^2 - l_5^2}{2 l_2 l_{P_i}}\right) \tag{4}$$

where $l_{P_i} = ||\overrightarrow{AP_i}||$, $0 \leq (\theta_i, \alpha_i) \leq 2\pi$ and $0 \leq \gamma_i \leq \pi$. The two possible values of the crank angle in equation (3) are as shown in figure 2.

It may be noted that in one crank rotation, the crank crosses each of l_{max} and l_{min} lines once, thus dividing the rotation into two parts. The sign of γ_i in one l_{max} to l_{min} part is opposite to that in the remaining l_{min} to l_{max} part. Thus for each phase we have two sets of θ_i. The positive sign in equation (3) represents anti-clockwise rotation of crank and the negative sign in equation (3) represents clockwise rotation of crank. If direction of rotation is not specified, appropriate θ_i must be chosen.

For each path point P_i, using the crank angle θ_i, we can now find coordinates of $B(x_{B_i}, y_{B_i})$ as,

$$\begin{aligned} x_{B_i} &= x_A + l_2 \cos(\theta_i) \\ y_{B_i} &= y_A + l_2 \sin(\theta_i) \end{aligned} \tag{5}$$

and the coordinates of point $C(x_{C_i}, y_{C_i})$ are given as,

$$\begin{aligned} x_{C_i} &= x_{B_i} + \frac{l_3}{l_5}\left[(x_{P_i} - x_{B_i}) cos(\beta) + (y_{P_i} - y_{B_i}) \sin(\beta)\right] \\ y_{C_i} &= y_{B_i} + \frac{l_3}{l_5}\left[(y_{P_i} - y_{B_i}) cos(\beta) - (x_{P_i} - x_{B_i}) \sin(\beta)\right] \end{aligned} \tag{6}$$

where β is the angle in the coupler link as shown in figure 1.

With the above definition and kinematic equations, we next formulate the objective functions used in our formulation.

3 Formulation of Objective Function

As mentioned earlier, the adjustable four-bar mechanisms can be broadly classified as a) driving side adjustable, and b) driven side adjustable. The driving side adjustable mechanisms can be sub-classified as adjustable crank length mechanism and adjustable l_5-link length mechanism. The driven side adjustable mechanisms can sub-classified as adjustable rocker link pivot mechanism, adjustable rocker link length mechanism, adjustable coupler link length mechanism and adjustable coupler angle mechanism. The design process for each of these types is divided into two stages. The first stage is to design the driving dyad and in the second stage the design of the remaining elements of the mechanism is done. In driving dyad design we find optimal location of crank pivot $A\,(x_A, y_A)$, length of crank, l_2, and the length l_5 in the coupler such that the workspace boundaries are tangential to the paths under consideration.

The main idea used in the formulation of the minimization objective function in the second stage is that the locus traced by the point C, as the point P moves along the coupler path, is a circular arc. The minimization objective function is the residual error obtained by *circle fitting* all the points C_i corresponding to coupler path points P_i. The algorithm used for least-squares circle fitting is from Gander et al. [23].

3.1 Driving side adjustable mechanisms

Stage I
In this stage the driving dyad parameters and fixed crank pivot location is determined. The workspace for the driving dyad changes with the adjustment on the driving side which is achieved by changing l_2 or l_5. The optimal location of the driving crank pivot A is to be determined. The objective functions for each type of adjustment are given below.

Type I: Adjustable crank length mechanism
The length, l_5, remains fixed for all the paths traced by the coupler point P. The change in the

workspace of the driving dyad is achieved by changing the length of the crank, l_2. For the case $l_2 < l_5$, the formulation of the objective function, for the i^{th} path, is given below.

$$
\begin{aligned}
l_5^i &= \frac{l_{max}^i + l_{min}^i}{2} \\
l_2^i &= \frac{l_{max}^i - l_{min}^i}{2} \\
l_5 &= \max\left\{l_5^1, l_5^2, \ldots, l_5^m\right\} \\
l_{21}^i &= l_{max}^i - l_5 \quad \text{and} \quad l_{22}^i = l_5 - l_{min}^i
\end{aligned}
\tag{7}
$$

where m is the total number of given paths to be traced by the coupler point of the four-bar mechanism and the lengths l_{max}^i and l_{min}^i are calculated using equation (2).

Since length l_5 remains fixed even after adjusting the mechanism, we must have for the i^{th} and j^{th} coupler paths,

$$l_5^i - l_5^j = 0 \qquad \forall i \neq j \quad \text{and} \quad i, j \in \{1, 2, \ldots, m\}$$

For each individual i^{th} coupler path the corresponding crank length l_2^i remains fixed. Hence, we must have

$$l_2^i - l_{21}^i = 0 \qquad \text{and} \qquad l_2^i - l_{22}^i = 0$$

From above we can write the optimization problem with i, j, and k indices representing different coupler paths as[2],

$$
\begin{aligned}
&Minimize: \\
&S\left(x_A, y_A\right) = \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \left(l_5^i - l_5^j\right)^2 + \sum_{k=1}^{m} \left(l_{21}^k - l_2^k\right)^2
\end{aligned}
\tag{8}
$$

Subject to the following constraints:

Constraint 1: Search space restriction for x_A and y_A

$$x_A \in [x_{min}, x_{max}] \qquad \text{and} \qquad y_A \in [y_{min}, y_{max}] \tag{9}$$

Constraint 2: The crank angle should always increase or decrease as P advances along the coupler curve. The conditions for the counter-clockwise and clockwise rotation of the crank respectively are,

$$
\begin{aligned}
&\theta_{(q+1)}^i - \theta_q^i > 0 \\
&\theta_{(q+1)}^i - \theta_q^i < 0 \\
&\text{for} \quad q = 1, 2, \ldots, N-1
\end{aligned}
\tag{10}
$$

where, N is the total number of points P_q^i on the given i^{th} coupler path and each θ_q is calculated using equation (3). It may be noted that one of the conditions in equation (10) also needs to be satisfied.

Constraint 3: The maximum l_5 must satisfy

$$l_5^i < l_{5max} \tag{11}$$

Type II: Adjustable l_5-link length mechanism

In this type of adjustment, the crank length l_2 remains fixed for all the paths traced by the coupler

[2]In the objective function $\left(l_{22}^k - l_2^k\right)^2$ can also be used in place of $\left(l_{21}^k - l_2^k\right)^2$.

point P. The change in the workspace of the driving dyad is achieved by changing the length l_5 in the coupler link. For $l_2 < l_5$ case, the formulation of the objective function is given below. For the i^{th} path, we have

$$\begin{aligned} l_5^i &= \frac{l_{max}^i + l_{min}^i}{2} \\ l_2^i &= \frac{l_{max}^i - l_{min}^i}{2} \\ l_2 &= \max\left\{l_2^1, l_2^2, \ldots, l_2^m\right\} \\ l_{51}^i &= l_{max}^i - l_2 \quad \text{and} \quad l_{52}^i = l_2 + l_{min}^i \end{aligned} \tag{12}$$

where, m is the total number of given paths to be traced by the mechanism. The lengths l_{max}^i and l_{min}^i are calculated using equation (2). Since l_2 remains fixed even after adjusting the mechanism, we must have for the i^{th} and j^{th} coupler paths,

$$l_2^i - l_2^j = 0 \qquad \forall i \neq j \quad \text{and} \quad i, j \in \{1, 2, \ldots, m\}$$

For each individual i^{th} coupler path, the corresponding length l_5^i remains fixed. Hence, we must have

$$l_5^i - l_{51}^i = 0 \qquad \text{and} \qquad l_5^i - l_{52}^i = 0$$

From above we can write the optimization problem with i, j, and k indices representing different coupler paths as[3],

$$\begin{aligned} &Minimize: \\ &S\left(x_A, y_A\right) = \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \left(l_2^i - l_2^j\right)^2 + \sum_{k=1}^{m} \left(l_{51}^k - l_5^k\right)^2 \end{aligned} \tag{13}$$

Subject to the constraints (9), (10), and (11) given above.

