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CLOSED-FORM ANALYTIC SYNTHESIS OF A FIVE-LINK SPATIAL MOTION GENERATOR

GEORGE N. SANDOR†
220 MEB, University of Florida, Gainesville, FL 32611, U.S.A.

DILIP KOHLI‡
University of Wisconsin, Milwaukee, WI 53202, U.S.A.

and

CHARLES F. REINHOLTZ§ and ASHITAVA GHOSAL§
University of Florida, Gainesville, FL 32611, U.S.A.

Abstract—In the synthesis of motion generating mechanisms we seek to determine the key dimensions of a preconceived type of single-degree-of-freedom mechanism which will guide one of its links through a sequence of finitely or infinitesimally separated arbitrarily prescribed positions. In space, each such finitely separated precision position can be specified by the radius vector of the origin and the Euler angles of orientation of a coordinate system, embedded in the guided link, with respect to a fixed coordinate system of reference.

The motion generator mechanism considered here consists of two grounded *R–S* links,¶ one grounded *C–S* link, a ternary *S–S–S* coupler and the *R–R–C* fixed frame. Motion of the coupler is to be prescribed. Each equation of synthesis is written by expressing the closure of the vector polygon covering the starting and one displaced position of one dyad formed by a grounded link and the coupler. The resulting system can be solved for unknown vectors defining the dyad in its starting position, in closed form for up to three precision positions.

1. INTRODUCTION

The central problem in kinematic synthesis of motion generators is to find the dimensions of the mechanism which will provide a given specified motion to one of the links of the mechanism. The motion of the link may be specified in terms of finitely separated, infinitesimally separated or multiply separated positions.

A dramatic growth in the development of graphical and analytical tools of kinematic synthesis in the last decades has facilitated the design of planar mechanisms for a variety of tasks. The techniques of kinematic synthesis developed in the late fifties and early sixties, and which at the time appeared suitable only for computation on rather sophisticated computers, are now being used by designers on programmable hand calculators to provide quick and better designs for fairly complex problems such as path generation with prescribed timing, motion generation defined by multiply separated positions, etc.

There has been a considerable amount of research work[1] in the development of analytical tools of kinematic synthesis of spatial mechanisms. Novodvorskii[2] formulated the synthesis problem of function generation using the *RSSR* mechanism, where input and output axes of the two revolute pairs were mutually perpendicular. Stepanoff[3] solved the generalized case of an *RSSR* mechanism with skewed, nonintersecting input and output axes. Levitskii and Shakavasian[4] applied a least squares technique for finite position synthesis of *RSSR* mechanisms up to eight precision positions. Rao *et al.*[5] used the principle of linear superposition to synthesize several function generating mechanisms for the maximum number of precision positions.

Wilson[6] derived the relationships to calculate centerpoint and sphericpoint curves for guiding a rigid body by means of an *R–S* link. Roth[7, 8] investigated the loci of special lines and points associated with spatial motion. Roth[9] and Chen and Roth[10, 11] proposed a general theory for computing the number and locus of points in a rigid body in finite or infinitesimal motion which have positions satisfying the constraints of binary or combined link chains.

Sandor[12] and Sandor and Bisshopp[13] introduced methods of dual number quaternions and stretch rotation tensors to find loop closure equations of spatial mechanisms. The methods proposed were general enough to include generation of space curves, etc. Suh[14, 15] employed 4×4 matrices for synthesis of spatial mechanisms where design equations are expressed as constraint equations in order to obtain constrained motion. Kohli and Soni[16] employed matrix methods to synthesize spherical four-link and six-link mechanisms for multiply separated positions of a rigid body in spherical motion. Recently, Kohli and Soni[17] used pair constraint geometry and successive screw displacements to synthesize mechanisms containing revolute, cylinder, prismatic, helical and spheric pairs.

In the present paper, we seek to develop procedures

†Research Professor of Mechanical Engineering.
‡Associate Professor of Mechanical Engineering.
§Graduate Research Assistants in Mechanical Engineering.
¶*R*: revolute, *C*: cylindric and *S*: spherical joint.

for the synthesis of an *RSSR–SC* five link spatial mechanism for motion generation.

