Abstract

Spacecraft appendages like the solar array, antenna are light weight and they are flexible. These are stowed during launch and deployed in the orbit. The deployment of these appendages is powered by pre-loaded torsion springs at joints which have a locking mechanism. The locking mechanism prevents further rotation of joint when it reaches the predefined angle. During locking the system loses its rotational degree of freedom and due to the inherent flexibility, vibration is induced in the system. In this paper the energy lost due to locking during the deployment of a two link and single link is estimated. The two link system has flexible links with revolute joints. Initially the system has two rotational degrees of freedom; after the first locking one rotational degree of freedom is lost and after second locking it behaves like a cantilever beam. The two link system reduces to a composite link by locking one joint resulting in a single rotational degree of freedom. Further with locking this link reduces a cantilever beam. The finite element method with Lagrange's equation is used for deriving the equations of motion. The momentum balance method is used to obtain the state variables just after locking. Experiments have been carried out. The results of mathematical model and experiments are compared for joint rotation, strain and tip acceleration during motion. The energy transfer after each locking is discussed.

Keywords : Flexible links, Momentum Balance Method, Locking

NOTATION

A_j	Cross sectional area of link *j*
b_j	Width of link *j*
$\boldsymbol{d}_j$	Nodal degrees of freedom of link *j*
E_j	Young's modulus of link *j*
E_a	Energy just after locking
E_b	Energy just before locking
E_i	Initial Energy
E_l	Energy lost due to locking
E_s	Energy spent by the system
$\boldsymbol{h}$	Corioli's and centripetal terms
I_j	Moment of inertia of the link cross section of link *j*
$\mathbf{K_C}$	Stiffness matrix for cantilever
$\mathbf{K_S}$	Stiffness matrix for single flexible link
$\mathbf{K_T}$	Stiffness matrix for two flexible links
K_{p1}	Torsion spring stiffness at joint *1*
K_{p2}	Torsion spring stiffness at joint *2*
ℓ_1	Length of link *1*
ℓ_2	Length of link *2*
$\mathbf{M_C}$	Mass matrix for cantilever
$\mathbf{M_S}$	Mass matrix for single flexible link
$\mathbf{M_T}$	Mass matrix for two flexible links
m_{pj}	Tip mass of link *j*
N	Cubic shape function

$\boldsymbol{q_f}$	Flexible degrees of freedom of the link
$\boldsymbol{q_r}$	Rigid body degrees of freedom
t_j	Depth of the link
T	Total kinetic energy
V	Total potential energy
w_j	Transverse deflection of the link *j*
$w_{j\ell}$	Tip deformation of link *j*
θ_1	First joint angle
θ_2	Second joint angle
θ_{10}	First joint pre rotation angle
θ_{20}	Second joint pre rotation angle
θ_{1in}	First joint initial angle
θ_{2in}	Second joint initial angle
θ_{1t-}	First joint angle just before locking
θ_{2t-}	Second joint angle just before locking
ρ_j	Density of the link *j*
τ_j	Torque at joint *j*
$\psi_{j\ell}$	Tip slope of link *j*

1. INTRODUCTION

Spacecraft appendages such as solar arrays and antennae are light weight and have large dimensions. These appendages have relatively low structural rigidity. In addition, the volume limitations inside the fairing of launch vehicles have necessitated the design of many appendages (for example, solar arrays and antennae) in a compact stowed manner during the launch, and mechanisms to deploy them in space [1]. Dynamic modelling and simulation of these structures including flexibility helps to predict the accurate behavior of the system during deployment.

Many flexible systems are subject to changes in kinematic behaviour during their operation. Some examples are locking of individual solar array of a satellite during deployment and retrieving an on-orbit satellite by a flexible robot arm. Such operations result in a redistribution of total momentum that may lead to impulsive forces and moments on the system. These impulsive forces may lead to large vibrations in the lightweight flexible structures. The large flexible systems such as solar arrays or antennae under go locking during deployment. The deployment for these systems is through torsion spring driven hinges. These hinges are designed based on the total torsional energy transferred to the hinge during locking. In this paper, the dynamics of flexible linkage with torsion spring driven hinges, used in deployment mechanisms of solar array of Indian Remote Sensing (IRS) satellite, is studied through mathematical simulation and experiments. The associated energy transfer due to locking is obtained. This study will help in designing the optimum weight array/antenna hinges for spacecraft.

Many researchers [2-7] have developed mathematical models and numerical simulation tools for studying the structural flexibility of the open and closed loop mechanisms during their motion. The deployment analysis of accordion type of solar array, stowed during launch and deployed in orbit, with rigid body assumption is discussed by Wie et al [8] and Nataraju and Vidyasagar [9]. When the solar array is deployed, locking results in impact at the joints and induces vibration in the system. Several researchers [10-17] have studied the post impact behaviour of open and close loop mechanisms with flexible links and validated their studies through experiments.

