Kinematics of Wheeled Mobile Robots on Uneven Terrain

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Abstract

This paper deals with the kinematic analysis of a wheeled mobile robot (WMR) moving on uneven terrain. It is known in literature that a wheeled mobile robot, with a fixed length axle and wheels modeled as thin disk, will undergo slip when it negotiates an uneven terrain. In this paper, we model the wheels as a torus and propose the use of a passive joint allowing a lateral degree of freedom. Furthermore, we model the mobile robot, instantaneously, as a hybrid-parallel mechanism with the wheel-ground contact described by differential equations which take into account the geometry of the wheel, the ground and the non-holonomic constraints of no slip. We present an algorithm to solve the direct and inverse kinematics problem of the hybrid-parallel mechanism. Simulation results show that the three-wheeled WMR with torus shaped wheels and passive joints can negotiate uneven terrain without slipping. Our proposed approach presents an alternative to variable length axle approach.

1 INTRODUCTION

In this paper we address the problem of motion of wheeled mobile robots on uneven terrain without kinematic slip. The motion of wheeled mobile robots (WMR) on flat terrain has been well studied in [1, 8]. Waldron [11] has argued that two wheels independently joined to a common axle cannot roll on uneven terrain without slip. The use of Ackerman steering and differential wheel actuation which works for conventional vehicles on flat terrain does not work because there is no instantaneous center compatible with both wheels. The lateral slip in WMR's is undesirable because it leads to *localization* errors thus increasing the burden on sensor based navigation algorithms. In addition, for planetary explorations, power is at a premium and such slipping leads to large wastage of power.

The problem of two wheels joined independently to an axle, moving on uneven terrain without slip, has been studied by Sreenivasan et. al. [2, 3, 4, 9, 10]. They have modeled the vehicles as hybrid series-parallel chains and using instantaneous rate kinematics showed that for prevention of slip, a) the line joining the wheel terrain contact points must be coplanar with the axle axis, or b) the wheels must be driven at identical speeds relative to the axle. For prevention of slip they have suggested the use of a variable length axle (VLA), wherein an unactuated prismatic joint is used in the axle to vary axle length. There are a few limitations of using a VLA – a) at high inclinations there is slipping due to gravity loading, and b) the dynamic slip due to inertial loading becomes large at higher speeds. To overcome the limitations in VLA, the use of an actuated VLA has been proposed. An actuated VLA, however, requires accurate measurement of slip to obtain the desired actuator output.

It may be noted that all the above mentioned work model the wheel as a thin disk. On a flat ground this is reasonable since the contact point always lies in a vertical plane passing through the center of the wheel. However on uneven terrain this is not the case in general and the contact point will vary along the lateral surface of a general wheel due to terrain geometry variations.

In this paper we have proposed an alternative to VLA for slip-free motion capability in wheeled mobile robots. Our alternative design is based on the following concepts:

- Each wheel is assumed to be a torus. The wheels and the ground are considered as rigid bodies and single point contact is assumed between the wheel and the ground. The equations describing the geometry of the wheel and the ground are assumed to be sufficiently smooth and continuous such that derivatives up to second-order exists and geometric properties such as curvature and torsion can be computed.

- The equations of contact between two arbitrary surfaces in single point contact, derived by Montana[6], are used to model the contact of a torus shaped wheel on an uneven terrain.

- The lateral rotational motion of the wheel is accommodated by a *passive* rotary joint. This allows the distance between the wheel-ground contact points to change without changing the axle length. Since this joint is passive, sensing or control is not required.

- *Instantaneously*, the wheeled mobile robot can be modeled as hybrid-parallel mechanism with a three-degree-of-freedom joint at the wheel-ground contact. Unlike a typical kinematic joint, the *no-slip* non-holonomic constraint leads to non-linear ordinary differential equations. The non-linear ODE's are derived for the torus and smooth ground pair by following Montana[6].

- The direct and inverse kinematics of the mobile robot is solved by integrating the ordinary differential equations and the holonomic constraints arising out of the hybrid-parallel mechanism.

