# Tip over stability analysis of a three-wheeled mobile robot capable of traversing uneven terrains without slip

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**A mobile robot traversing an uneven terrain can undergo tip over instability when one or more wheels of the mobile robot losses contact with the uneven terrain. In this paper, we study the tip over stability of a three wheeled mobile robot. The three wheeled mobile robot studied in this paper has torus shaped rear wheels and have the ability of lateral tilting – a condition required for slip free motion on uneven terrain. The torus shaped wheels and slip free motion makes the dynamics and tip over stability analysis more difficult and interesting. In this paper, the force-angle stability measure technique is used to analyze and detect tip over instability. Simulation results of the stability analysis shows that the wheeled mobile robot with lateral tilt of rear wheels is capable of moving on certain kinds of rough terrains without tip over.**

***Keywords- wheeled mobile robot, uneven terrain, lateral tilt, tip over stability***

## I. INTRODUCTION

A wheeled mobile robot (WMR) will not move on uneven terrain without slip [1]. Slip free motion occurs if the length of the axle connecting two wheels can change [2] or for a fixed length axle the wheels are allowed to tilt in a lateral direction [3-4]. In this paper we consider a three wheeled mobile robot moving on uneven terrain with rear wheels possessing lateral tilt capability.

Tip over or roll over instability occurs when vehicle body undergoes a rotation which results in reduction in number of ground contact points. During the tip over all remaining point lie on a single line and is called the tip over axis. Control is then lost and finally, if the situation is not reversed the vehicle overturns. It is well known that for tip over stability low centre of gravity (C.G) height is desirable, large weight is stabilizing at low speed and destabilizing at high speed. In the rough terrain conditions, typically the speeds are low and large weight is a stabilizing factor. In order to detect and prevent tip over instability we have to define instantaneous stability margin. McGhee [5] defined stability margin as shortest horizontal distance between C.G and support pattern boundary projected on horizontal plane -- this measure is insensitive to top heaviness. Davidson [6] used screw theory to study the effect of for angular loads. Messuri and Klein [7] used the concept of minimum work required to tip over. Ghasempoor and Sepehri [8] included inertial and external load with assumption of constant load magnitude and direction throughout the tip over motion. Ollero and Heredia [9] analyzed the stability of general class of path tracking algorithm for straight paths and paths of constant curvatures, considering the pure delay in the control loop. They stated that the path tracking implementation typically involves a pure delay in control loop that can significantly affect its stability. They defined stable limits as minimum look ahead to make motion stable and maximum look ahead to make motion unstable. Huang and Sugano [10] came up with stability degree and valid stable region on zero moment point (ZMP) criterion. The ZMP is defined as the point on the ground about which the sum of all the moments of active forces is equal to zero. If the ZMP lies inside the support polygon then the mobile robot is stable. The stability degree is the quantitative measure of a stable extent of mobile manipulator according to the relationship between the ZMP position and the stable region. The longer the minimal distance is from the ZMP to the boundary, the larger is stability degree. The maximum value of stability degree is one. The valid stability region is the area in which the stability degree will not become negative under the disturbance of assumed condition.

Papadopoulos and Rey [11] proposed a new tip-over stability measure called force-angle stability measure. The force-angle stability measure, $\forall$, is given by the minimum of all the angles, $\theta$, made by resultant force through C.G. to the tip over axis normal, weighted by the magnitude of net force vector for heaviness sensitivity. The critical tip over stability occurs when $\theta$ goes to zero and therefore net force coincides with tip over axis normal. This is a simple graphical interpretation, easy to compute, sensitive to top heaviness and applicable to systems operating over uneven terrain subjected to inertial and external forces. We have used this force-angle stability measure in this paper.

This paper is organized as follows: Description of wheel lateral tilt for slip free motion in given section II, the force-angle stability measure is presented in section III, followed by simulation of wheeled mobile robot on uneven terrains in section IV. The discussion on the simulation results are presented in V followed by conclusion in Section VI.

## II. WHEEL LATERAL TILT

The primary difference between our WMR and commonly available WMR's moving on uneven terrains is the lateral tilting capability of the two rear wheels. The rear wheels can tilt by about 30 degrees on either side as shown in figure 1. The other main difference is the use of three wheels as opposed to large number of wheels used in most mobile robots for uneven terrains. The lateral tilting of the two rear wheels in is shown schematically in Figure 1.