Flexible systems are modelled by finite element method, assumed mode method and very rarely by lumped parameter methods. In the finite element method, the elastic members are discretized with finite elements with their nodes having translational and rotational degrees of freedom. The deflection in the element is approximated by polynomial shape functions discussed in Usoro et al [2] and Anthony et al [3]. The authors [3-7] have used the assumed mode method and represented the link flexibility by the finite number of modes, in terms of eigen functions which are transcendental and time dependent mode amplitudes. The modelling of structural flexibility using finite element method provides a systematic modelling technique for mechanisms with complex geometries and the boundary conditions can be incorporated in a straightforward way. Khulief and Shabana [10] used the momentum balance method for constrained mechanical systems with interconnected rigid and flexible bodies. The joints are simulated by constraint equations using Lagrange multipliers. The intermittent motion is monitored by an event predictor algorithm. The authors [10] extended their theory in [11] to a general multibody system subject to kinematic structure changes by a mixed set of Lagranges co-ordinates. Yigit et al [12,13] used the momentum balance method for a rotating beam impacting on a horizontal surface and validated through experiments. Nagaraj et al [14] simulated locking using the conservation of momentum principle. The analytical simulation of strain and joint rotation did not match well with the experiment. Nagaraj et al [15,16] used the momentum balance method for the flexible system undergoing locking during motion and validated through experiments. The simulation results were in good agreement with experiments. In this paper the theory developed in the above literature is used and validated by experiments for the solar array hinge of IRS satellite with flexible links. The joint rotation, strain in the link and acceleration at the link tip are measured. Finally, the energy transfer is estimated during locking.

2. MODELLING OF FLEXIBLE LINKS

In this section the mathematical model for the flexible two link system is described. A brief description of the system is presented below.

2.1 System description

The systems considered are

i) *Two link set up* : The schematic representation of a two link setup is shown in Figure 1a. The links are supported by air bearings at the ends and the motion of links is on a flat horizontal table there by gravity effects can be neglected. Each link is actuated by means of pre-loaded torsion spring mounted at the joint. The two flexible links are folded with a known initial angle and released. The motion of each link is indicated by the arrows in Figure 1b. The first locking takes place, when the angle between first link and the second link is zero, as shown in Figure 1c. After the first locking the second joint loses its rotational degree of freedom and the two flexible links form a single composite link. This composite link rotates about the first joint as shown in Figure 1d. A second locking takes place, when the first link rotates through 92.5 degrees as shown in Figure 1e. After the second locking the first joint loses its rotational degree of freedom, and the system behaves as a cantilever beam clamped about the first joint.

ii) *Composite link set up* : The schematic representation of a composite link setup is shown in Figure 2a. In this setup the second link is locked to the first link about second joint to form a single flexible composite link. The single composite link rotates about the first joint as shown in Figure 2a. The locking takes place, when the link rotates through 92.5 degrees as shown in Figure 2b. After locking the joint loses its rotational degree of freedom, and the composite link behaves as a cantilever beam clamped about the joint.

iii) *Single link set up* : The schematic representation of a single link setup is shown in Figure 3a. This set up is similar to the composite link set up with second link and second joint is disassembled. The motion and locking are similar to composite link set up except, link starts its rotation from zero degree. The composite link and single link are subsets of a two link system.

2.2 Assumptions

The following assumptions are made in the mathematical formulation.

- The experimental configuration is designed to eliminate the possibility of torsion. Hence, the torsional effects are neglected.
- The flexible links can deflect only in the horizontal plane as they are supported on the air bearing. Hence, the bending due to gravity is negligible.
- Each flexible link is long and slender and the Euler-Bernoulli beam theory can be used to model the transverse elastic deflection.
- Stress induced due to the motion of flexible link is within the elastic limit.

The transverse deflection of a link is approximated by means of finite element method [**2-3**]. The first link has n_1 elements and the second link has n_2 elements. The local transverse deflections for link *1* and link *2* are denoted by w_1 and w_2 respectively. The transverse deflection, w_j, for an element is expressed as a linear combination of cubic shape function $\boldsymbol{N}$ and nodal degrees of freedom $\boldsymbol{d_j}$ for link j as

$$w_j = \boldsymbol{N}\,\boldsymbol{d_j} \tag{1}$$

The nodal degrees of freedom, $\boldsymbol{d_j}$ for an n^{th} element of j^{th} link are given by

$$\boldsymbol{d_j} = \{w_{j,n}, \Psi_{j,n}, w_{j,n+1}, \Psi_{j,n+1)}\}^T \tag{2}$$

where, $w_{j,n}$, $w_{j,n+1}$ are the translational degrees of freedom and $\Psi_{j,n}$,$\Psi_{j,n+1}$ rotational degrees of freedom for the n^{th} element of j^{th} link.

2.3 Kinematics of flexible two link system

In this section, equations of motion for the system presented in Figure 1b are formulated. The two flexible links rotate about the two spring driven revolute joints. Each flexible link has transverse deflection during motion. The representation of two flexible links with joints is shown in Figure 4(a) with the original and the deformed configuration. The coordinate system $O\ X\ Y$ is the inertial reference frame. The body fixed coordinate system $O_j\ X_j\ Y_j$ is attached to the link j with origin at O_j and X_j axis along the link j. The deflection w_j is described in [**2**] relative to the co-ordinate system that follows the rigid body motion of link.