Our objectives in developing the design techniques to synthesize such a mechanism are:

(i) to keep the mathematics on a level which will allow an average machine designer to readily adapt and use the procedure in designing a spatial *RSSR–SC* motion generator;

(ii) to obtain a closed-form solution of the synthesis equations, and

(iii) to develop the description of the mechanism and the procedure of synthesis in simple terms readily adaptable by the designer for interactive computation with visual graphic display.

2. THE FIVE-LINK SPATIAL MOTION GENERATOR

Figure 1 shows a schematic of the "mechanism of preconceived type" to be synthesized. It consists of the two grounded links with fixed-axis revolutes at A_0 and B_0 and moving spheric joints A and B, (binary R–S links 2 and 3); the one grounded link with fixed-axis cylindric joint C_0 and spheric joint C (binary C–S link 4); the coupler with three spheric joints A, B and C (ternary S–S–S link 5) and the fixed frame (ternary R–R–C link 1). The fixed orthonormal coordinate system of reference $OXYZ$ is embedded in link 1, and the moving orthonormal coordinate system $oxyz$ is embedded in the coupler (link 5). Observe that the use of only these two coordinate systems differs sharply from many conventional space-mechanism theories in which each and every link requires at least one separate local coordinate system.

The degrees of freedom of the *RSSR–SC* mechanism may be found using

$$F = 6(n-1) - 5R - 4C - 3S \qquad (1)$$
$$= 6(5-1) - 5(2) - 4(1) - 3(3) = 1,$$

where n is the total number of links and R, C and S are the numbers of revolute, cylindric and spheric joints.

This is a single-degree-of-freedom mechanism with one link fixed in contrast to multi-degree-of-freedom robotic manipulators. Therefore, the rotational position of, say, link 2 determines the position of all other moving links.

The desired body motion is prescribed by specifying successive positions of the moving origin, o, and corresponding orientations of the moving $oxyz$ coordinate system.

3. VECTOR DESCRIPTION OF THE MECHANISM

In Fig. 1 the fixed vectors $\mathbf{a}_0$, $\mathbf{b}_0$ and $\mathbf{c}_0$ locate the two grounded revolutes at A_0 and B_0 and the grounded cylindric joint at C_0. The directions of the axes of these joints are denoted by unit vectors $\hat{\mathbf{u}}_a$, $\hat{\mathbf{u}}_b$ and $\hat{\mathbf{u}}_c$. Vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ represent the grounded links 2, 3 and 4, expressed in $OXYZ$. Observe that vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are perpendicular to unit vectors $\hat{\mathbf{u}}_a$, $\hat{\mathbf{u}}_b$ and $\hat{\mathbf{u}}_c$ respectively. Owing to the R-joints at A_0 and B_0, link vectors $\mathbf{a}$ and $\mathbf{b}$ can only rotate about their respective revolute axes $\hat{\mathbf{u}}_a$ and $\hat{\mathbf{u}}_b$. Link vector $\mathbf{c}$ can not only rotate about $\hat{\mathbf{u}}_c$, but can also translate along $\hat{\mathbf{u}}_c$ albeit always remaining attached and perpendicular to it. Vectors $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$, embedded in the coupler, locate spheric joints A, B and C with respect to the similarly embedded moving coordinate system $oxyz$. These spheric joints must remain connected to the three grounded dyads. Therefore, "vector dyads" $(\mathbf{a}, \boldsymbol{\alpha})$, $(\mathbf{b}, \boldsymbol{\beta})$ and $(\mathbf{c}, \boldsymbol{\gamma})$ must remain connected at their respective tips throughout the motion.