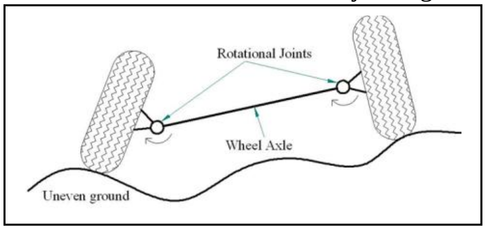


*Figure 1- Wheel lateral tilt concept*

It may be noted that for lateral tilt ability the wheel-ground contact must be a *point contact* and hence the wheel must be in the shape of a *torus*.

It is shown by Nilanjan and Ghosal [3-4] that with the capability of lateral tilt, a three wheeled mobile robot can traverse an uneven terrain without slip. In this work, we use the same concept and analyze such a WMR for tip over stability. The force-angle tip is used in this paper and this is discussed next.

## III. FORCE STABILITY MEASURE

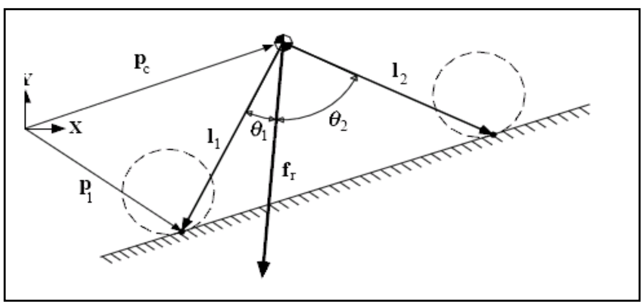


*Figure 2: Planar force angle stability measure [11]*

### *A. Tip over axis and its normal*

Figure 2 shows force-angle stability measure for planar system whose centre of mass is subjected to net force $\mathbf{f_r}$. This force makes an angle $\theta_1$ and $\theta_2$, with the two tip over axis normals $\mathbf{I_1}$ and $\mathbf{I_2}$. The force angle stability measure, $\forall$, is given by minimum of the two angles weighted by the magnitude of the force vector for heaviness sensitivity as given below

$$\forall = \theta_1 \, \| \, \mathbf{f_r} \, \| \quad (3.1)$$

Of all the vehicle contact points with the ground, it is only necessary to consider those outermost points which form a convex support polygon when projected onto the horizontal plane, and these points are referred to as ground contact points. In this present case we have only three ground contact points. The locations of the three ground contact point are

$$\mathbf{P}_i = [\, P_x \; P_y \; P_z \,]^T, \; i = 1, 2, 3; \quad (3.2)$$

and $\mathbf{P_c}$ represent the location of the vehicle center-of-mass in reference frame{O}. The ground contact points are numbered in clockwise direction. The line which joins the ground contact points are the candidate tip over mode axes $\mathbf{\tilde{a}_i}$=1,2,3. The i$^{th}$ tip over mode axis is given by

$$\mathbf{\tilde{a}_i = P_{i+1} - P_i} \quad (3.3)$$

$$\mathbf{\tilde{a}_n = P_1 - P_n} \quad (3.4)$$

for i= 1, 2, 3; and n=3. The ground contact numbering system are required in order to obtain a set of tip over axes whose direction all coincide with that of stabilizing moments.

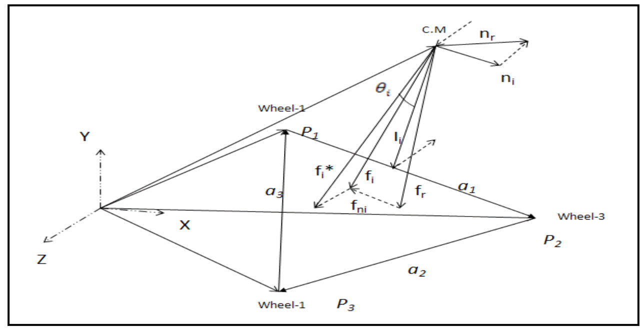


*Figure 3: Force angle stability measure for 3-WMR [11]*

Figure 3 shows the tip over axis along with the tip over axis normal. The component of resultant force $\mathbf{f_r}$ along tip over axis $\mathbf{\tilde{a}_2}$ has been shown along with the force angle stability margin $\theta_2$. A tripped tip over of the vehicle occurs when one of the ground contact points encounters an obstacle or a sudden change in the ground conditions. In a tripped tip over the vehicle undergoes a rotation about an axis which is some linear combination of the tip over mode axes associated with the single remaining ground contact point. In a tripped instability the force-angle stability measure will go to zero and then become negative for each contributing tip over mode axis so that it is not required to identify the exact tip over mode axis. For the tip over mode axis $\mathbf{\tilde{a}}$, the unit vector $\mathbf{\hat{a}} = \mathbf{\tilde{a}} / \| \mathbf{\tilde{a}_i} \|$, the tip over axis normals $\mathbf{I_i}$ which intersect the vehicle center-of-mass are given by subtracting portion which is lying along the $\mathbf{\tilde{a}_i}$ from ($\mathbf{P_{i+1} - P_i}$), for i=1, 2, 3 and $I_d$ is 3×3 identity matrix.