The position vector $\boldsymbol{r_{01}}$ of point Q_1 on the link-*1* from the origin of the inertial frame is

$$\boldsymbol{r_{01}} = \mathbf{R}_0^1 \boldsymbol{r_1} \tag{3}$$

where, $\mathbf{R}_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\boldsymbol{r_1} = \begin{Bmatrix} x_1 \\ \mathrm{w}_1 \\ 0 \end{Bmatrix}$

The position vector $\boldsymbol{r_{m1}}$ of the tip mass m_{p1} is given by

$$\boldsymbol{r_{m1}} = \mathbf{R}_0^1 \boldsymbol{P_1} \tag{4}$$

where, $\boldsymbol{P}_1 = \{\ell_1, w_{1\ell}, 0\}^T$

The position vector $\boldsymbol{r_{02}}$ of point Q_2 on the link-*2* from the origin of the inertial frame is given by

$$\boldsymbol{r_{02}} = \mathbf{R}_0^1 \boldsymbol{P_1} + \mathbf{R}_0^2 \boldsymbol{r_2} \tag{5}$$

where, $\mathbf{R}_0^2 = \begin{bmatrix} \cos(\theta_1 + \psi_{1\ell} + \theta_2) & -\sin(\theta_1 + \psi_{1\ell} + \theta_2) & 0 \\ \sin(\theta_1 + \psi_{1\ell} + \theta_2) & \cos(\theta_1 + \psi_{1\ell} + \theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and $\boldsymbol{r_2} = \{x_2, w_2, 0\}^T$

The position vector $\boldsymbol{r_{m2}}$ of the tip mass m_{p2} of the second link is given by

$$\boldsymbol{r_{m2}} = \mathbf{R}_0^1 \boldsymbol{P_1} + \mathbf{R}_0^2 \boldsymbol{P_2} \tag{6}$$

where, $\boldsymbol{P_2} = \{\ell_2, w_{2\ell}, 0\}^T$

2.3.1 Kinetic Energy

The total kinetic energy includes both flexible and rigid body motion of links and tip masses. The total kinetic energy T is given by

$$T = \sum_{j=1}^{2}\left(\sum_{i=1}^{n_j} \frac{1}{2}\int_0^{\ell_j} \rho_j A_j \left(\frac{d\boldsymbol{r_{0j}}}{dt}\right)^T \left(\frac{d\boldsymbol{r}_{0j}}{dt}\right) dx_j + \frac{1}{2} m_{pj} \left(\frac{d\boldsymbol{r_{mj}}}{dt}\right)^T \left(\frac{d\boldsymbol{r_{mj}}}{dt}\right)\right) \tag{7}$$

where, ρ_j is the density, A_j is the cross sectional area, ℓ_j is the element length for i^{th} element and n_j is the total number of elements in the link j. The above equation can be written as

$$T = \frac{1}{2}\dot{\boldsymbol{q}}^T \mathbf{M_T}\, \dot{\boldsymbol{q}} \tag{8}$$

where, $\boldsymbol{q} = \{\boldsymbol{q_r}, \boldsymbol{q_f}\}^T$, $\boldsymbol{q_r} = \{\theta_1, \theta_2\}$ is the rigid body and $\boldsymbol{q_f}$ are transverse nodal degrees of freedom of the links and $\mathbf{M_T}$ is the mass matrix.

2.3.2 Potential energy

The potential energy arises from two sources - the strain energy due to the transverse deflection of flexible links and potential energy of the torsion springs mounted at the joints.

The potential energy V_i of an element i of the link j is given by

$$V_i = \sum_{j=1}^{2}\left(\sum_{i=1}^{n_j} \frac{1}{2}\int_0^{\ell_i} E_j I_j \left(\frac{d^2 w_j}{dx_j^2}\right)^T \left(\frac{d^2 w_j}{dx_j^2}\right) dx_j\right) \tag{9}$$

The total potential energy V of the links is given by

$$V = \frac{1}{2} \boldsymbol{q}_f{}^{\boldsymbol{T}} \mathbf{K}_{\mathbf{T}} \, \boldsymbol{q}_f \tag{10}$$

The potential energy V_s due to torsion spring at i th joint is given by

$$V_S = \sum_{i=1}^{2} \frac{1}{2} K_{pi} (\theta_{io} - \theta_i)^2 \tag{11}$$

2.3.3 Torque at joints

The torque, τ_i , acting at the joints due to the spring force exerted by the rocker arm on the cam mounted at joint j and frictional torque acting at the joint j is computed by the principle of virtual work. The virtual work, δW, due to the torque τ_j is given by

$$\delta W = \sum_{j=1}^{2} \tau_j \delta\theta_j \tag{12}$$

2.3.4 Equations of motion

The equations of motion are derived based on the Lagrangian formulation. The equations of motion for the two link flexible system are obtained by using equations (7) to (12) in Lagrange's equation and can be written in a matrix form as

$$\begin{bmatrix} \mathbf{M}_{\mathbf{Tr}} & \mathbf{M}_{\mathbf{Trf}} \\ \mathbf{M}_{\mathbf{Trf}} & \mathbf{M}_{\mathbf{Tf}} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{q}}_r \\ \ddot{\boldsymbol{q}}_f \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{h}_{Tr}(\boldsymbol{q}_r, \dot{\boldsymbol{q}}_r, \boldsymbol{q}_f, \dot{\boldsymbol{q}}_f) \\ \boldsymbol{h}_{Tf}(\boldsymbol{q}_r, \dot{\boldsymbol{q}}_r, \boldsymbol{q}_f, \dot{\boldsymbol{q}}_f) \end{Bmatrix} + \begin{bmatrix} \mathrm{K}_{\mathrm{p}} & 0 \\ 0 & \mathbf{K}_{\mathbf{T}} \end{bmatrix} \begin{Bmatrix} \theta \\ \boldsymbol{q}_f \end{Bmatrix} = \begin{Bmatrix} \tau_{\mathrm{T}} \\ 0 \end{Bmatrix} \tag{13}$$