4. SPECIFYING THE PRESCRIBED MOTION

In Fig. 2 the first and the jth prescribed positions of the moving coordinate system are shown as $o_1x_1y_1z_1$ and $o_jx_jy_jz_j$. Their origins o_1 and o_j are located by the given position vectors $\mathbf{o}_1$ and $\mathbf{o}_j$, while their

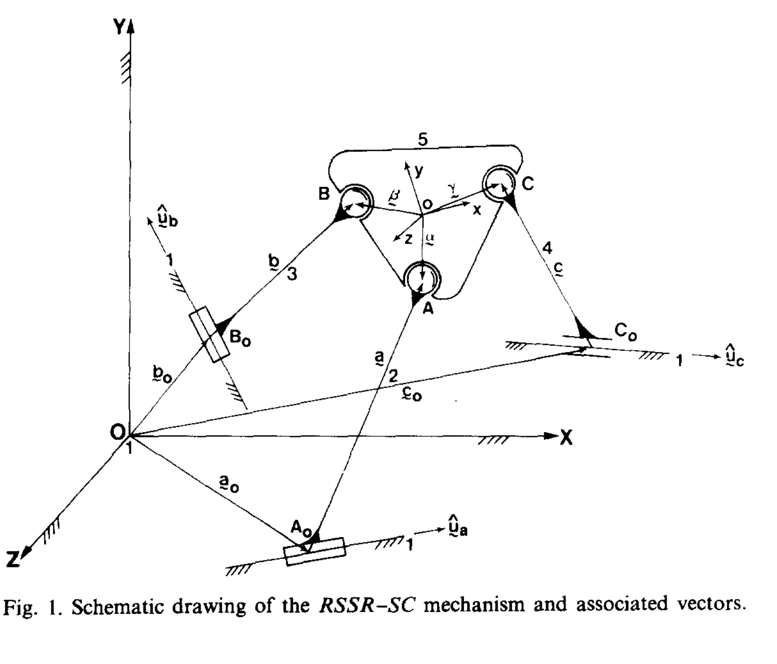

Fig. 1. Schematic drawing of the *RSSR–SC* mechanism and associated vectors.

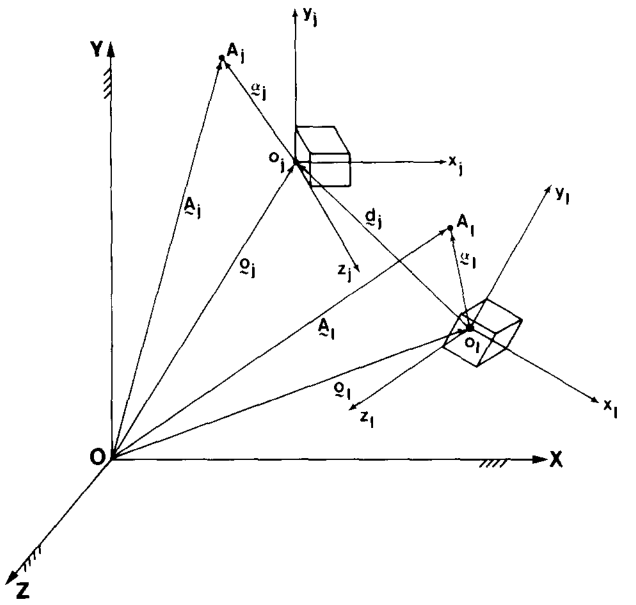

Fig. 2. The moving and the fixed coordinate systems.

orientations are specified by the given rotation matrices $[R_1]$ and $[R_j]$ in terms of Euler angles with respect to the fixed $OXYZ$ frame of reference. In order to express these conditions mathematically, let $\mathbf{V}^1$ be a vector expressed in the moving system in the first position and $\mathbf{V}^0$ be the same vector expressed in the fixed global coordinate system. The relation between $\mathbf{V}^1$ and $\mathbf{V}^0$ is given by

$$\mathbf{V}^0 = [R_1]\mathbf{V}^1. \tag{2}$$

Or, in general when $\mathbf{V}^j$ is the vector expressed in the moving system in the jth position,

$$\mathbf{V}_j^0 = [R_j]\mathbf{V}^j. \tag{3}$$

Thus, by specifying $\mathbf{o}_1$, $\mathbf{o}_j$, $[R_1]$ and $[R_j]$ for $j = 2, 3, \ldots, n$, we have prescribed n discrete positions of the coupler (link 5 in Fig. 1).