In both types of adjustment, (x_A, y_A) are the optimization variables. The optimization is carried out using the SQP algorithm. It is known that the SQP converges to the local minimum nearest to the starting point. Hence, to get the best solutions the search space is divided into several sub-intervals and the mid-point of the each sub-interval is taken as the starting point for the optimization in the corresponding sub-interval. The method gives a single solution for each sub-interval and hence we get a large number of solutions for all the sub-intervals. To sort out the best driving dyads we select solutions which have objective function value, S, less than a user chosen maximum value of error S_{max}. The *Constraint 3* given in (11) above may be applied separately after the optimization. The best solutions for the driving dyad are used to synthesize the remaining part of the four-bar mechanism. It should be noted that Stage I optimization only gives the possible locations of fixed pivot A. The exact location of A and the remaining mechanism parameters are determined after performing Stage II optimization.

Stage II

In this stage, the dimensions of the coupler link l_3, rocker link l_4 and coupler angle β as well as the location of the fixed pivot D are determined. For the adjustable type of mechanisms stated above, the location of the fixed pivot is independent of the path and remains fixed throughout. The locus of C will be circular arcs belonging to a common circle for all the paths traced by point P of the four-bar mechanism. The common circle will be centered at D and has radius l_4, and a single circle is fitted through all the points C obtained from all the given coupler paths. Using the crank length and length l_5, the location of point C corresponding to each of the j^{th} path point P_j^i on the

[3] In the objective function $\left(l_{52}^k - l_5^k\right)^2$ can also be used in place of $\left(l_{51}^k - l_5^k\right)^2$.

i^{th} coupler path can be found out using the formulation in equation (6). The coupler angle β and the coupler link length l_3 are the parameters which are optimally determined. The optimization problem can be formulated as,

$$Minimize:$$
$$f\left(l_3,\beta\right)=\sum_{i=1}^{m}\sum_{j=1}^{N}\left(\sqrt{\left(x_{C_j}^{i}-a\right)^2+\left(y_{C_j}^{i}-b\right)^2}-r\right)^2 \tag{14}$$

where, (a,b) is the center and r is radius of the circle on which the locus of C lies.

Subject to the following constraints:
Constraint 1:

$$l_2^i<l_3\leq l_{3max} \tag{15}$$

Constraint 2:

$$0\leq\beta<2\pi \tag{16}$$

Constraint 3:

$$l_2^i<l_4 \quad \text{and} \quad l_2^i<l_1 \tag{17}$$

Constraint 4: For link CD to be a rocker, the angular sweep of link CD should be less than π radians.

$$\psi_{max}-\psi_{min}<\pi \tag{18}$$

Constraint 5:
Grashof's criterion for crank-rocker type mechanism [4] should be satisfied for each i^{th} path.

The objective function f in (14) is the least-squares error obtained by circle fitting the points C_j. The least-squares circle fitting algorithm given in Gander et al. [23] is used to determine (a,b,r). As mentioned earlier, the SQP algorithm is used for optimization with the search space divided into several sub-intervals with mid-point of the each sub-interval is used as the starting point of optimization in the respective sub-interval. Once the optimal circle is determined we get the remaining parameters as,

$$\begin{aligned}\text{fixed pivot}\quad D&=(a,b)\\ l_4&=r\\ l_1&=\sqrt{(x_A-a)^2+(y_A-b)^2}\\ \delta&=\text{atan2}\left(b-y_A,a-x_A\right)\qquad\text{with}\quad\delta\in[0,2\pi)\end{aligned} \tag{19}$$

The Stage II is performed for each of the selected driving dyad. The combined best solution after performing Stages I and II is the minimum sum of both cost functions, i.e., $(S+f)_{min}$. Miscellaneous constraints related to the actual physical characteristics of the application in which the mechanism is applied may also be incorporated during the optimization process or during the selection of mechanism.

3.2 Choice of S_{max}

As stated in (3) and (4), θ_i depends on l_2 and l_5 through γ_i. Once the optimal location of pivot A is determined, l_2 and l_5 are calculated for a particular coupler curve. In adjustable crank-length case, pivot A is optimally determined such that the parameter l_5 calculated as in (7) remains same for all the coupler curves. The changeable crank-length parameter for a particular coupler curve can be l_{21} or l_{22} and we choose the value used in the corresponding objective function to ensure least error in γ_i. Pivot A is searched in multiple sub-intervals and a list of solutions with objective function values less than S_{max} is passed to Stage II. The value of S_{max} decides the number of solutions we want to pass to the Stage II and S_{max} is chosen by the examining the list of solutions obtained after Stage I[4] so that the above mentioned conditions are approximately satisfied. The value of S_{max} for adjustable l_5 is chosen in a manner similar to the adjustable crank case.

3.3 Driven side adjustable mechanisms

The driven side adjustable mechanism is also designed in two stages. Referring to [6], the Stage I is given as below.

Stage I
To first design the driving dyad ABP, the optimal driving crank pivot location A needs to be determined. As the workspace of the driving dyad remains fixed for all the given paths, l_{max} and l_{min}, remain fixed throughout. The optimization problem with i and j indices representing different coupler paths can be formulated as follows

$$\begin{aligned} & Minimize: \\ & S\left(x_A, y_A\right) = \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \left[\left(l^i_{max} - l^j_{max}\right)^2 + \left(l^i_{min} - l^j_{min}\right)^2 \right] \end{aligned} \tag{20}$$

Subject to the constraints (9), (10), and (11) given earlier.

The optimization variables are (x_A, y_A). The procedure for search and selection of best driving dyads is same as described in Stage I of driving side adjustable mechanisms given in sub-section 3.1 with $l^i_2 = l_2$ and $l^i_5 = l_5$ as l_2 and l_5 are same for all coupler curves. Once the optimal pivot point A is determined, we can find l_2 and l_5 for the case $l_2 < l_5$ as

$$l_2 = \frac{l_{max} - l_{min}}{2} \quad \text{and} \quad l_5 = \frac{l_{max} + l_{min}}{2}$$

where $l_{max} = \max\left\{l^1_{max}, l^2_{max}, \ldots, l^m_{max}\right\}$ and $l_{min} = \min\left\{l^1_{min}, l^2_{min}, \ldots, l^m_{min}\right\}$.

Stage II
The remaining mechanism parameters are optimally determined in the same way as described in Stage II of driving side adjustable mechanisms but, the objective functions are different for different type of adjustment. The sub-classifications and optimization problem for each sub-type are given below.