The first term contains the configuration dependent mass matrix $\mathbf{M_T}$ and the second derivative of the generalised coordinates. This matrix can be separated into the sub-matrix containing rigid body terms $\mathbf{M_{Tr}}$, the sub-matrix containing the terms involving the coupling of the rigid body and flexible variables $\mathbf{M_{Trf}}$, and the sub-matrix containing the terms involving only the flexible variables $\mathbf{M_{Tf}}$. The second term is the vector of Coriolis and centripetal terms. The third term is the stiffness term and this can be separated into contribution from the torsion springs $\mathbf{K_p}$ and the flexible links $\mathbf{K_T}$. The term on the right hand side τ_T is the external torque computed from equation(12). The equations of motion described above are integrated numerically with the given initial conditions and the torque, till the generalised coordinate of the second joint becomes zero. At this stage the flexible link *2* locks with the link *1* at the joint *2* and loses its rotational degree of freedom as shown in Figure 1(c).

2.4 Mathematical model for locking

The first locking takes place in the two link setup when the angular rotation of the second joint is zero as described above. The angular velocity of the first joint and the velocity of flexible variables after first locking is evaluated using the momentum balance method.

Momentum balance method : The momentum balance method employed in studies [**10-13** and **15-16**] is based on the impulse momentum law . It assumes that

- The impact occurs instantaneously and thus neglects the duration of impact.
- The system configuration is continuous during impact
- The velocities are bounded during impact

The equation of momentum balance method is presented in references [**15-16**] and is given by

$$\mathbf{B}_1 \Delta \dot{\boldsymbol{Z}}_1 = \boldsymbol{F}_1 \tag{14}$$

where, $\mathbf{B}_1$ contains mass matrix terms, $\Delta \dot{\boldsymbol{Z}}_1 = \{\Delta\dot{\theta}_1, \Delta\dot{\boldsymbol{q}}_f, \mathrm{H}_1\}^{\mathrm{T}}$, H_1 is the impulse at joint *2*, $\mathbf{F}_1$ is the terms containing product of joint velocity $\dot{\theta}_2$ and mass matrix terms associated with joint *2*. The velocity after locking is given as

$$\dot{\theta}_{1+} = \dot{\theta}_{1-} + \Delta\dot{\theta}_1$$

$$\dot{\boldsymbol{q}}_{f+} = \dot{\boldsymbol{q}}_{f-} + \Delta\dot{\boldsymbol{q}}_f \tag{15}$$

The negative sign in the subscripts indicates the state variables just before locking and the positive sign indicates the state variables just after locking.

2.5 Modelling for single flexible composite link

The two flexible links described earlier locks, when the joint rotation of link 2 reaches zero degree. The system reduces to a single flexible composite link as shown in Figure 1(c). The generalised coordinates and the velocities computed in the previous section forms the initial condition to the single flexible composite link. This link now continues its rotation about the first joint as shown in Figure 1(d). The equations of motion can be derived on the lines described in section 2.3. The deformed and undeformed single flexible composite link is shown in Figure 4b.

The flexural deformations *v* is defined relative to the co-ordinate system that follows the rigid body motion of link as described earlier. The position vector of the deflected link and the tip mass is computed as described earlier.

The equation of motion for this configuration is similar to equation (13) and is given by

$$\begin{bmatrix} \mathbf{M}_{\mathbf{sr}} & \mathbf{M}_{\mathbf{srf}} \\ \mathbf{M}_{\mathbf{srf}} & \mathbf{M}_{\mathbf{sf}} \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\boldsymbol{q}}_f \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{h}_r(\theta_1, \dot{\theta}_1, \boldsymbol{q}_f, \dot{\boldsymbol{q}}_f) \\ \boldsymbol{h}_f(\theta_1, \dot{\theta}_1, \boldsymbol{q}_f, \dot{\boldsymbol{q}}_f) \end{Bmatrix} + \begin{bmatrix} \mathrm{K}_{\mathrm{p1}} & 0 \\ 0 & \mathbf{K}_{\mathbf{s}} \end{bmatrix} \begin{Bmatrix} \theta \\ \boldsymbol{q}_f \end{Bmatrix} = \begin{Bmatrix} \tau_1 \\ 0 \end{Bmatrix} \quad (16)$$

The equations of motion derived above are integrated numerically till the generalised coordinate of the first joint reaches 92.5 deg. At this stage, the first joint locks by losing its rotational degree of freedom. The velocity of flexible variables just after locking is evaluated based on the momentum balance method as described earlier [15,16]. The single flexible composite link now reduces to a cantilever beam. The above mathematical model is used for the composite link setup and single link setup respectively with appropriate initial conditions.

2.6 Modelling of cantilever beam

The composite link described earlier locks, when the rotation of the first joint reaches 92.5 deg as mentioned earlier. The system now reduces to a simple cantilever beam with clamped boundary condition at the first joint as shown in Figures 1e, 2b and 3b respectively.