5. THE DYADIC DISPLACEMENT POLYGON

Figure 3 shows three of the five links of the mechanism of Fig. 1. They are:

Link 1, the fixed link with coordinate system $OXYZ$ and with vectors $\mathbf{a}_0$ locating the point A_0 on the fixed revolute axis $\hat{\mathbf{u}}_a$;

Link 2, the grounded R–S link with vector $\mathbf{a}_1$ connecting its moving spheric joint A to the fixed revolute in position 1 and

Link 5, the coupler, with embedded coordinate system $oxyz$ and embedded vector $\boldsymbol{\alpha}$ connecting mov-

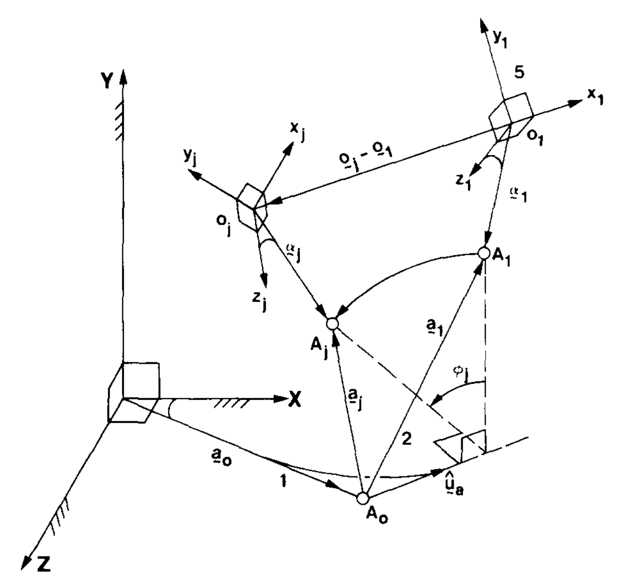

Fig. 3. Schematic of the 1st and the jth positions of links 2 and 5.

ing origin o to the spheric joint A, also in position 1, marked $o_1x_1y_1z_1$ and $\boldsymbol{\alpha}_1$, where $\boldsymbol{\alpha}_1$ is expressed in the fixed $OXYZ$ system (see eqn (6)).

In addition to these first or starting positions, Fig. 3 also shows the jth displaced positions of links 2 and 5, as follows:

Link 5, with its origin o translated by the vector $\mathbf{o}_j - \mathbf{o}_1$ from position 1 to position j and its embedded coordinate system $o_jx_jy_jz_j$, including the embedded vector $\boldsymbol{\alpha}$, rotated from orientation 1 to orientation j (from $\boldsymbol{\alpha}_1$ to $\boldsymbol{\alpha}_j$); and

Link 2, with vector $\mathbf{a}$ rotated about $\hat{\mathbf{u}}_a$ from position $\mathbf{a}_1$ to position $\mathbf{a}_j$ by the angle ϕ_j.

Note that in Fig. 3 we have the following known features:

Prescribed are: $\mathbf{o}_1$, $\mathbf{o}_j$, $o_1x_1y_1z_1$, and $o_jx_jy_jz_j$. Unknown are: $\boldsymbol{\alpha}_j$, $\mathbf{a}_1$, $\mathbf{a}_j$, $\mathbf{a}_o$; in addition, as will be shown later, for 3 prescribed positions $\boldsymbol{\alpha}_1$ may be assumed arbitrarily.

We now define the *dyadic displacement polygon* as the closed vector polygon with vertices A_o, A_1, o_1, o_j and A_j. Closure of this polygon can be written by expressing all its vector sides in the $OXYZ$ system and equating their sum with zero. Going counterclockwise in Fig. 3, we have

$$\mathbf{a}_1 - [R_1]\boldsymbol{\alpha} + \mathbf{o}_j - \mathbf{o}_1 + [R_j]\boldsymbol{\alpha} - \mathbf{a}_j = 0. \quad (4)$$

In eqn (4) $\mathbf{o}_j$, $\mathbf{o}_1$, $[R_1]$ and $[R_j]$ are given, and all other quantities are unknown. However, as will be shown, for $j = 2, 3$ (i.e. 3 prescribed positions), $\boldsymbol{\alpha}$ is a free choice.