Type I: Adjustable rocker link pivot mechanism
In this type, the position of fixed pivot D is variable, but the length of link CD remains same. Since m-paths are to be traced, the movable pivot C will trace $m-$different circular arcs on $m-$different

[4] After extensive simulation it was found that $S_{max} \leq 10^{-2}$ was adequate.

circles with centres $D_1, D_2, \ldots, D_m$ but with the same radius r. The number of unknown parameters is $9 + (D_2, D_3, \ldots D_m)$ or $9 + 2(m-1)$ or $7 + 2m$ and optimization variables are, $u = (l_3, \beta)$. The optimization problem with index i representing the coupler path, index j representing the path point on the i^{th} coupler path, (a_i, b_i) denoting the center of the circles and r denoting the common radius on which the loci of C lies, can be stated as,

$$Minimize:$$

$$f(l_3, \beta) = \sum_{i=1}^{m}\sum_{j=1}^{N}(\sqrt{(a_i - x^i_{C_j})^2 + (b_i - y^i_{C_j})^2} - r)^2 \tag{21}$$

Subject to the constraints (15), (16), (18), satisfaction of Grashoff's criterion for crank-rocker given above and (17) replaced by $l_2 < l_4$ and $l_2 < l_1^i$. The points $C_j\left(x_{C_j}, x_{C_j}\right)$ are determined using equation (6) given above. The cost function f is the least-square residue error obtained during circle fitting. To get the minimum f in the sub-interval, we need to get $(a_1, b_1, a_2, b_2, \ldots, a_m, b_m, r)$ at the optimum point in the sub-interval. During each iteration in the sub-interval, the optimization problem is converted into a non-linear least-squares problem with $(a_1, b_1, a_2, b_2, \ldots, a_m, b_m, r)$ as unknowns. The non-linear least-squares problem is solved using Gauss-Newton method [24] which needs a starting value for the unknowns. The procedure for obtaining the unknowns is as follows:

Step 1: The set of C_is for each coupler paths are separately circle fitted to obtain (a_{is}, b_{is}, r_{is}) for each path.

Step 2: Non-linear least-squares problem given in (21) is formed with $(a_1, b_1, a_2, b_2, \ldots, a_m, b_m, r)$ as unknown variables. The starting value of the unknowns for the centre are the output of the Step 1. The starting value for the radius r may be either $\min\{r_{1s}, r_{2s}, \ldots, r_{ms}\}$ or mean r_{is}. Step 2 gives the values for the unknowns for the corresponding optimization iteration in the sub-interval. Using these values and equation (21) we can calculate cost function f for each optimization iteration.

The remaining mechanism parameters can be calculated using,

$$\begin{aligned} &D_i = (a_i, b_i), \quad l_4 = r, \\ &l_1^i = \sqrt{(x_A - a_i)^2 + (y_A - b_i)^2}, \\ &\delta_i = \text{atan2}\,(b_i - y_A, a_i - x_A) \qquad \text{with} \quad \delta_i \in [0, 2\pi) \end{aligned} \tag{22}$$

Type II: Adjustable rocker link length mechanism
In this type, the length of link CD is variable but fixed pivot D remains unchanged for all the m paths. Hence, the movable pivot C will trace m different circular arcs on m different circles with same center D but different radii. The number of unknown parameters are $9 + (l_4^2, l_4^3, \ldots l_4^m)$ or $8 + m$, and the optimization variables are $u = (l_3, \beta)$. The optimization problem with index i representing the coupler path, index j representing the path point on the i^{th} coupler path, (a, b) denoting the common center of the circles and r_i denoting the radius of the circles on which the loci of C lies, can be stated as

$$Minimize:$$

$$f(l_3, \beta) = \sum_{i=1}^{m}\sum_{j=1}^{N}(\sqrt{(a - x^i_{C_j})^2 + (b - y^i_{C_j})^2} - r_i)^2 \tag{23}$$

Subject to the constraints (15), (16), (18), satisfaction of Grashoff's criterion for the crank-rocker mechanism given above and (17) replaced by $l_2 < l_4^i$ and $l_2 < l_1$. The points $C_j\left(x_{C_j}, x_{C_j}\right)$ are determined using equation (6) given above.

Similar to Type I formulation, here we have to find the values of unknowns $(a, b, r_1, r_2, \ldots, r_m)$. The task is to find the starting values of the unknown variables for the Gauss-Newton method. The Step 1 to find the starting values is same as for Type I above. The Step 2 is as given below:

Step 2: Non-linear least-squares problem formed using equation (23). The starting values for the unknowns are $(a_s, b_s, r_{1s}, r_{2s}, \ldots, r_{ms})$, where,

$$a_s = \frac{\sum_{i=1}^{m} a_{is}}{m} \quad \text{and} \quad b_s = \frac{\sum_{i=1}^{m} b_{is}}{m}$$

The remaining mechanism parameters can be calculated using

$$\begin{aligned} &D = (a, b) \qquad l_4^i = r_i, \\ &l_1 = \sqrt{(x_A - a)^2 + (y_A - b)^2}, \\ &\delta = \text{atan2}\,(b - y_A, a - x_A) \qquad \text{with} \quad \delta \in [0, 2\pi) \end{aligned} \tag{24}$$

Type III: Adjustable coupler link length mechanism
In this type, the length of link BC is variable and all the other parameters remain unchanged for all the m paths. Hence, the movable pivot C will trace circular arcs belonging to the same circle. The number of unknown parameters are $9 + \left(l_3^2, l_3^3, \ldots, l_3^m\right)$ or $8 + m$ and the optimization variables are, $u = (l_3^1, l_3^2, \ldots, l_3^m, \beta)$. The optimization problem can be stated as

$$\begin{aligned} &Minimize: \\ &f\left(l_3^1, l_3^2, \ldots, l_3^m, \beta\right) = \sum_{i=1}^{m}\sum_{j=1}^{N}(\sqrt{(a - x_{C_j}^i)^2 + (b - y_{C_j}^i)^2} - r)^2 \end{aligned} \tag{25}$$

Subject to the constraints (16), (17), (18), satisfaction of Grashoff's criterion given above and (15) replaced by $l_2 < l_3^i \leq l_{3max}$ and where $C_j\left(x_{C_j}, x_{C_j}\right)$ are determined using equation (6) given above. The remaining mechanism parameters can be calculated using equations (19).

Type IV: Adjustable coupler angle mechanism
In this type, the coupler angle β is variable and all the other parameters remain unchanged for all the paths. Hence, the movable pivot C will trace circular arcs belonging to the same circle. The number of unknown parameters are $9 + (\beta_2, \beta_3, \ldots, \beta_m)$ or $8 + m$, and the optimization variables are, $u = (l_3, \beta_1, \beta_2, \ldots, \beta_m)$. The remaining procedure is same as in Type III except constraint in (16) is replaced by $0 \leq \beta_i < 2\pi$. The solution selection criterion is same as for driving side adjustable mechanisms given in sub-section 3.1.

The above optimization formulations are for $l_5 > l_2$. The same formulations are also valid for $l_5 < l_2$ case.

4 Selection of Adjustment Type

In this section we provide some rationale for the designer in the selection of the adjustment method. The selection can be based on the qualitative nature of the coupler curves and their change when one of the parameter is changed. The selection can also be based on minimizing the error between the desired and the path actually obtained by the adjustable mechanism. Both these aspects are discussed next.

4.1 Qualitative Behavior of Coupler Paths

The effect of various parameters on the coupler curve have been studied by several authors (see, for example the Atlas of coupler curves of planar four-bar mechanism by Hrones and Nelson [25] and the work in the references [26, 27]). The main observations are summarized below.

- The workspace boundaries are changed whenever crank-length l_2 or length l_5 is changed. For crank-length adjustable mechanisms for the case $l_5 > l_2$ using equation (2) we have, $l_{max}^2 > l_{max}^1$ and $l_{min}^2 < l_{min}^1$ for $l_2^2 > l_2^1$. The first workspace is completely enclosed by the second workspace.

- The increase or decrease in crank-length correspondingly increases or decreases the size (height and width) of the coupler curve but the shape of the curve is not much changed. In this case, the bigger coupler curve completely encloses the smaller one as shown in figure 3(a). The observation regarding crank-length adjustment also hold for the $l_5 < l_2$ with the difference that $l_{min}^2 > l_{min}^1$.