### *B. Net resultant force*

The net force acting on the centre of mass of the wheeled mobile robot which would participate in a tip over instability $\mathbf{f_r}$ is given by [11]:

$$\mathbf{f_r} = \sum(\mathbf{F_{grav} + F_{disp} - F_{inertial}}) \quad (3.5)$$

where $\mathbf{F_{inertial}}$ forces due to the linear acceleration, $\mathbf{F_{grav}}$ is the gravitational loads, $\mathbf{F_{disp}}$ are any other external forces action directly on the vehicle. Similarly, for the net moment $\mathbf{n_r}$ acting about centre of mass is given by [11]

$$\mathbf{n_r} = \sum(\mathbf{N_{grav} + N_{disp} - N_{inertial}}) \quad (3.6)$$

For a given tip over axis $\mathbf{\tilde{a}_i}$ we are only concerned with those components of $\mathbf{f_r}$ and $\mathbf{n_r}$ which acts about the tip over axis. The component of $\mathbf{f_r}$ and $\mathbf{n_r}$ about the tip over axis $\mathbf{\tilde{a}_i}$ is given by

$$\mathbf{f_i = (I_d - \tilde{a}_i \tilde{a}_i^T) f_r} \quad (3.7)$$

$$\mathbf{n_i} = (\tilde{\mathbf{a}}_i \tilde{\mathbf{a}}_i^{\mathbf{T}})\, \mathbf{n_r} \qquad (3.8)$$

for i=1, 2, 3 and $I_d$ is 3×3 identity matrix.

Since the Force Angle stability measure is based on the computation of the angle between the net force vector and each of the tip over axis normals, it is necessary to replace the moment $\mathbf{n_i}$ with an equivalent force couple $\mathbf{f_{ni}}$ for each tip over axis. The equivalent force couple must necessarily lie in the plane normal to the moment $\mathbf{n_i}$**.** The most judicious choice of the infinite possible force couple locations and directions in this plane, is that pair of minimum magnitude where one member of the couple passes through the center of mass and the other through the line of the tip over axis. One of the forces of equivalent force couple acting on the centre of mass is given by

$$\mathbf{f}_{ni} = (\hat{\mathbf{I}}_i \times \mathbf{n}_i) / \| \mathbf{I}_i \| \qquad (3.9)$$

where $\hat{\mathbf{I}}_i$ is the tip over axis normal, defined by equation (3.4). The new net force vector $\mathbf{f}_i^*$ for the $i^{th}$ tip over axis is thus

$$\mathbf{f}_i^* = \mathbf{f}_i + \mathbf{f}_{ni} \qquad (3.10)$$

## C. *Force-Angle stability measure*

The candidate angle for the force angle stability are then given by [11]

$$\theta_i = \beta_i \cos^{-1} (\mathbf{f}_i^* . \mathbf{I}_i) \qquad (3.11)$$

where i=1, 2, 3, and $-\Pi \leq \theta_i \geq \Pi$ The sign of $\theta_i$ is determined by $\beta_i$ as follows

$$\beta_i = +1 \text{ if } (\mathbf{f}_i^* . \mathbf{I}_i).\ \hat{\mathbf{a}} < 0$$

$$\beta_i = -1 \text{ if } (\mathbf{f}_i^* . \mathbf{I}_i).\ \hat{\mathbf{a}} \geq 0 \qquad (3.12)$$

For i=1, 2, 3, the appropriate sign of the angle measure associated with each tip over axis is determined by establishing whether or not the net force vector lies inside the support pattern. The overall force-angle stability measure is then given by $\forall_i = \min(\theta_i) \| \mathbf{f_r} \|$ for i=1,2,3.