6. FINDING A SUITABLE REVOLUTE AXIS FOR LINK 2

The preceding discussion shows that positions of a body in space are specified by giving the location and orientation of the moving coordinate system embedded in the body at each position. Thus, we have given:

$$\mathbf{o}_j \text{ and } [R_j], \quad j = 1, 2, 3. \quad (5)$$

Now we turn our attention to the R–S dyad shown in vector form in Fig. 3. By assuming a value for $\boldsymbol{\alpha}$ we have located the spherical joint relative to the moving frame $oxyz$. We can transform $\boldsymbol{\alpha}$ to be expressed in the fixed $OXYZ$ frame as:

$$\boldsymbol{\alpha}_j = [R_j]\boldsymbol{\alpha}, \quad j = 1, 2, 3 \quad (6)$$

which is known, because $[R_1]$ and $[R_j]$ were prescribed.

Referring to Fig. 2, the vector $\mathbf{A}_j$ in the $OXYZ$ system can be expressed as the sum of $\mathbf{o}_j$ and $[R_j]\boldsymbol{\alpha}$, or

$$\mathbf{A}_j = \mathbf{o}_j + [R_j]\boldsymbol{\alpha}, \quad j = 1, 2, 3. \quad (7)$$

For an R–S dyad with three prescribed positions, the vectors $(\mathbf{A}_j - \mathbf{A}_1)$, $j = 2, 3$, must lie in a plane perpendicular to the axis of the revolute joint. Thus, the unit vector along the axis of the revolute joint, $\hat{\mathbf{u}}_a$, is given by the following relation:

$$\hat{\mathbf{u}}_a = (\mathbf{A}_2 - \mathbf{A}_1) \times (\mathbf{A}_3 - \mathbf{A}_1)/|(\mathbf{A}_2 - \mathbf{A}_1) \times (\mathbf{A}_3 - \mathbf{A}_1)|. \quad (8)$$

7. FINDING THE LOCATION OF THE FIXED REVOLUTE A_0

Since the grounded link $\mathbf{a}$ rotates about the revolute axis $\hat{\mathbf{u}}_a$, no kinematic generality is lost by making it perpendicular to $\hat{\mathbf{u}}_a$. Therefore, we can say that the vectors $\mathbf{a}_j$, $j = 1, 2, 3$, also lie in the plane defined by $(\mathbf{A}_j - \mathbf{A}_1)$ for $j = 2, 3.$. In addition, the axis $\hat{\mathbf{u}}_a$ which is normal to this plane intersects it at A_o, located by the unknown vector $\mathbf{a}_o$ in the $OXYZ$ system. This is the unknown position of the revolute joint A_o in the global coordinate system. The point A_o can be found by locating the center of the circle which passes through the three points A_1, A_2 and A_3. This can be done in the following way.

Referring to Fig. 4, first find the unit vectors $\hat{\mathbf{p}}_2$ and $\hat{\mathbf{p}}_3$ which are perpendicular to $(\mathbf{A}_2 - \mathbf{A}_1)$ and $(\mathbf{A}_3 - \mathbf{A}_2)$, respectively, and also to $\hat{\mathbf{u}}_a$.

$$\hat{\mathbf{p}}_2 = \hat{\mathbf{u}}_a \times (\mathbf{A}_2 - \mathbf{A}_1)/|\mathbf{A}_2 - \mathbf{A}_1| \quad (9)$$

$$\hat{\mathbf{p}}_3 = \hat{\mathbf{u}}_a \times (\mathbf{A}_3 - \mathbf{A}_2)/|\mathbf{A}_3 - \mathbf{A}_2|. \quad (10)$$

Now let the lines of action of these two vectors line up along the perpendicular bisectors of $(\mathbf{A}_2 - \mathbf{A}_1)$ and $(\mathbf{A}_3 - \mathbf{A}_2)$ respectively. Then these lines will intersect at A_0. By denoting the perpendicular distance from $(\mathbf{A}_2 - \mathbf{A}_1)$ to A_0 along $\hat{\mathbf{p}}_2$ as λ_2 and similarly the distance from $(\mathbf{A}_3 - \mathbf{A}_2)$ to A_0 along $\hat{\mathbf{p}}_3$ as λ_3, we express

$$\mathbf{a}_0 = \lambda_2\hat{\mathbf{p}}_2 + \mathbf{A}_1 + \frac{\mathbf{A}_2 - \mathbf{A}_1}{2}$$
$$= \lambda_3\hat{\mathbf{p}}_3 + \mathbf{A}_2 + \frac{\mathbf{A}_3 - \mathbf{A}_2}{2}. \quad (11)$$