- For l_5 length adjustable mechanisms for the case $l_5 > l_2$ using equation (2) we have, $l_{max}^2 > l_{max}^1$ and $l_{min}^2 > l_{min}^1$ for $l_5^2 > l_5^1$. The first workspace and the second workspace have some area in common.

- The change in l_5 also changes the size of the coupler curve but the shape is almost retained. The bigger size coupler curves do not completely enclose the smaller size coupler curves (see figure 3(b)). The observation regarding l_5 length adjustment also hold for the $l_5 < l_2$ with the change that $l_{min}^2 < l_{min}^1$.

- By changing only the base link-length, the shape is changed but the size is not much changed. This is shown in figure 3(c).

- By changing the base link angle δ, both the shape and size of the coupler curve is retained but the position of the curve in the plane is changed as shown in figure 3(d).

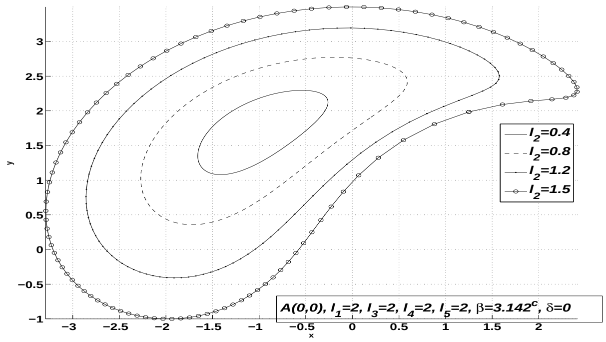
- By changing both the base link-length and base angle, the shape, size and position of the curve in the plane is changed (see figure 3(e)).

- By changing the output side link-length i.e length of rocker l_4, the shape and size of the curve are almost retained but the position of the curve in the plane is changed as shown in figure 3(f).

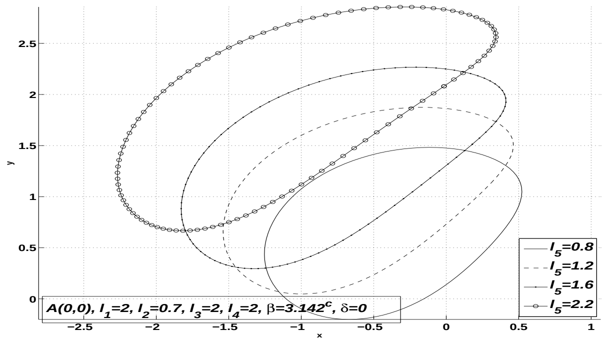
- By changing the coupler link-length l_3, the shape and position of the curve in the plane is almost retained, but the size of the curve changed markedly as shown in figure 3(g).

- By changing the coupler angle β, the shape, size and position of the curve in the plane are all changed markedly (see figure 3(h)).

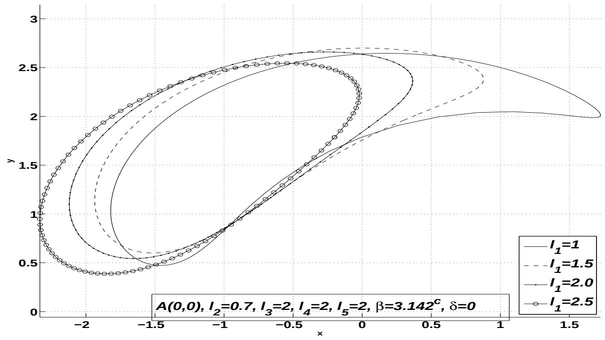
It maybe noted that all the cases are qualitative and the figures are for a typical four-bar mechanism. The chosen fixed as well as the variable dimensions are stated in the respective figures. The above cases often help in deciding the type of adjustment required in a four-bar mechanism to achieve multiple path generation.



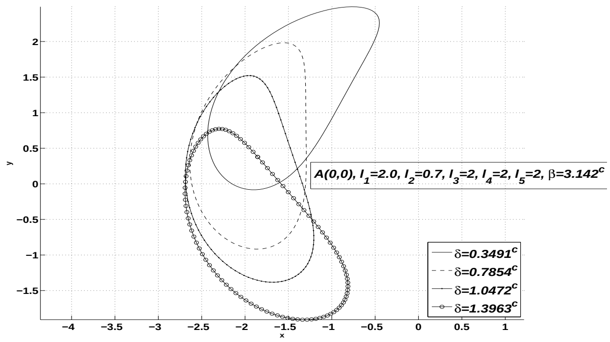
(a) Crank-length adjustment



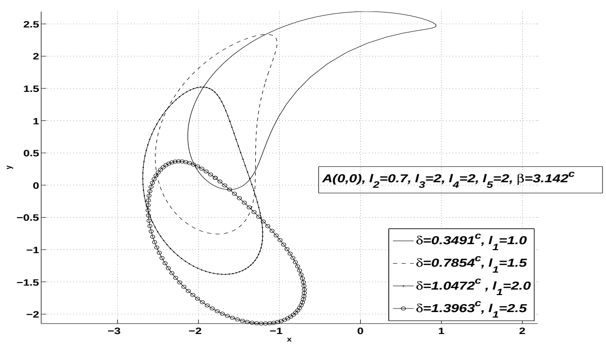
(b) l_5 length adjustment



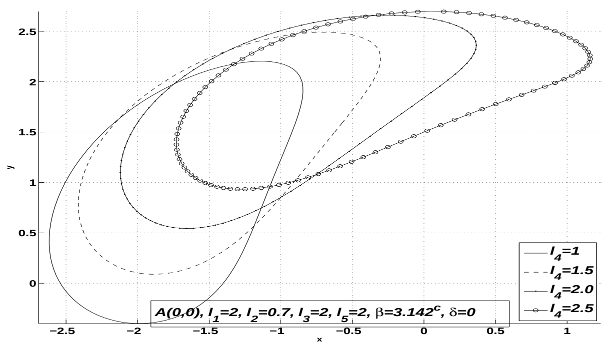
(c) Base link-length adjustment



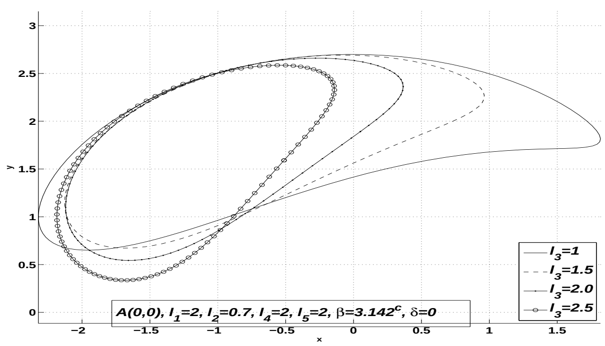
(d) Base link angle adjustment



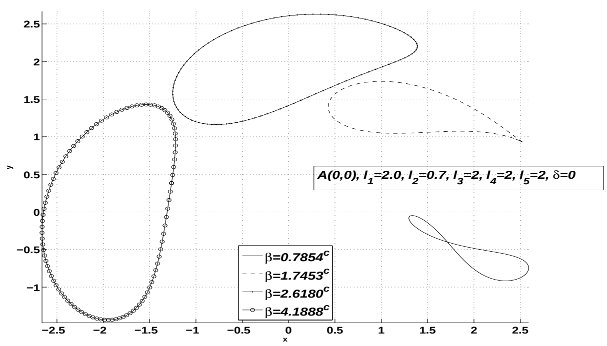
(e) Base link-length and angle adjustment