This scalar is thus an instantaneous measure of the tip over stability margin of the system. The magnitude of a positive ∀ describes the magnitude of the tip over stability margin of a stable system. Critical tip over stability occurs when ∀= 0. Negative values of ∀ indicate that a tip over instability is in progress. Here it is to be noted that the minimum angle is weighted by $\| \mathbf{f_r} \|$ in order to obtain heaviness sensitivity and not by $\| \mathbf{f_i} \|$ which would introduce discontinuities in ∀ whenever the tip over axis index **i** associated with $\min(\theta_i)$ changes.

$$\forall_i = \theta_i \ \| \ \mathbf{f_r} \ \| \qquad (3.13)$$

## D. *Algorithm for tip over analysis*

The stability analysis has been performed in the following steps:

1) Modeling and simulation of wheeled mobile robot
2) Evolution of all the possible tip-over mode axes and its normal.
3) Evolution of resultant of all the forces and moments at each instant and its equivalent force through centre of mass
4) Visualization of point of intersection of resultant force with ground
5) Evolution of the force angle stability measure

# IV. SIMULATION

The robot consists of three toroidal wheels attached with rotary joints to the rigid platform. One of the two rotary joints at rear wheel of the wheeled mobile robot is passive while other being actuated. The passive joint allows lateral tilt of the torodial wheels i.e. rotation of wheel about an axis perpendicular to the axle and lying in the plane of the platform. The in-plane rotation at rear wheel is provided by a motor or actuator. The front wheel can be steered by a motor or actuator about an axis perpendicular to the plane of top platform, and it has no lateral tilt capability. The static and dynamic resistance at wheel terrain contact is 0.8 and 0.9 respectively. The WMR weights 50kg. Mass of each wheel is 2kg. The robot is in XZ plane and gravity is in negative Y direction. Lateral tilt of wheel is allowed for a maximum of 30 degrees on either side. The numerical values for stiffness and damping coefficients for spring and damper has been taken to $K_{pi}$=16.2376 Nm/rad and $K_{vi}$=0.574 Nms/rad in simulation. The three wheeled mobile robot line diagram is shown in Figure 4.

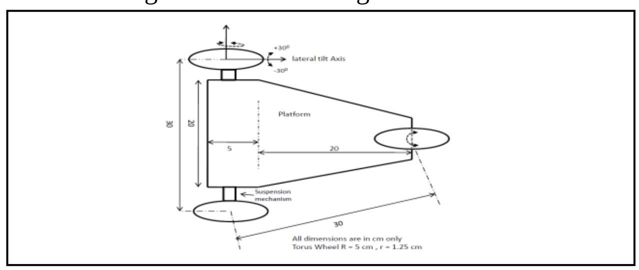


*Figure 4: Three Wheeled Mobile Robot line diagram*

# V. RESULTS AND DISCUSSIONS

In this section we presented simulation results of WMR on different terrains. We have tested our wheeled mobile robot for tip over stability measure. The path followed by the wheeled mobile robot includes straight line and curvilinear.

## A. *On Flat terrain without steering*

On flat terrain, force angle stability margin of 3-WMR moving with 1m/sec constant velocity on flat terrain has been studied for a straight line motion.

For straight line motion, it is trivial that stability margin do not change with time, if the velocity of the vehicle is constant. It can be observed in figure 5. that stability margin about axis-2 and axis-3 is same and constant, about axis-1 is bit high because C.M is chosen a bit far away to axis-1 compared to axis-1 and axis-2.

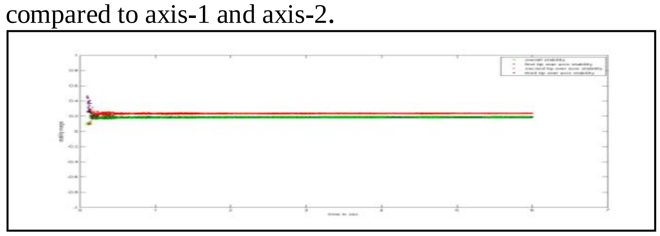

Figure 5: WMR moving on flat terrain without steering

*B. On Flat terrain with sudden steering*

On a flat terrain, the force-angle stability margin of the WMR, moving with input velocity 2.8 m/sec for rear wheels has been studied for straight line motion with instantaneous steering to left at $8^{th}$ unit of time.

It is trivial that vehicle tries to tip-over about tip-over axis-3 due to instantaneous steering to left. The stability margin reduces about tip-over axis-3 and increases about tip-over axis 2 and 1 as shown in Figure 6. In between the simulation it is observed that tip-over stability reduces about tip-over axis 2 and 3and increases about axis-1 due to the sudden deceleration by which resultant force is moved forward approaching near to axis-2 and axis-3.