We demonstrate our approach with a 3-wheeled vehicle and show by simulation that slip free motion can be achieved without a passive or actuated VLA. This is the main contribution of this paper. The paper is organized as follows: in the next section, we obtain the contact equations for a single wheel moving on uneven terrain by following Montana [6]. Then, we present our approach of modeling of the vehicle as a hybrid-parallel manipulator instantaneously and derive the kinematic equations. This is followed by simulation results, illustrating the capability of the vehicle to negotiate uneven terrain without slip. In the last section we present the conclusions and scope of future research.

2 KINEMATIC MODELING OF SINGLE WHEEL

From differential geometry of a surface given in the parametric form $(x, y, z)^T = \mathbf{X}(u, v)$, the metric, curvature and torsion for such a surface is defined by

$$[M] = \begin{bmatrix} |\mathbf{X}_u| & 0 \\ 0 & |\mathbf{X}_v| \end{bmatrix} \qquad [K] = \begin{bmatrix} -\mathbf{X}_{uu} \cdot \mathbf{n}/|\mathbf{X}_u| & -\mathbf{X}_{uv} \cdot \mathbf{n}/|\mathbf{X}_v| \\ -\mathbf{X}_{uv} \cdot \mathbf{n}/|\mathbf{X}_u| & -\mathbf{X}_{vv} \cdot \mathbf{n}/|\mathbf{X}_v| \end{bmatrix}$$

$$[T] = [\mathbf{X}_v \cdot \mathbf{X}_{uu}/|\mathbf{X}_u| \qquad \mathbf{X}_v \cdot \mathbf{X}_{uv}/|\mathbf{X}_v|] \qquad \text{where } \mathbf{n} = \frac{\mathbf{X}_u \times \mathbf{X}_v}{|\mathbf{X}_u \times \mathbf{X}_v|}$$

where $\mathbf{X}_{(.)}$ and $\mathbf{X}_{(.)(.)}$ denote first and second partial derivatives. We assume that we have a digital elevation model (DEM) of the ground i.e. n available measured data points given in the form $(x, y, z)_i, \quad i = 1, 2, ..., n$. For our analysis, we require a surface representation which is at least $\mathcal{C}^2$ continuous. Without loss of generality we represent the uneven surface using a bi-cubic patch given by

$$\mathbf{X}(u, v) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} u^i v^j \qquad (u, v)\epsilon[0, 1]$$

The coefficients a_{ij} are determined if 16 data points are known (for details, see [7]). For our simulation purposes we have assumed synthetic ground data and have used in-built functions in Matlab[5] (Spline Tool Box) to generate a bi-cubic patch from the n given data points.

Figure 1 shows a torus wheel on an uneven ground. The frames $\{0\}$ and $\{w\}$ are fixed to the ground and wheel respectively. The frames $\{1\}$ and $\{2\}$ are the Gaussian frames at the point of contact on the ground and wheel, respectively, fixed with respect to the body frames. The four parameters (u_1, v_1), (u_g, v_g) (point of contact on surfaces 1 and 2 in $\{w\}$ and $\{0\}$ respectively) and the angle ψ between the X-axis of $\{1\}$ and $\{2\}$ are the five degrees of freedom between the two contacting surfaces. The angle ψ is chosen such that a rotation by angle $-\psi$ aligns the two X-axes. The equation of the torus shaped wheel in $\{w\}$ in terms of (u_1, v_1) can be written as

$$(x, y, z) \quad = \quad (r_1 \cos u_1, \cos v_1(r_2 + r_1 \sin u_1), \sin v_1(r_2 + r_1 \sin u_1)) \tag{1}$$

and the equation of the uneven ground is given by $(x, y, z) = (u_g, v_g, f(u_g, v_g))$. We can arrive at frame $\{w\}$ form $\{0\}$ by using standard 4×4 homogeneous transformation matrices which are not given here due to lack of space. Using the contact equations of Montana[6] we have,