(f) Rocker link-length adjustment



(g) Coupler link-length adjustment



(h) Coupler angle adjustment

Figure 3: Qualitative behavior of coupler curves with adjustments of parameters

4.2 Error Computation

This sub-section deals with the quantitative approach towards selection of the adjustable mechanism. This is useful when the selection of adjustment method cannot be ascertained qualitatively. For example, in case of driven side adjustment when the workspace remains the same and we cannot qualitatively identify the adjustment method then, the Stage II optimization process is applied for each type of adjustment and adjustment type with minimum optimization cost function value f is chosen. It may happen that one or more of the optimization constraints in Stage II of design procedure may be violated disqualifying the particular adjustment method. The procedure is illustrated in example 2 in section 5. With respect to figure 1, input-output equation [4, 28] for the planar four-bar mechanism is given as,

$$\psi_i = -\delta + 2\tan^{-1}\left(\frac{-R \pm \sqrt{Q^2 + R^2 - S^2}}{S - Q}\right) \tag{26}$$

where,

$$\begin{aligned} Q &= 2l_1 l_4 \cos(\delta) - 2l_2 l_4 \cos(\delta + \phi_i) \\ R &= 2l_1 l_4 \sin(\delta) - 2l_2 l_4 \sin(\delta + \phi_i) \\ S &= l_1^2 + l_4^2 + l_2^2 - l_3^2 - 2l_1 l_2 \cos(\delta)\cos(\delta + \phi_i) - 2l_1 l_2 \sin(\delta)\sin(\delta + \phi_i) \end{aligned} \tag{27}$$

The coordinates of point $B(x_{B_i}, y_{B_i})$, $D(x_D, y_D)$ and $C(x_{C_i}, y_{C_i})$ can be written as,

$$\begin{aligned} x_{B_i} &= x_A + l_2 \cos(\delta + \phi_i) \\ y_{B_i} &= y_A + l_2 \sin(\delta + \phi_i) \\ x_D &= x_A + l_1 \cos(\delta) \\ y_D &= y_A + l_1 \sin(\delta) \\ x_{C_i} &= x_D + l_4 \cos(\delta + \psi_i) \\ y_{C_i} &= y_D + l_4 \sin(\delta + \psi_i) \end{aligned} \tag{28}$$

The coordinates of coupler point $P_{C_i}(x_{P_{C_i}}, y_{P_{C_i}})$ $(i = 1, 2, \ldots, 100)$ can be written as,

$$\begin{aligned} x_{P_{C_i}} &= x_{B_i} + \frac{l_5}{l_3}\left[(x_{C_i} - x_{B_i})\cos(\beta) - (y_{C_i} - y_{B_i})\sin(\beta)\right] \\ y_{P_{C_i}} &= y_{B_i} + \frac{l_5}{l_3}\left[(x_{C_i} - x_{B_i})\sin(\beta) + (y_{C_i} - y_{B_i})\cos(\beta)\right] \end{aligned} \tag{29}$$

where we assume that the generated path using the synthesized mechanism is given in terms of 100[5] points.

Let $P(x_{P_j}, y_{P_j})$ be a data point of a coupler path. The error in the j^{th} data point for the k^{th} coupler path is given by,

$$\begin{aligned} e_j^k &= \min\{e_{j1}, e_{j2}, \ldots, e_{ji}, \ldots, e_{j100}\} \\ e_{ji} &= \sqrt{(x_{P_{C_i}} - x_{P_j})^2 + (y_{P_{C_i}} - y_{P_j})^2}, \quad i = 1, 2, \ldots, 100, \quad j = 1, 2, \ldots, N_r, \quad k = 1, 2, \ldots, m \end{aligned} \tag{30}$$

[5]The number 100 is arbitrarily chosen for determining the maximum error between the coupler curves obtained from the given path points and the coupler curves obtained from the synthesized adjustable mechanism. It can be chosen to be more or less depending on the complexity of the coupler curve. The number 100 was found to be reasonable after extensive simulations.

where N_r is the number of data-points given initially to represent each coupler path and m is the number of coupler paths to be traced. The maximum error and the total error for the k^{th} coupler path is given as,

$$\begin{aligned} E^k_{max} &= \max\{e^k_1, e^k_2, \ldots, e^k_j, \ldots, e^k_{N_r}\} \\ E^k_{path} &= \sum_{j=1}^{N_r} e^k_j \end{aligned} \tag{31}$$

The total error for all the m coupler paths is given by,

$$E_{Total} = \sum_{k=1}^{m} E^k_{path} \tag{32}$$

If the adjustment cannot be chosen from using the minimum value of f then once the mechanism parameters are obtained from the optimization procedures (see sections 3.1 and 3.3), the total error for each adjustment can be computed as above and the adjustment method with minimum E_{Total} can be used.

5 Results and Discussion

To demonstrate the use of the proposed method we give two examples, one for each adjustable driving side mechanism and adjustable driven side mechanism. Simulations were done using 64-bit MATLAB R2011b on computer with Intel Core-2-Quad 2.40 GHz processor and 4 GB RAM. The optimization was done using 'fmincon' function of MATLAB [29].

5.1 Example 1

The first example we present is the simulation of a human stride. This may be used for designing the adjustable four-bar mechanism needed to generate the basic stride path for an exercising machines such as the one given in [30]. The human stride traces an approximate "D" shaped curve [26]. The straight portion of the "D" represents the stride-length. In this example, we design a single adjustable mechanism which can trace three different stride-lengths. The stride lengths are selected as 40 cm, 54 cm and 70 cm and the height of the "D" is chosen to be one-fifth of the stride-length with reference to the Hoecken's mechanism (see [26]). Since the workspace is different for each different stride-length, we need a crank or an adjustable mechanism where l_5 is changeable (see section 4) and we choose to design a crank-length adjustable mechanism. The curved portion of the "D" is assumed to be part of an ellipse and for the three paths, the curved portions are represented as below.

$$\begin{aligned} \text{Path 1} &= (a_1 \cos t,\ 4 + b_1 \sin t) \\ \text{Path 2} &= (a_2 \cos t,\ 2.25 + b_2 \sin t) \\ \text{Path 3} &= (a_3 \cos t,\ b_3 \sin t) \\ \text{with} \quad &\in [0, \pi], \quad a_1 = 20, \quad b_1 = \frac{2a_1}{5} \\ a_2 = 27, \quad b_2 &= \frac{2a_2}{5}, a_2 = 35, \text{ and } \quad b_3 = \frac{2a_3}{5} \end{aligned}$$

The straight portion of the "D" for the three paths are on $y = 4$ for Path 1, $y = 2.25$ for Path 2 and $y = 0$ for Path 3. Each path is specified by 20 data points. The path is further refined to 50