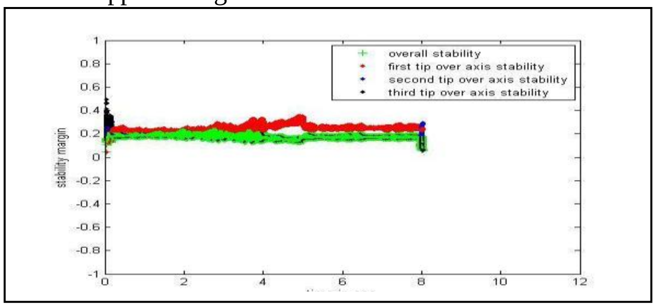


*Figure 6: WMR on flat terrain with sudden steering to left*

*C. On Uneven terrain with low slope and smooth peaks of height 200-300 mm*

The wheeled mobile robot has been tested for tip-over stability on a surface with low slope and smooth peaks of height 200-300 mm shown in the figure 7. The Force stability analysis as a function of time is shown in figure8. It is observed that WMR will move on this kind of surfaces without tip over.

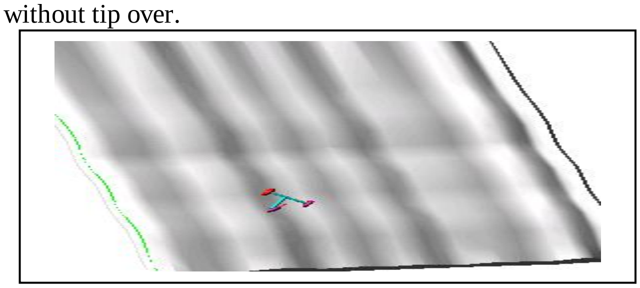

Figure 7 Low slope smooth peak surface

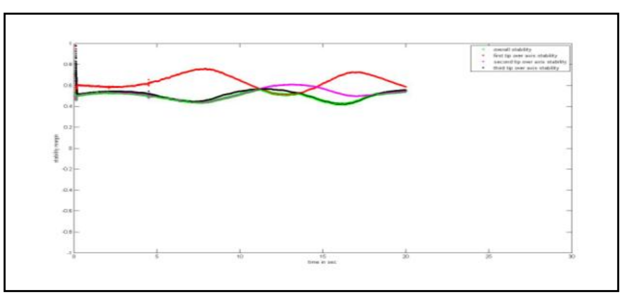

*Figure 8: WMR on low slope with smooth peaks of 200-300mm surface*

*D. On Uneven terrain with low slope with smooth peaks of height 700-900 mm*

The wheeled mobile robot has been tested for tip-over stability on a surface with low slope with high peaks of height 700-900mm. Figure 9. refers the force angle stability measure of this simulation. From the graph it is clear that WMR can move on high peak surfaces too with variation in stability margin.

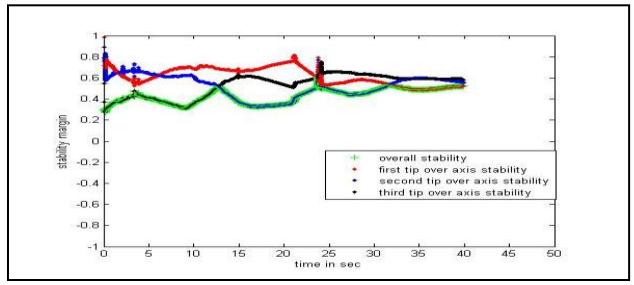


*Figure 9: WMR on low slope with smooth peaks of height 700-900mm surface*

*E. On Uneven terrain with low slope and sharp peaks*

The wheeled mobile robot has been tested for tip-over stability on a surface with low slopes and sharp peaks plot is shown below in Figure 10. It is clear that sudden valleys make the variation in stability measure. The robot is stable in this kind of surfaces.

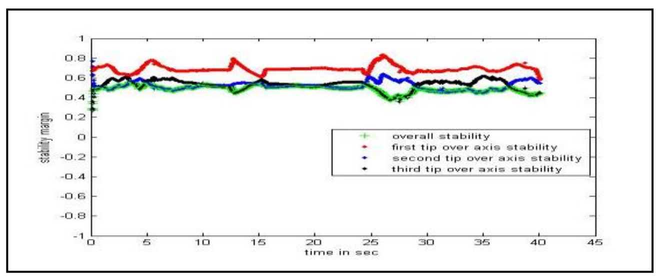


*Figure 10: On uneven terrain with low slope and sharp peaks*

*F. On Uneven terrain with high slope and smooth peaks*

The force stability measure of wheeled mobile robot tested on a surface with high slope and smooth peaks shown in the Figure 11. The WMR stability margin is shown in Figure 12. It is clear that highly varying stability margin means highly stable for some time and less stable for some time. Still the robot is moving without tip over.