$$\begin{aligned}
(\dot{u}_1, \dot{v}_1)^T &= [M_1]^{-1}([K_1] + [K^*])^{-1}[(-\omega_y, \omega_x)^T - [K^*](v_x, v_y)^T] \\
(\dot{u}_g, \dot{v}_g)^T &= [M_g]^{-1}[R_\psi]([K_1] + [K^*])^{-1}[(-\omega_y, \omega_x)^T + [K_1](v_x, v_y)^T] \\
\dot{\psi} &= \omega_z + [T_1][M_1](\dot{u}_1, \dot{v}_1)^T + [T_g][M_g](\dot{u}_g, \dot{v}_g)^T \\
0 &= v_z
\end{aligned} \tag{2}$$

$$\text{where} \quad [K^*] = [R_\psi][K_g][R_\psi]^T, \qquad [R_\psi] = \begin{pmatrix} \cos(\psi) & -\sin(\psi) \\ -\sin(\psi) & -\cos(\psi) \end{pmatrix},$$

ω_x, ω_y and ω_z are the angular velocity and v_x, v_y and v_z are the linear velocity components of $\{2\}$ relative to $\{1\}$, expressed in $\{2\}$. For rolling without slip v_x, v_y should be zero. In equation 2, $[M_1], [K_1], [T_1]$ are the metric, curvature and the torsion of the wheel respectively and $[M_g], [K_g], [T_g]$ are the corresponding properties of the ground.

3 KINEMATIC MODELING OF WMR

We now consider the modeling of a 3-wheeled vehicle moving on uneven terrain without slip. For this, we assume that the rear wheels have a degree-of-freedom at the wheel axle joint allowing lateral tilt. The front wheel can be steered and it has no lateral tilt capability. In this configuration, we can model the vehicle instantaneously as an equivalent hybrid-parallel mechanism as shown in figure 2. As mentioned in equation (2), at the wheel-ground contact point, we have one holonomic constraint, $v_z = 0$, which ensures wheel-ground contact is always maintained. Moreover, at each instant, we have 2 non-holonomic constraints which prevents instantaneous sliding, and these are $v_x = 0$ and $v_y = 0$. Intuitively, this suggests us to model the wheel ground contact point, *instantaneously*, as a three-degree-of-freedom (DOF) joint. It may be noted that this joint is different from a three-DOF spherical joint since

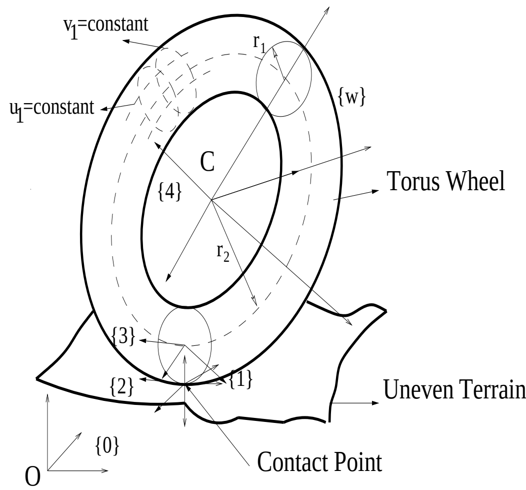

Figure 1: TORUS WHEEL ON UNEVEN TERRAIN.

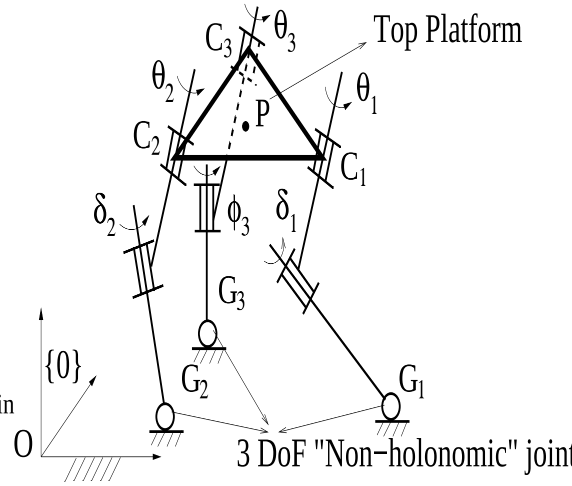

Figure 2: EQUIVALENT INSTANTANEOUS MECHANISM FOR THE 3-WHEELED WMR.