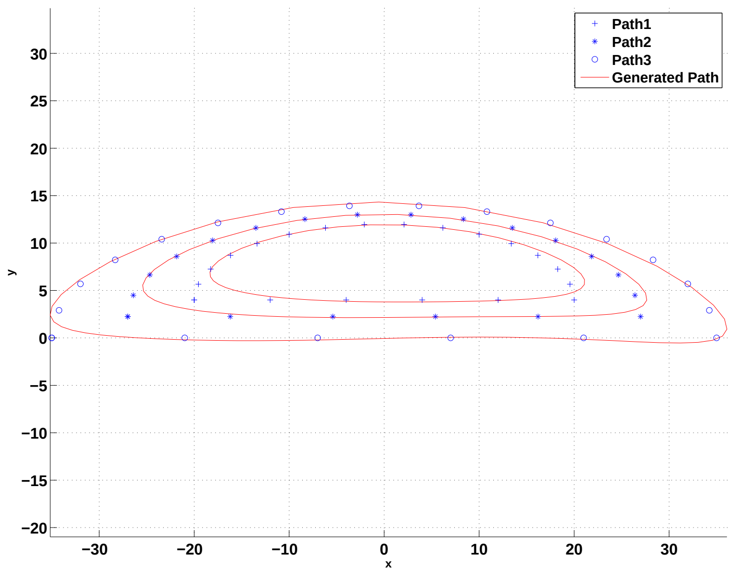


Figure 4: Example 1 – Generated and desired paths

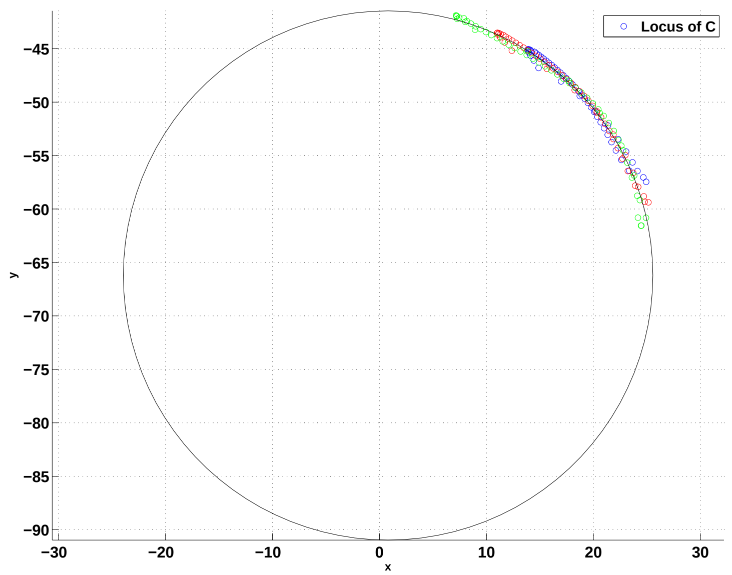


Figure 5: Example 1 – Locus of C

points using spline interpolation.

Stage I optimization
We have $x_A \in [-100, 100]$ and $y_A \in [-100, 100]$. Both intervals are divided into 40 sub-intervals each. Applying the theory given in sub-section (3.2), the value of S_{max} is chosen to be 0.01, $l_{5max} < 90$ and $l_2^i < 20$ which results in 21 possible locations of pivot A. Output of the optimization are:

Value of the cost function, $S = 0.0023$
Coordinates of $A\,(15.271, -80.0000)$ $\quad l_2^1 = 5.2428, \quad l_2^2 = 6.991, \quad l_2^3 = 9.236, \quad l_5 = 89.246$

If we apply coupler extension-link adjustment method, the lowest value of function S is 25.563 which contradicts the theory given in sub-section (3.2) and is disqualified immediately This also shows the importance of qualitative analysis of the adjustable methods.

Stage II optimization
In this case, $l_3 \in [20, 150]$ is divided into 15 sub-intervals and $\beta \in [0, 2\pi]$ is divided into 4 sub-intervals. The output of the second optimization are:
Value of the cost function, $f = 11.0268$
$l_1 = 19.980, \quad l_3 = 29.740, \quad l_4 = 24.750, \quad \beta = 0.3146$ rad, $\quad \delta = 2.3803$ rad where all dimensions are in centi-metres (cm).

The three coupler paths share all the parameters except the crank-length l_2. Figure 4 shows the generated and desired paths and figure 5 shows the locations of point C corresponding to the points P on the coupler curves at the optimum point. The circle plotted is the best fitting circle as per [23] for the distribution of C's at the optimum point. The error at each path point is calculated as per section 4.2. The maximum error for each path is as follows:
For path 1, $E^1_{max} = 2.8592$ which is 7.15% of the stride length,
For path 2, $E^2_{max} = 3.0108$ which is 5.58% of the stride length, and
For path 3, $E^3_{max} = 1.5172$ which is 2.17% of the stride length.
The time taken for Stage I is 67 seconds and for Stage II is 1150 seconds.

5.2 Example 2

This second example has been originally studied in [6]. We show that our approach using less number of search variables yields similar results. The given 20 data points are refined to 50 points using spline interpolation. Both the coupler curves have the same workspace but, the method of adjustment is not clear. To choose the adjustment method we apply Stage II for each type of adjustment.

Stage I optimization
We have $x_A \in [-20, 20]$ and $y_A \in [-20, 20]$. Both intervals are divided into 20 sub-intervals each. Applying the theory given in sub-section (3.2) the value of S_{max} is chosen to be 0.003 which results in 9 possible locations of pivot A.

Stage II optimization
The interval $l_3 \in [3, 20]$ divided into 4 sub-intervals and $\beta \in [0, 2\pi]$ is divided into 4 sub-intervals. The value of cost function and total error for each type is given in Table 1.

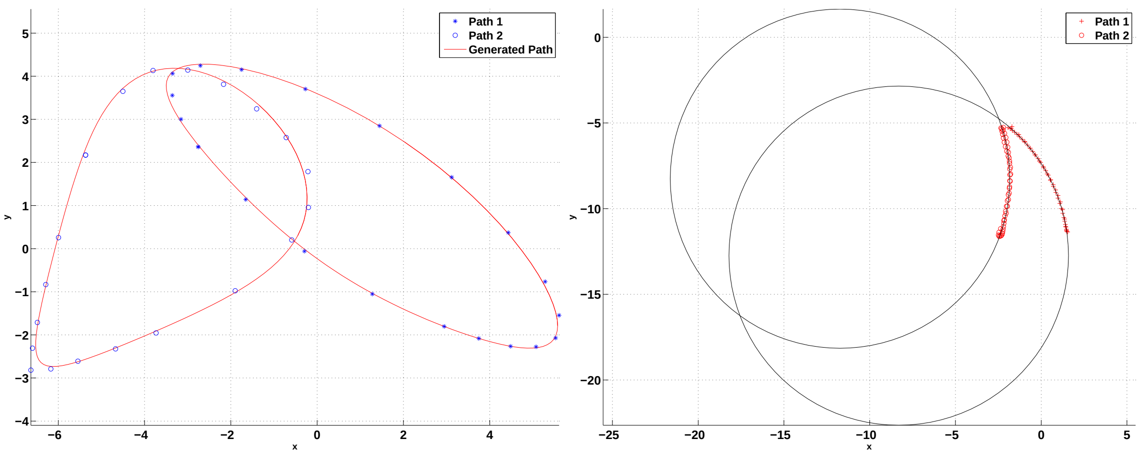
Type I and Type II mechanism give acceptable result but Type III and Type IV are disqualified due to constraint violation in Stage II optimization. Output after both stages of optimization for each type of adjustment is tabulated in Table 1. Since, value of f as well as E_{Total} is the least for adjustable pivot or Type I adjustment, we choose it to synthesize the mechanism.

The two coupler paths share all the parameters except the fixed rocker link pivot D. Figure 6(a) shows the generated and desired paths and figure 6(b) shows the locations of point C corresponding to the points P on the respective coupler curves at the optimum point. The circles plotted is the best fitting circles as per [23] for the distribution of C's at the optimum point for both the paths. The error at each path point is calculated as per section 4.2.