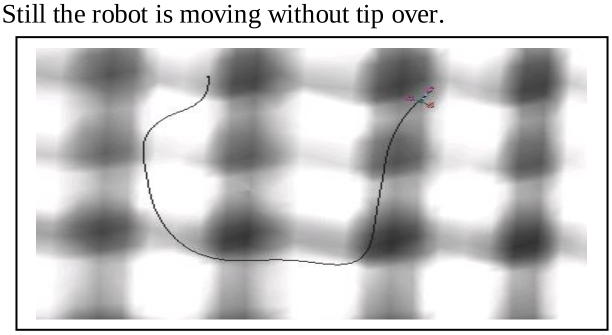

Figure 11: High slope smooth peak surface

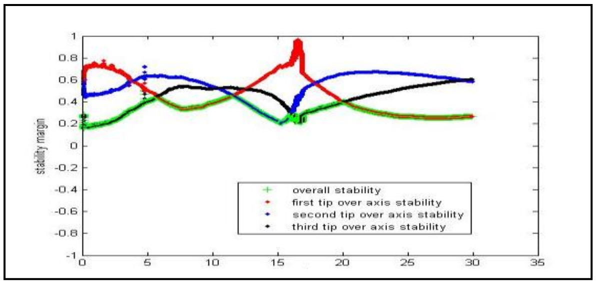


*Figure 12: On surface with high slope with smooth peaks*

### G. On Uneven terrain of high slope and sharp peaks

The wheeled mobile robot has been tested for tip-over stability moving on a surface with high slopes and sharp peaks. The stability plot has been shown in figure 13. Tip over axes are changing quickly due to high slope and sharp peak surface characters. Thus WMR navigation for this surface is typical.

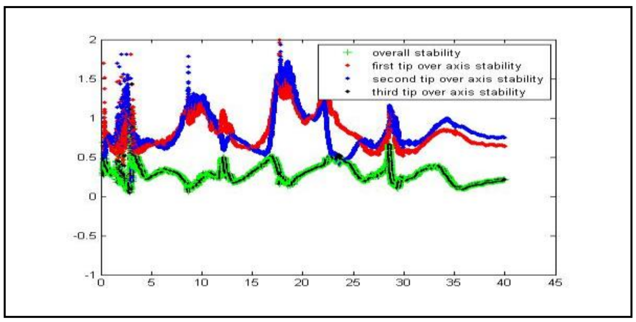


*Figure 13: on surface with high slope and sharp peaks*

### H. On a peak of 1600mm

The wheeled mobile robot has been tested for tip-over stability moving on a surface with deep valley. The stability plot has been shown in Figure 14. It can be observed that stability margin reduces and almost goes to zero while the WMR climbing down from the peak valley due to the level difference of wheel-ground contacts.

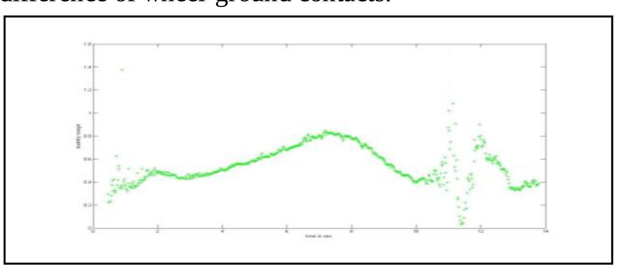

*Figure 14: On surface with deep valley*

## VI. CONCLUSION

In this paper, we have presented the tip over analysis for a three-wheeled mobile robot capable of traversing uneven terrain without slip. The WMR has been tested for tip over stability following straight line path and curvilinear path on different terrains. The stability margin reduces with increase in difference of level of the wheel-ground points. When WMR moves on flat terrain the stability margin remains constant. On uneven terrains of low slope surfaces the stability margin reduces with increase in peak height of surface. On uneven terrain of high slope surfaces the robot is near to unstable condition. From the simulations we can conclude that the wheeled mobile robot is capable of moving on limited rough terrain without tip over.

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