the two non-holonomic constraints restrict the motion at any instant *only* in terms of achievable velocities[1]. In addition, the wheel-axle joints allowing rotation of the wheel, lateral tilt and steering respectively are modeled as 1 DOF rotary joints. Using Gruebler's formula we obtain the degrees of freedom of the top platform as 3, and we choose rotation at the two rear wheels, θ_1 and θ_2, and the steering at the front wheel, ϕ_3, as the actuated variables. The two lateral tilts at the rear wheels, δ_1 and δ_2, and the rotation of the front wheel θ_3 are taken to be the passive variables which are to be computed.

As the vehicle is subjected to non-holonomic no-slip constraints, the kinematics problem is formulated in terms of the first derivatives of the kinematic variables. The kinematic variables are obtained by integration since the no-slip constraints are non-integrable. The direct and inverse kinematics problem for the 3-DOF vehicle can be stated as follows:

Direct Kinematics Problem: Given the actuated variables, $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\phi}_3$, and the geometrical properties of the ground and wheel, find the orientation of the top platform in terms of a rotation matrix ${}^0_p[R]$ (or a suitable parametrization of it) and

[1] As known in literature, non-holonomic constraints restrict only the space of achievable velocities and *not* the positions. A wheel or a thin disk undergoing rolling without slip, with $v_x = v_y = 0$, can reach any position in a plane and the only constraint is that of not leaving the plane and loosing contact

the position of the center of the platform (or any other point of interest).

Inverse Kinematics Problem: Given any three of the velocities of the top platform $V_x, V_y, V_z, \Omega_x, \Omega_y, \Omega_z$ (V_x, V_y, V_z are the components of the linear velocity vector of the center of the platform or any other point of interest and $\Omega_x, \Omega_y, \Omega_z$ are the components of the angular velocity vector of the platform) and the geometric properties of the ground and wheel, find the two drive inputs to the rear wheels ($\dot{\theta}_1$, $\dot{\theta}_2$) and the steering input to the front wheel ($\dot{\phi}_3$). [2]

To solve the above problems we proceed as follows:

Generate surface: As described in section 2, we use 2-D cubic splines to reconstruct the surface from elevation data. From the interpolated surface we find expressions for the metric, curvature and torsion form for the ground. We also obtain expressions for the metric, curvature and torsion form for the torus shaped wheel.

Form contact equations: For each wheel we write the 5 differential equations (see equation (2)) in the 15 contact variables u_i, v_i, u_{g_i}, v_{g_i}, and ψ_i, where $i = 1, 2, 3$. Since the wheels undergo no-slip motion, we set $v_x = v_y = 0$ for each of the wheels. It may be noted that ω_x, ω_y and ω_z in the contact equations for each wheel are the three components of angular velocities of frame $\{2\}$ with respect to frame $\{1\}$ and are unknown. These are related to the angular velocity of the platform $\Omega_x, \Omega_y, \Omega_z$ and the input and passive joint rates. In the fixed coordinate system, $\{0\}$, we can write

$$^0(\omega_x, \omega_y, \omega_z)^T =^0 (\Omega_x, \Omega_y, \Omega_z)^T -^0 \omega_{input} \quad (3)$$