The time taken by Stage I is 11 seconds and by Stage II is 54 seconds. The optimization in Stage II has been carried out with 5, 6, 7 and 8 sub-intervals for the variable β and it was observed that

Parameter	Mechanism Type I		Mechanism Type II	
	Path 1	Path 2	Path 1	Path 2
Pivot A	$(0.000, -18.000)$		$(0.000, -18.551)$	
l_1	9.850	15.290	10.739	
l_2	3.0562		3.0645	
l_3	9.874		10.000	
l_4	9.992		7.1874	5.0508
l_5	19.398		19.9342	
β	6.266 rad		0.0000 rad	
δ	2.583 rad	2.451 rad	2.2026 rad	
Pivot D	$(-8.352, -12.771)$	$(-11.783, -8.259)$	$(-6.3422, -9.8856)$	
S	0.000733		0.000641	
f	0.157		34.884	
E_{path}	1.987	1.813	15.532	12.673
E_{Total}	3.800		28.205	

Table 1: Comparison of Type I and Type II adjustments



(a) Generated and desired path

(b) Locus of C

Figure 6: Example 2

the final solution does not change. However, as expected the time taken for optimization increases with the increase in the number of sub-intervals.

6 Conclusion

The article presents a new optimization based methodology for synthesising adjustable planar four-bar mechanisms for approximate multi-path generation. The adjustments can be made in both the driving side and driven side of the mechanism. The objective functions used in the optimization process for synthesising various parameters uses the least possible number of variables – the most common type of adjustments, the crank and coupler extension length adjustment for the driving side and rocker-link and rocker-link fixed pivot adjustment for the driven side of a four-bar mechanism uses only two optimization variables for optimization at each stage. The method uses least-squares based circle fitting technique. Various constraints are suggested to sort the appropriate mechanism and the method does not require starting point to be given by the user for the SQP optimization used. Several criteria regarding selection of the choice of adjustment method has been discussed and presented. The suggested method is capable of synthesising adjustable mechanism for more than two paths. The proposed method is illustrated using two examples and for these examples, the results are within reasonable error limits.

Acknowledgement

The authors would like to thank the anonymous reviewers for their comments. Their comments have helped in significantly improving the quality of this manuscript.

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List of Table Captions

Parameter	Mechanism Type I		Mechanism Type II	
	Path 1	Path 2	Path 1	Path 2
Pivot A	$(0.000, -18.000)$		$(0.000, -18.551)$	
l_1	9.850	15.290	10.739	
l_2	3.0562		3.0645	
l_3	9.874		10.000	
l_4	9.992		7.1874	5.0508
l_5	19.398		19.9342	
β	6.266 rad		0.0000 rad	
δ	2.583 rad	2.451 rad	2.2026 rad	
Pivot D	$(-8.352, -12.771)$	$(-11.783, -8.259)$	$(-6.3422, -9.8856)$	
S	0.000733		0.000641	
f	0.157		34.884	
E^{sum}	1.987	1.813	15.532	12.673
E_{Total}	3.800		28.205	

Table 1: Comparison of Type I and Type II adjustments

List of Figure Captions

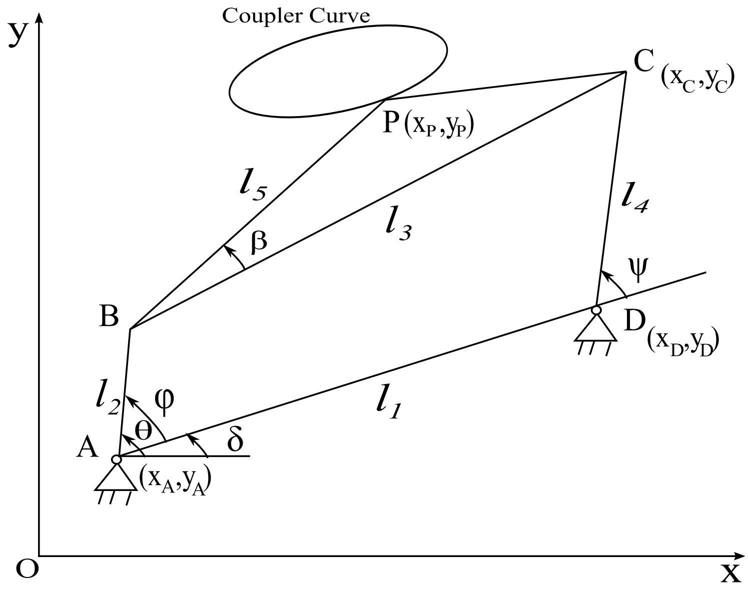


Figure 1: Schematic of a four-bar mechanism

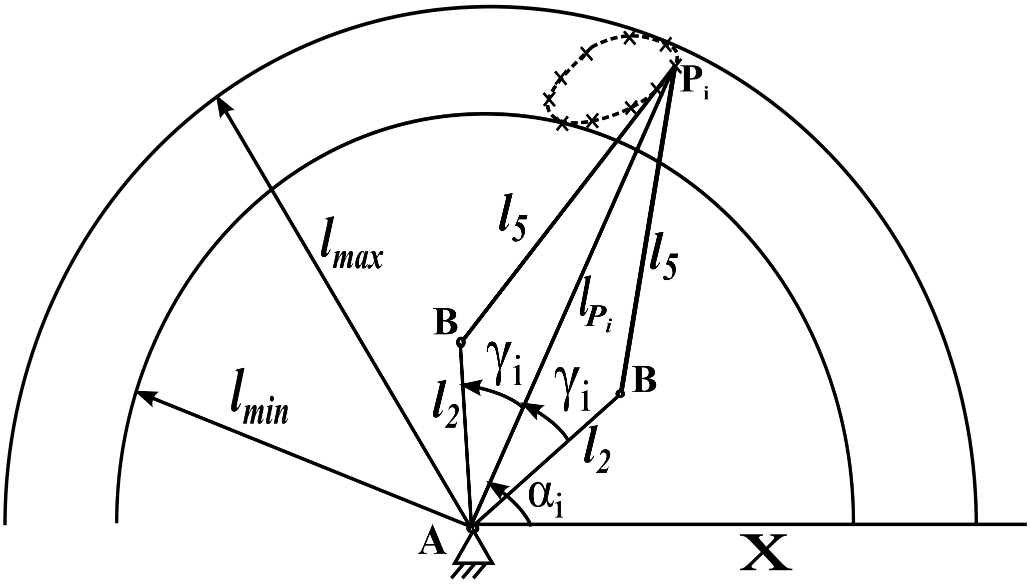
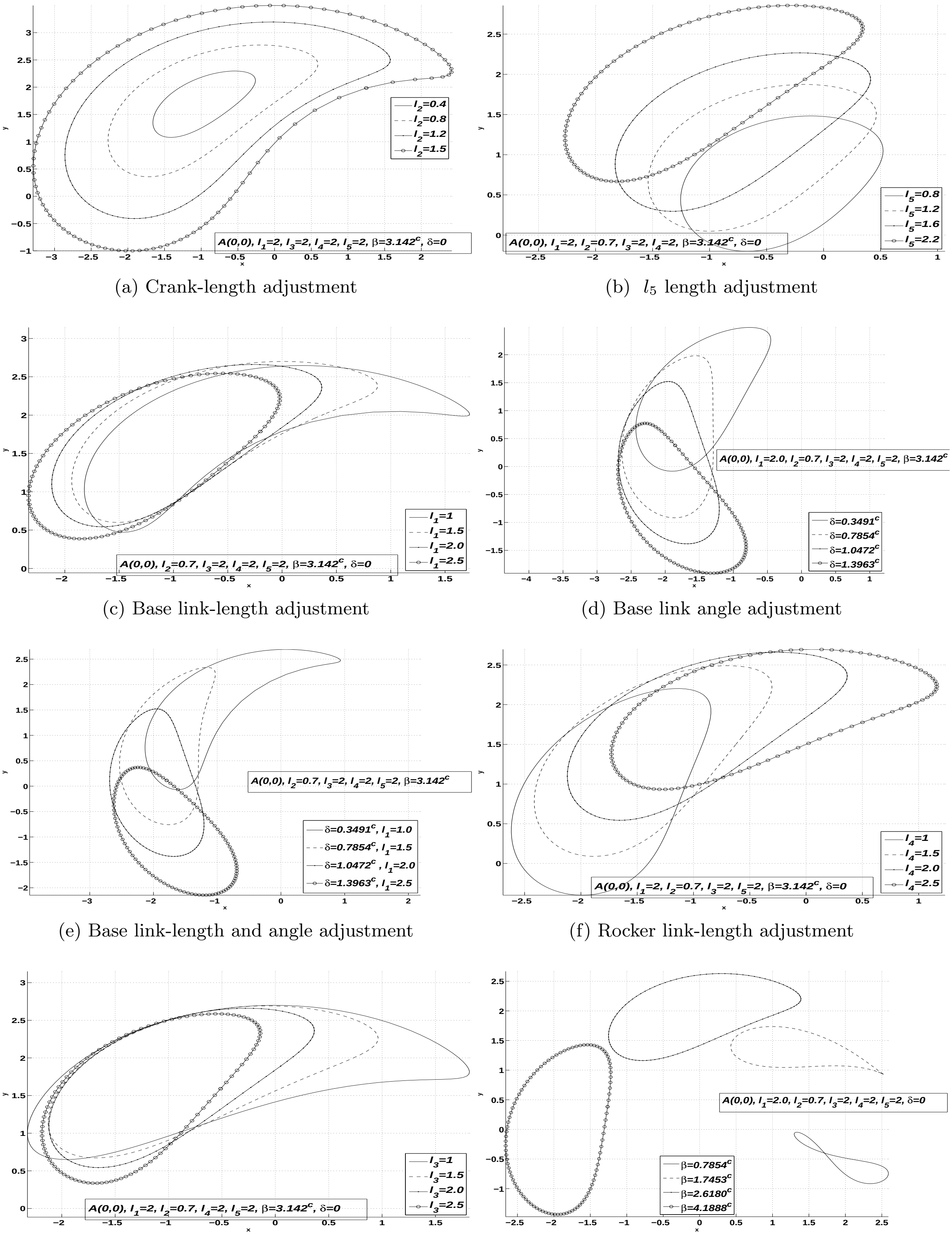