The equation of motion for the cantilever beam is given by

$$\mathbf{M}_{\mathbf{c}}\, \ddot{\boldsymbol{q}}_{\boldsymbol{f}} + \mathbf{K}_{\mathbf{c}}\, \boldsymbol{q}_{\boldsymbol{f}} = 0 \tag{17}$$

where $\mathbf{M_c}$ is the mass matrix and $\mathbf{K_c}$ is the stiffness matrix of cantilever. These equations are integrated to get the response after second locking.

2.7 Energy transfer after locking

The locking takes place in two stages when the joint reaches a predefined angle as mentioned earlier. The locking involves the transfer of energy. In this section the energy of the system just before and after locking is evaluated.

2.7.1 Two flexible links

In the two link setup the two flexible links rotate independently about the joints from the initial configuration. The first locking constrains the further rotation of the joint 2. The system behaves like a flexible composite link rotating at joint 1 after first locking. The energy before first locking is computed as follows.

The initial energy is given by

$$\mathrm{E_i} = \sum_{\mathrm{j=1}}^{2} \frac{1}{2}\mathrm{K}_{pj}(\theta_{\mathrm{jo}} - \theta_{\mathrm{jin}})^2 \tag{18}$$

The energy spent from the initial configuration up to locking is given by

$$E_s = \sum_{\mathrm{j=1}}^{2} \left\{ \frac{1}{2}\mathrm{K}_{pj}(\theta_{\mathrm{jo}} - \theta_{\mathrm{jt-}})^2 + \tau_{\mathrm{j}}\theta_{\mathrm{jt-}} \right\} \tag{19}$$

The energy of the system just before locking at time t_{-} is given by

$$E_b = E_i - E_s \qquad (20)$$

This energy is converted to the kinetic energy and the flexural strain energy of the link after first locking.

The energy just after first locking is given by

$$E_a = \frac{1}{2}\dot{\boldsymbol{q}}_f{}^{\mathrm{T}}\mathbf{M}_{\mathrm{S}}\,\dot{\boldsymbol{q}}_f + \frac{1}{2}\boldsymbol{q}_f{}^{T}\mathbf{K}_{\mathrm{s}}\,\boldsymbol{q}_f \qquad (21)$$

where, $\boldsymbol{q_f} = \{\theta_{1+}, \boldsymbol{q}_{f+}\}$ are the magnitudes of joint rotation and flexible nodal degrees of freedom just after locking

The percentage energy loss due to locking is given by

$$E_l = \frac{(E_b - E_a)100}{E_b} \qquad (22)$$

2.7.2 Single flexible link

In the two link setup, the composite link rotates about joint 1 after first locking as described earlier. The second locking occurs when joint 1 rotates through a predetermined angle as described earlier. The energy before second locking for the two link system, the energy before first locking for the composite link and percentage energy loss are computed as described in the previous section

3. EXPERIMENTAL SETUP AND INSTRUMENTATION

The dynamics of flexible links undergoing locking is studied experimentally by simulating Two link set up, Composite link set up and Single link set up.

The experimental set up of two links is shown in Figure 5. The two flexible aluminum links are constrained to slew in the horizontal plane on a granite table. Each joint is driven by pre-loaded torsion spring made of beryllium copper. The first joint is supported rigidly by the side of a granite table. The second joint and the tip of second link are floated on the granite table by air bearings. Each joint is provided with a locking mechanism which prevents further rotation of the joint when the joint reaches a predefined angle. The experimental set-up of single flexible composite link is shown in Figure 6. In this set up the second link is locked to the first link and forms the composite link. The composite link rotates about the first joint. The experimental set up of single link is shown in Figure 7.

3.1 Instrumentation

A brief description of instrumentation of the experimental set up is presented.

3.1.1 Joint angle measurement

The rotation of each joint is measured by a continuous turn 10K Ohms potentiometer mounted at the joint. The input voltage of the potentiometer is 5V and the output is recorded in a microcomputer through Analog to Digital Converter (ADC). The accuracy of the potentiometer is 0.5%.

3.1.2 Strain measurement

The strain at the base of each flexible link is measured using strain gauges. The gauge factor is *2.135*. A full bridge circuit having two gauges each on the two opposite surfaces is mounted at the root of each link. The strain signals pass through a preamplifier and the output is recorded in a microcomputer.

3.1.3 Acceleration measurement

The variable capacitance low mass accelerometer (Endevco 7290-10) is used to measure the tip acceleration of each link during motion. The accelerometers are mounted at the tip of each link. The accelerometer signals pass through a signal conditioner and the output is recorded in a microcomputer through an ADC.

4. RESULTS AND DISCUSSIONS

In this section the numerical simulation results from the developed mathematical model and experimental results are compared. The first order differential equations of motion were solved [18] by a variable step, variable order (of interpolation) predictor-corrector Adams algorithm. The experiments are carried out six times for each setup and the results of one typical experiment are presented for brevity. The joint rotation, strain and acceleration obtained from experiments are compared with the mathematical model for the single link set up, composite link set up and two link set up.

The input data is presented in Tables 1, 3 and 5 for the cases considered in this study. Referring to the experimental set-up (Figures 5 to 7) the flexural rigidity of each link is affected by the presence of flanges at the end of each link. Hence, the link 1 is stiff at both ends and link 2 is stiff at one end. The actual flexural rigidity and the equivalent cross section for each link is arrived by modelling the complete region covered by the flanges and the associated components by finite element method. Hence, the equivalent thickness is presented in the above Tables.