where $^0\omega_{input} = {}^0[\dot{R}]_{in}{}^0[R]_{in}{}^T$ with $^0[R]_{in}$ given by ${}^0_w[R][R(\mathbf{e_2}, \delta_\mathbf{i})][\mathbf{R}(\mathbf{e_1}, \theta_\mathbf{i})]$ for $i = 1, 2$ and by ${}^0_w[R][R(\mathbf{e_3}, \phi_\mathbf{i})][\mathbf{R}(\mathbf{e_1}, \theta_\mathbf{i})]$ for $i = 3$, and $\mathbf{e_1} = (\mathbf{1}, \mathbf{0}, \mathbf{0})^\mathbf{T}$, $\mathbf{e_2} = (\mathbf{0}, \mathbf{1}, \mathbf{0})^\mathbf{T}$, $\mathbf{e_3} = (\mathbf{0}, \mathbf{0}, \mathbf{1})^\mathbf{T}$. The above equation (3) couples all 5 sets of ODE's and we get a set of 15 coupled ODE's in 21 variables. These are the 15 contact variables $u_i, v_i, u_{g_i}, v_{g_i}, \psi_i$ ($i = 1, 2, 3$), the 3 wheel rotations $\theta_1, \theta_2, \theta_3$, the 2 lateral tilts δ_1, δ_2, and the front wheel steering ϕ_3.

Relate angular velocity of wheels and platform: The angular velocity and the linear velocity of the center of the platform are expressed in terms of the 15 wheel variables $u_i, v_i, u_{g_i}, v_{g_i}, \psi_i$ ($i = 1, 2, 3$). If γ, β, α be a 3-2-1 Euler angle parametriza-

[2] Instead of the angular velocities $\Omega_x, \Omega_y, \Omega_z$, we may use the Euler angle rates $\dot{\alpha}, \dot{\beta}, \dot{\gamma}$, where γ, β, α is a 3-2-1 Euler angle parametrization of orientation.

tion representing the orientation of the platform we have

$$
\begin{aligned}
{}^0\Omega_x &= \dot{\alpha}\cos(\beta)\cos(\gamma) - \dot{\beta}\sin(\gamma) = f_1(u_i, v_i, u_{g_i}, v_{g_i}, \psi_i) \\
{}^0\Omega_y &= \dot{\alpha}\cos(\beta)\sin(\gamma) + \dot{\beta}\cos(\gamma) = f_2(u_i, v_i, u_{g_i}, v_{g_i}, \psi_i) \\
{}^0\Omega_z &= \dot{\gamma} - \dot{\alpha}\sin(\beta) = f_3(u_i, v_i, u_{g_i}, v_{g_i}, \psi_i)
\end{aligned} \tag{4}
$$

If x_c, y_c, z_c denotes the coordinates of the center of the platform in $\{0\}$. The linear velocity of the center of the platform is given by

$$
{}^0(V_x, V_y, V_z)^T = (\dot{x_c}, \dot{y_c}, \dot{z_c})^T = V_{w_i} +^0 (\Omega_x, \Omega_y, \Omega_z)^T \times^0 R_{c_i} \tag{5}
$$

where i stands for any one of the 3 wheels 1,2 or 3 and ${}^0R_{c_i}$ is the point of attachment of the wheel to the platform from the center of the platform expressed in frame $\{0\}$.

Form holonomic constraint equations: In addition to the contact equations, for the 3 wheels to form a vehicle the distance between the 3 points C_1, C_2, C_3 (refer to figure 2) must remain constant. These holonomic constraint equations can be written as

$$
(\overrightarrow{OC_1} - \overrightarrow{OC_2})^2 = {l_{12}}^2; (\overrightarrow{OC_1} - \overrightarrow{OC_3})^2 = {l_{13}}^2; (\overrightarrow{OC_3} - \overrightarrow{OC_2})^2 = {l_{32}}^2; \tag{6}
$$

where $\overrightarrow{OC_1}$, $\overrightarrow{OC_2}$, $\overrightarrow{OC_3}$ are the position vectors of the center of the three wheels, C_1, C_2, C_3, respectively from the origin **O** of the fixed frame and l_{ij} is the distance between center of wheels i and j respectively.