Figure 2: Driving dyad, workspace and crank angle



(a) Crank-length adjustment

(b) l_5 length adjustment

(c) Base link-length adjustment

(d) Base link angle adjustment

(e) Base link-length and angle adjustment

(f) Rocker link-length adjustment

(g) Coupler link-length adjustment

(h) Coupler angle adjustment

Figure 3: Qualitative behavior of coupler curves with adjustments of parameters

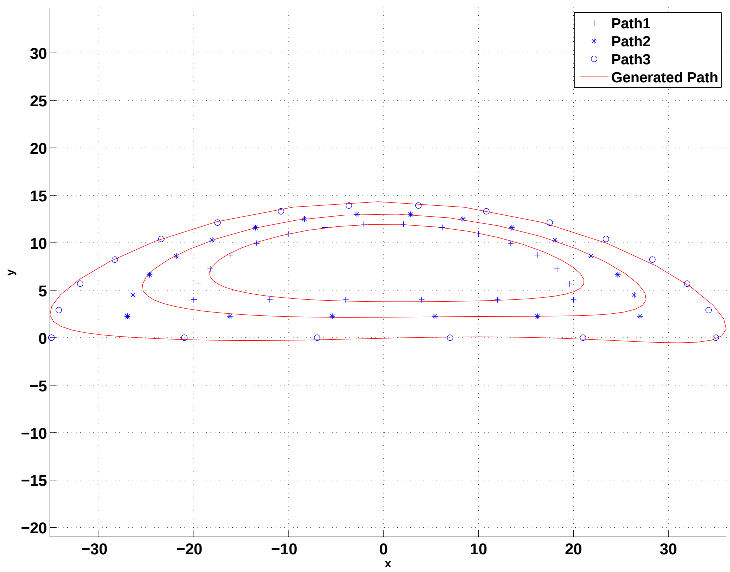


Figure 4: Example 1 – Generated and desired paths

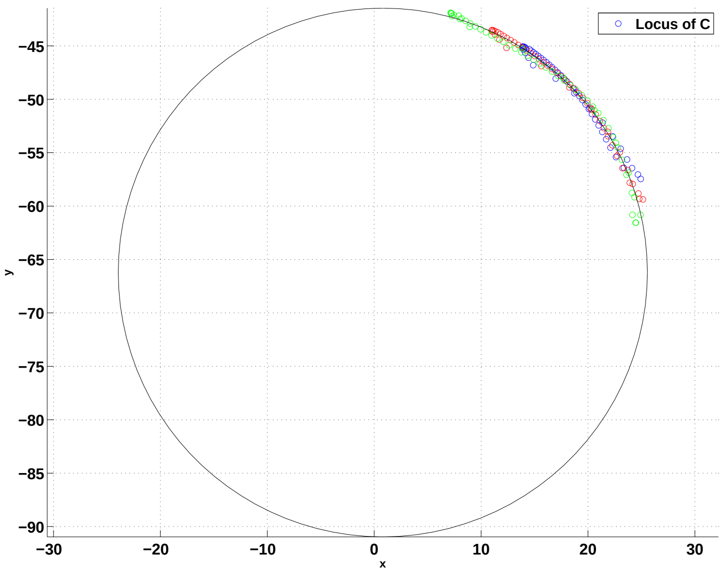
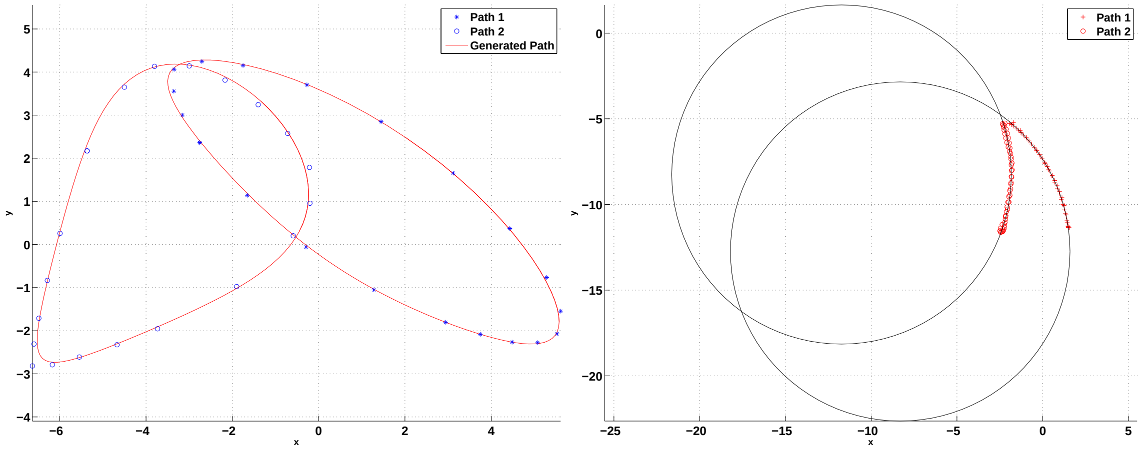


Figure 5: Example 1 – Locus of C



(a) Generated and desired path (b) Locus of C

Figure 6: Example 2