4.1 Single link set up

The link is released from initial configuration. The link locks when it deploys through a predetermined angle as shown in Figure 3. The differential Equations (16) are solved from the input data and the initial conditions given in Table 1. Figure 8(a) presents the rotation of the joint. The mathematical model and experimental results of the rotation of joint shows smooth and continuous variations up to locking. It can be seen from the above figure that the mathematical model results are close to the experimental results. The time for locking from the experiment is 3.63 sec and from the mathematical model is 3.48 sec, when the joint rotation θ_1 reaches 92.5 degrees. Now the joint loses its rotational degree of freedom. The velocities due to locking are computed from momentum balance method. The single flexible link at this stage reduces to a cantilever. The differential equations(17) are numerically integrated with the initial conditions obtained from the momentum balance method after locking.

The strain in the flexible link is measured in micro strains and figure 8(b) presents the experimental and the mathematical model results of the strain behavior. It is observed that the magnitude of strain is very small from initial configuration up to locking, because the flexural strain energy of the link is small. The strain gauges are not sensitive to acquire such small values of strain. The abrupt increase in the strain values is observed just after locking. The kinetic energy of the link just before locking is transformed to the flexural strain energy and the flexural kinetic energy. This induces large strain in the link. The strain predicted by the mathematical model is in close agreement and also in phase with the experimental data. The peak strain from the experiments is 363 micro strains and from the mathematical model is 325 micro strains.

Figure 8c presents the acceleration at the tip of link. The acceleration in link is enhanced after locking due to transfer of energy as described above. It is observed that the experimental acceleration shows high frequency components due to locking. This is not observed in the mathematical model because of the use of proportional damping in the equations. The average acceleration from the experiment is 15.0 m/s^2 and from mathematical model is 8.0 m/s^2. It was observed from the strain response plot that the locking is not exciting the higher modes of the system, but higher modes are seen in the acceleration response.

The energy before and after locking are presented in Table 2. It is observed that all the kinetic energy of flexible link gets transferred to the cantilever after locking.

4.2 Composite link set up

The composite link is released from the initial configuration. The link locks when it deploys through a predetermined angle as shown in Figure 2. The differential equations (16) are solved from the input data and the initial conditions mentioned in Table 3. Figure 9a presents the rotation of joint. The behavior of joint rotation is similar to that of single link case. The time for locking in the experiment is 5.5 sec when the joint rotation θ_1 reaches 92.5 degrees. This time is higher than the predicted 5.28 sec from the mathematical model. The velocities after locking are computed from momentum balance method. The composite link at this stage reduces to a cantilever beam. The differential equations (17) are numerically integrated with the initial conditions obtained from the momentum balance method after locking.

Figure 9b presents the strain at the root and at the mid span of composite link. It is observed that the magnitude of strain is very small from initial configuration up to locking, because the torque at the joint is small and the flexural strain energy of the link is small. The strain gauges are not sensitive to acquire such small values of strain. The strain behavior is similar to the single link case except that locking introduces higher frequencies in the system. The peak strain in the experiment is 295 micro strains at the root of composite link and 153 microstrains at the mid span of composite link. The peak strain from the mathematical model at these locations are 332 micro strains and 155 microstrains respectively.

Figures 9c presents the acceleration at the mid span and the tip of composite link. The acceleration is enhanced after locking due to transfer of energy as described above. It is observed that the acceleration shows high frequency components due to locking. The acceleration from the mathematical model for the above locations are 1.65 m/s^2 and 2.1

m/s^2 respectively. The average acceleration from the experimental data for these locations is 3.0 m/s^2 and 2.5 m/s^2.

The energy before and after locking are presented in Table 4. It is observed that all the kinetic energy of flexible composite link gets transferred to the cantilever after locking.

4.3 Two link set up

The two flexible links move independently and lock at the joints at the predetermined angle as shown in Figure 1. The differential equations (13) are solved from the input data and the initial conditions given in Table 5. Figures 10a presents the rotation of joints. The joint rotations shows smooth and continuous variations up to the first locking. The mathematical model results are close to the experimental results in the initial motion and deviates slightly at the end of first locking. The time for the first locking from the mathematical model is 2.82 sec, when the joint rotation θ_2 reaches zero degree. This is lower than predicted 3.13 sec by the experiment. After first locking the joint 2 loses its rotational degree of freedom. The discontinuity in the velocities due to first locking are computed from the momentum balance method by using equations (14) and (15). The differential equation (16) are solved numerically till the joint rotation θ_1 reaches 92.50 degrees. The two flexible links rotate like a single flexible composite link about the first joint as shown in Figure 1(d). After first locking, the first joint angle shows oscillations in both mathematical and experimental results. The second locking takes place when the composite link rotates by 92.5 degrees. The time for second locking in the experiment is 5.52 sec. This time is slightly higher than the mathematical model, which is predicting 5.43 sec. The differential equations (17) are numerically integrated with the initial conditions obtained from the momentum balance method for second locking. Now the composite link vibrates like a cantilever.