Solution of the Direct Kinematics problem

From steps 1, 2, 4 we have 15 first order ODE's and 3 algebraic constraint equations for the 18 unknown variables ($\dot{\theta}_1, \dot{\theta}_2, \dot{\phi}_3$ are given). This system of differential algebraic equations (DAE's) can be converted to 18 ODE's in 18 variables by differentiating the constraint equations. It is to be noted that the 18 ODE's have been derived symbolically using the symbolic manipulation package Mathematica [12]. Using an ODE solver, we solve the set of 18 ODE's numerically, with the initial conditions obtained as outlined below. Once we have obtained $u_i, v_i, u_{g_i}, v_{g_i}, \psi_i, i = 1, 2, 3$ and $\delta_1, \delta_2, \theta_3$ we can obtain the rotation matrix of the platform ${}^0_p[R]$. The position vector of the center of the platform $\overrightarrow{OP}$ with respect to the fixed frame, $\{0\}$, denoted by (x_c, y_c, z_c) is given by (see figure 2)

$$
(x_c, y_c, z_c)^T = \overrightarrow{OC_i} +^0_p [R]\overrightarrow{C_iP} \qquad \text{for any } i = 1, 2 \text{ or } 3. \tag{7}
$$

Solution of the Inverse Kinematics problem

From steps 1,2,3,4 we have 21 first order ODE's and 3 algebraic constraints for 24 unknowns (in this case we assume $\dot{x}_c, \dot{y}_c, \dot{\gamma}$ are given). This set of DAE's is also converted to ODE's and integrated using initial conditions determined as discussed below. Numerical solution gives the 15 contact variables $u_i, v_i, u_{g_i}, v_{g_i}, \psi_i, i = 1, 2, 3$, the 3 actuated variables $\theta_1, \theta_2, \phi_3$ and the 3 passive variables $\delta_1, \delta_2, \theta_3$. The other 3 platform variables z_c, α, β are also obtained.

Initial conditions

To solve the set of ODE's in direct or inverse kinematics problem, we have to choose the initial conditions such that it satisfies the holonomic constraint equations. For the direct kinematics among the 18 variables we can choose $\delta_1 = 0, \delta_2 = 0, \theta_3 = 0$, v_1, v_2, v_3 to be $3\pi/2$ and the position of point of contact of any one wheel in $\{0\}$. The other two wheels must also be in contact with the ground. Hence, for each wheel, we have

$$\overrightarrow{OC_i} + {}^{\mathbf{0}}_{\mathbf{w}}[\mathbf{R}].\overrightarrow{C_iG_i} = \overrightarrow{OG_i}; \qquad i = 1, 2, 3 \tag{8}$$

Converting them to unit vectors we have two independent equations for each wheel. In addition, for each of the three wheels, $\cos(\psi_i) = \hat{e}_{1_i}.\hat{e}'_{1_i} \quad i = 1, 2, 3$, where $\{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ and $\{\hat{e}'_1, \hat{e}'_2, \hat{e}'_3\}$ are the coordinate axes of reference frames $\{2\}$ and $\{1\}$ respectively in $\{0\}$ (refer to figure 1). In addition there are 3 holonomic constraint equations given by equation (6). This gives us 10 nonlinear equations in 10 variables and we can solve them numerically.

For inverse kinematics problem, in addition, we have to obtain the initial values of θ_1, θ_2, ϕ_3, α, β, z_c. We can choose $\theta_1 = 0$, $\theta_2 = 0$, $\phi_3 = 0$ (or any other initial desired heading). As we know $u_i, v_i, u_{g_i}, v_{g_i}, \psi_i, i = 1, 2, 3$, we can obtain the position vector of the center of the 3 wheels and we can get the rotation matrix ${}^0_p[R]$ of the platform. Once ${}^0_p[R]$ is known, we obtain the 3-2-1 Euler angle sequence, γ, β, α. The position of the center of the platform is given by equation 7 and we have x_c, y_c and z_c at the initial instant.