Figures 10b presents the behavior of strain in the links. Figures 10c presents the accelerations in the links. It is observed that the magnitude of strain and tip acceleration are very small up to first locking because the torque acting at link 1 and link 2 are small and the flexural energy of the links are small. The strain and tip accelerations are enhanced due to the first locking. The peak strain in the experiment at root and at the mid span of composite link are 130 microstrains and 1100 microstrains respectively. The peak strains predicted from the mathematical model, for the above locations, are 490 microstrains and 495 microstrains. The peak accelerations at the mid span and the tip of composite link is 30 m/s^2 and 27 m/s^2 . The enhancement of this strain and tip acceleration shows oscillatory response in the first joint rotation. The magnitude of strain at the mid span of composite link is much larger than at root because the locking is taking place at the second joint. The kinetic energy of link 2 just before locking is transformed to the flexural strain energy and the flexural kinetic energy. This induces large strain at the middle of composite link. The strain at middle of composite link predicted by the experimental model is large compared to the mathematical model just after first locking. The results of the strain at root predicted by the mathematical model is large when compared to the experimental results just after first locking. After second locking the magnitude of strain at the root is larger than at the mid span of cantilever because the locking is taking place at the first joint. The peak strain at root from experiments is 703 micro strains and from mathematical model is 610 micro strains. The strain is enhanced because of additional energy transfer due to locking and additional moment at the joint due to the length of the link getting doubled after first locking. The peak strain in the mid span of cantilever from the experiments is 545 microstrains and from mathematical model is 580 microstrains. It is observed that the locking induces higher modes in the system. It is observed from Figure 10c that the acceleration in link is slightly enhanced due to second locking but the presence of high frequency components are shown in the response. The peak acceleration at the mid span and tip of cantilever are 26 m/s^2 and 16 m/s^2. It is observed that both first and second locking are inducing higher modes in the system.

The energy before and after each locking is computed by using equations (18) to (27). The results are presented in Table 6. It is observed that a slight amount of energy loss is occurring during first locking and negligible loss of energy is occurring during second locking.

The results of mathematical model are compared with experimental data. The locking time, joint rotation and strain behavior match well all the experiments, except after first locking for two link set up. The acceleration of link was in phase with experimental data and showed high frequency components in the experimental data. It is observed that the locking excites the higher modes in the system when the link length is large. It is also observed that all the energy gets transmitted after locking when the system rotates about a single joint. Some energy loss is observed when the system is having two rigid body degree of freedom.

The possible application of this study is discussed here. The large antenna of satellite is deployed about a revolute hinge joint and lock at the end of the deployment. The shock moment during locking is evaluated by assuming a statically equivalent triangular load distribution. The strain energy of this load is equated to the kinetic energy of the antenna, assuming the antenna is not flexible during deployment [**17**]. It was observed from the energy analysis of single flexible link that the energy is not lost during locking. Hence, all the input energy can be used for evaluating the shock moment. The solar array will have a yoke with many panels connected by hinges. These panels and yoke lock at the end of deployment. It was observed in the energy analysis of two link set up that when more than one link is involved some energy is lost during locking. Hence, in evaluating the shock moment at the hinges, this energy loss can be accounted. Hence, it is likely that the shock moment may get reduced and in turn the hinge mass may get reduced.

It can be observed from the above figures that the increase in locking time compared with the mathematical model and the decrease in amplitude of strain in every cycle indicates the presence of damping and friction. The damping arises in the experimental setup due to friction in the joints and potentiometer shaft, structural damping of the link material and friction at the air bearings The friction at the joints was measured and modelled as a constant opposing torque and structural damping was modelled as a proportional damping from the strain response curves. The other damping and friction are difficult to estimate and were not taken into account in the numerical simulations.

5. CONCLUSIONS

The energy transfer for the flexible links subjected to change in kinematic configuration due to locking has been described in this paper. The transverse deflection of flexible link is modelled by finite element method. The momentum balance method is used for locking. The experiments were carried out for two flexible links, single flexible composite link and single flexible link with IRS hinges. The results of locking time, joint rotation, strain behavior of the links and tip acceleration of the links from mathematical model and experiment are in good agreement. These comparisons show that modelling for flexible link under going locking during motion using momentum balance method gives reasonably good results. The energy transfer is computed for each locking. It was observed that almost all the input energy is transformed to flexural energy and kinetic energy for composite link and single flexible link after locking. It was observed that small percentage of energy was lost for two flexible links after first locking. The locking induces the moment on the hinges. These moments are to be computed based on the total energy of the system. The hinges are designed based on these moments.

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Table 1 Input parameters for single link set up

Description	*symbol*	*Magnitude*
Length of the link (m)	ℓ_1	0.923
Cross sectional area (m^2)	A_1	1.66134e-04
Thickness of link (m)	t_1	4.15335e-03
Area moment of inertia (m^4)	I_1	2.388e-10
Flexural rigidity (Nm^2)	$E_1 I_1$	16.717
Young's modulus (N/m^2)	E_1	0.7e11
Link density (Kg/m^3)	ρ_1	2700.0
Tip mass (Kg)	m_{p1}	0.716
Torque at joint 1 (Nm)	τ_1	0.07
Torsion spring stiffness (Nm/rad)	K_{P1}	0.06323
Pre rotation angle at joint 1 (degrees)	θ_{10}	267.5
Initial angle at joint 1 (degrees)	θ_{1in}	0.0
Initial Conditions : $\theta_1 = 0.0, \dot{\theta}_1 = 0.0, \boldsymbol{q}_f = 0.0, \dot{\boldsymbol{q}}_f = 0.0$		