4 NUMERICAL SIMULATION AND RESULTS

We have tested our algorithm on various synthetically generated surfaces and for various types of inputs. We present on representative result due to the constraints of space. The uneven ground (surface) used in the simulations is shown in figure 3. The geometrical parameters of the vehicle used are a) Length of the rear axle $= 2l_a = 2m$, b) Distance of center of front wheel from middle of axle $= l_s = 1.5m$, and c) Two radii of the torus shaped wheel are $r_1 = 0.05m, r_2 = 0.25m$. The center of the vehicle is $(1/3)l_s$ from the center of the axle.

For the direct kinematics problem the inputs are chosen as $\dot{\theta}_1 = 0.5$ rad/sec, $\dot{\theta}_2 = 0.4$ rad/sec, $\dot{\phi}_3 = 0$ rad/sec. The variation of lateral tilts of the rear wheels

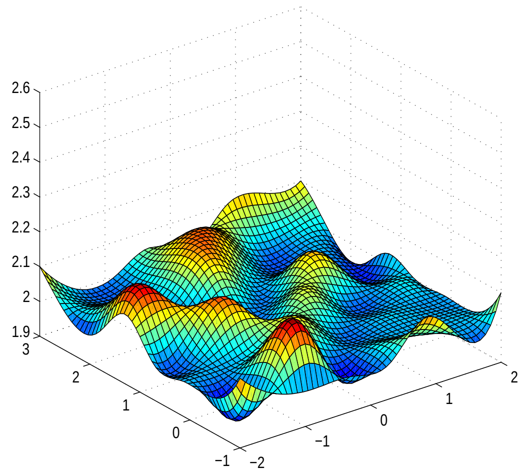

Figure 3: UNEVEN TERRAIN USED FOR SIMULATIONS.

The initial conditions for direct kinematics are: $\delta_1 = 0$ rad, $\delta_2 = 0$ rad, $\theta_3 = 0$ rad, $u_1 = 1.586967$ m, $v_1 = 3\pi/2$ rad, $u_{g_1} = 0.983772$ m, $v_{g_1} = -0.037978$ m, $\psi_1 = -3.140963$ rad, $u_2 = 1.547598$ rad, $v_2 = 3\pi/2$ rad, $u_{g_2} = -la$, $v_{g_2} = 0$ m, $\psi_2 = -3.144127$ rad, $u_3 = 1.578296$ rad, $v_3 = 3\pi/2$ rad, $u_{g_3} = 0.001578$ m, $v_{g_3} = 1.549151$ m, $\psi_3 = -3.143452$ rad. For inverse kinematics, in addition, we have $\theta_1 = 0$ rad, $\theta_2 = 0$ rad, $\phi_3 = 0$ rad, $\alpha = 0$ rad, $\beta = 0.009$ rad, $z_c = 2.3131$ m.

when the 3-wheeled WMR is moving on the uneven terrain is shown in figure 4, and the locus of the wheel center's, wheel-ground contact points and the center of the platform is shown in figure 5. For the inverse kinematics problem, the inputs are $\dot{x_c} = 0.01255$ m/s, $\dot{y_c} = 0.12865$ m/s, $\dot{\gamma} = 0.0159$ rad/s. Figures 6, 7 show variation of lateral tilt, and the locus of the wheel-ground contact points and the center of the platform respectively, when the 3-wheeled WMR moves on the same uneven terrain shown in figure 3. In all the above cases, the no-slip conditions at the wheel ground contacts and the holonomic constraints are satisfied.

5 CONCLUSION

In this paper, we have studied kinematics of a three-wheeled mobile robot with torus shaped wheels and passive joints allowing lateral tilt of the wheels. Numerical simulation results clearly show that the WMR can negotiate uneven terrain without kinematic slip. Our analysis is valid for any (uneven) surface representation which provides up to second derivatives efficiently and accurately. Future work is being carried out on improved terrain modeling and representation. We are also investigating the ability of the vehicle to traverse uneven surfaces when joint limits are imposed on the lateral degrees of freedom. Finally, the dynamics of the vehicle on uneven surface is also being investigated.

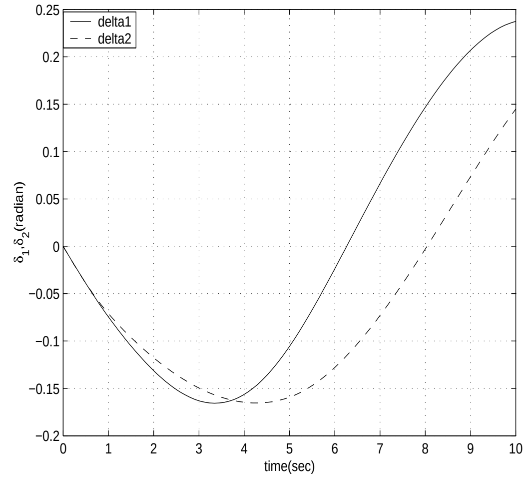

Figure 4: VARIATION OF LATERAL TILTS FOR 3-WHEELED WMR.

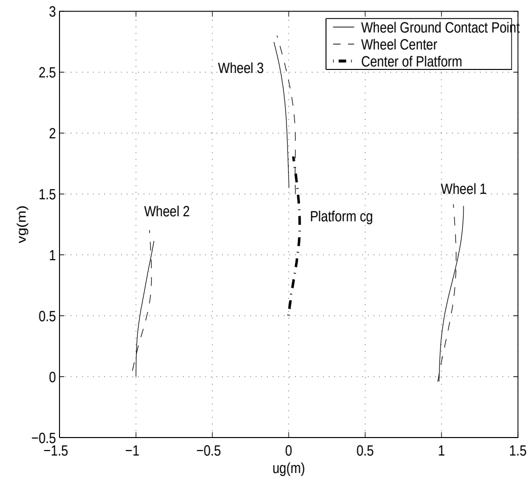

Figure 5: PLOT OF CENTER OF WHEELS, WHEEL GROUND CONTACT POINT AND PLATFORM CG FOR 3-WHEELED WMR.

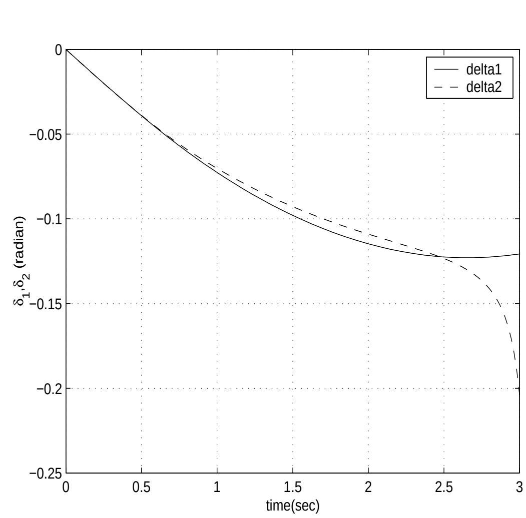

Figure 6: VARIATION OF LATERAL TILTS FOR 3-WHEELED WMR.

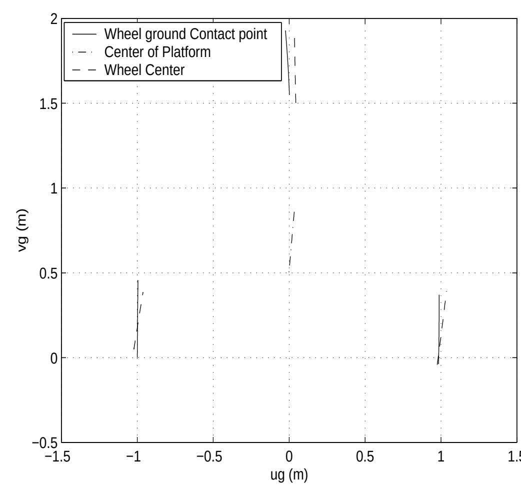

Figure 7: PLOT OF CENTER OF WHEELS, WHEEL GROUND CONTACT POINT AND PLATFORM CG FOR 3-WHEELED WMR.

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