Table 2 Energy transfer (Nm) for single link set up

Initial energy	E_i	0.689
Energy spent from initial configuration up to locking	E_s	0.408
Energy just before locking	E_b	0.281
Energy of cantilever just after locking		
i) Strain Energy	V	0.141e-02
ii) Kinetic energy	T	0.279
iii) Total energy	$E_a = V+T$	0.281

Table 3 Input parameters for composite link set up

Description	*Link 1*		*Link 2*	
	Symbol	*Magnitude*	*symbol*	*Magnitude*
Length of the link (m)	ℓ_1	*0.94601*	ℓ_2	*0.92301*
Width of the link (m)	b_1	*0.040*	b_2	*0.040*
Thickness of the link (m)	t_1	*0.004204*	t_2	*0.0040102*
Flexural rigidity (Nm^2)	$E_1 I_1$	*17.3366*	$E_2 I_2$	*15.0478*
Youngs modulus (N/m^2)	E_1	*0.7e011*	E_2	*0.70e11*
Link density (Kg/m^3)	ρ_1	*2700.0*	ρ_2	*2700.0*
Tip mass (Kg)	m_{p1}	*0.716*	m_{p2}	*0.5831*
Torque at joint (Nm)	τ_1	*0.070*	τ_2	*0.1050*
Torsion spring stiffness (Nm/rad)	K_{P1}	*0.06323*	K_{P2}	*0.0643*
Pre rotation angle (degrees)	θ_{10}	*267.50*	θ_{20}	*270.0*
Initial angle at joint (degrees)	θ_{1in}	*0.0*	θ_{2in}	*0.0*
Initial Conditions : $\theta_1 = 32.3\deg(0.,5637\,\boldsymbol{rad}), \dot{\theta}_1 = 0.0, \boldsymbol{q}_f = 0.0, \dot{\boldsymbol{q}}_f = 0.0$				

Table 4 Energy transfer (Nm) for composite link set up

Initial energy	E_I	0.481
Energy spent from initial configuration up to locking	E_s	0.375
Energy just before locking	E_b	0.106
Energy of cantilever just after locking		
i) Strain Energy	V	0.50e-03
ii) Kinetic energy	T	0.105
iii) Total energy	$E_a = V+T$	0.106

Table 5 Input parameters for two link set up

Description	*Link 1*		*Link 2*	
	Symbol	*Magnitude*	*symbol*	*Magnitude*
Length of the link (m)	ℓ_1	*0.94601*	ℓ_2	*0.92301*
Width of the link (m)	b_1	*0.040*	b_2	*0.040*
Thickness of the link (m)	t_1	*0.004204*	t_2	*0.0040102*
Flexural rigidity (Nm^2)	$E_1 I_1$	*17.3366*	$E_2 I_2$	*15.0478*
Youngs modulus (N/m^2)	E_1	*0.7e011*	E_2	*0.70e11*
Link density (Kg/m^3)	ρ_1	*2700.0*	ρ_2	*2700.0*
Tip mass (Kg)	m_{p1}	*0.716*	m_{p2}	*0.5831*
Torque at joint (Nm)	τ_1	*0.070*	τ_2	*0.1050*
Torsion spring stiffness (Nm/rad)	K_1	*0.06323*	K_2	*0.0643*
Pre rotation angle (degrees)	θ_{10}	*267.50*	θ_{20}	*270.0*
Initial angle at the joint (degrees)	θ_{1in}	*0.0*	θ_{2in}	*180.0*
Initial Conditions $\theta_1 = 0.0, \dot{\theta}_1 = 0.0, \theta_2 = 180.0\deg(3.1416\,\boldsymbol{rad}), \dot{\theta}_2 = 0.0, \boldsymbol{q_f} = 0.0, \dot{\boldsymbol{q}}_f = 0.0$				

Table 6 Energy transfer (Nm) for two link set up

Initial energy	E_I	2.621
Energy spent from initial configuration up to locking	E_s	1.507
Energy just before locking	E_s	1.114
Energy of cantilever just after locking		
i) Strain Energy	V	0.006
ii) Kinetic energy	T	0.968
iii) Total energy	$E_a = V+T$	0.974
Energy lost	E_l	12.61%
Initial energy	E_I	0.663
Energy spent from initial configuration up to locking	E_s	0.408
Energy just before locking	E_b	0.255
Energy of cantilever just after locking		
i) Strain Energy	V	0.22e-02
ii) Kinetic energy	T	0.249
iii) Total energy	$E_a = V+T$	0.251
Energy lost	E_l	0%---

Figure 1: Schematic representation of two link set up

Figure 2: Schematic representation of composite link set up

Figure 3: Schematic representation of single link set up

Figure 4(a): Representation of two flexible links

Figure 4(b): Representation of composite flexible link

Figure 5: Two link experimental setup

Figure 6: Composite link experimental setup

Figure 7 : Single link experimental setup

Figure 8 Response of joint rotation, root strain and tip acceleration for one link set up

Figure 9(a) Response of joint rotation for composite link set up

Figure 9(b) Response of strain at root and mid span of cantilever for the composite link set up

Figure 9(c) Response of acceleration at mid span and tip of cantilever for composite link set up

Figure 10(a) Response of joint 1 and joint 2 rotations for two link set up

Figure 10(b) Response of strain at root and mid span of the composite link for the two link set up

Figure 10(c) Response of acceleration at mid span and tip of composite link for the two link set up

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