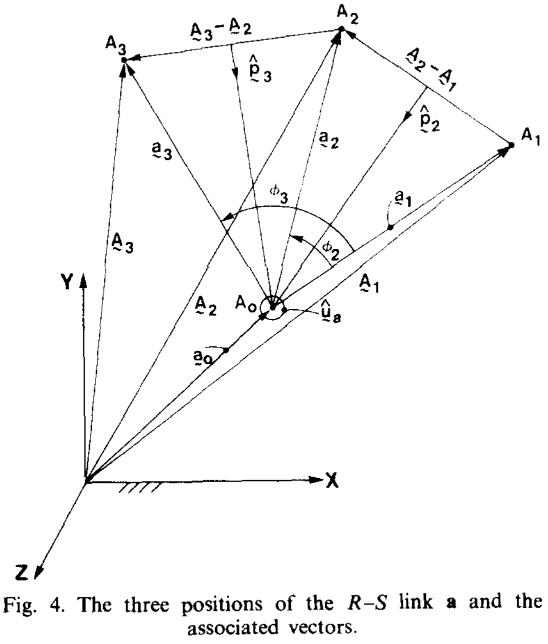

Fig. 4. The three positions of the R–S link $\mathbf{a}$ and the associated vectors.

To solve for λ_3, first pre-cross-multiply the second and third expressions in eqn (11) with $\hat{\mathbf{p}}_2$ to give

$$\lambda_2\hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_2 + \hat{\mathbf{p}}_2 \times \mathbf{A}_1 + \hat{\mathbf{p}}_2 \times \frac{\mathbf{A}_2 - \mathbf{A}_1}{2} =$$

$$\lambda_3\hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_3 + \hat{\mathbf{p}}_2 \times \mathbf{A}_2 + \hat{\mathbf{p}}_2 \times \frac{\mathbf{A}_3 - \mathbf{A}_2}{2}. \tag{12}$$

Observe that in eqn (12) the first term on the left side is zero, which eliminates λ_2.

Next take the dot product of both sides of eqn (12) with $\hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_3$ and divide by $\hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_3$ dot $\hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_3$, making the coefficient of λ_3 equal to one. Solving explicitly for λ_3 gives:

$$\lambda_3 = \frac{\hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_3 \cdot \left[\hat{\mathbf{p}}_2 \times \mathbf{A}_1 + \hat{\mathbf{p}}_2 \times \frac{\mathbf{A}_2 - \mathbf{A}_1}{2} - \hat{\mathbf{p}}_2 \times \mathbf{A}_2 - \hat{\mathbf{p}}_2 \times \frac{\mathbf{A}_3 - \mathbf{A}_2}{2}\right]}{\hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_3 \cdot \hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_3} \tag{13}$$

The value of λ_3 thus obtained can be substituted into eqn (11) to find $\mathbf{a}_0$:

$$\mathbf{a}_0 = \lambda_3\hat{\mathbf{p}}_3 + \mathbf{A}_2 + \frac{\mathbf{A}_3 - \mathbf{A}_2}{2}. \tag{14}$$

Knowing $\mathbf{a}_0$, $\mathbf{a}_1$ can be found by

$$\mathbf{a}_1 = \mathbf{A}_1 - \mathbf{a}_0. \tag{15}$$

This result gives the location of the fixed R joint and the moving S joint of link 2 in the starting position. This completely determines the first R–S dyad consisting of $\mathbf{a}_1$ and $\boldsymbol{\alpha}_1$.

8. FINDING THE DIRECTION AND LOCATION OF THE FIXED REVOLUTE OF LINK 3

The second R–S dyad is found in an identical manner to the first R–S dyad, except that $\boldsymbol{\beta}$, B and b are substituted for $\boldsymbol{\alpha}$, A and a with all subscripts and underscores in eqns (4)–(15). Note that, like $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ can also be assumed arbitrarily at the designer's discretion.

9. FINDING THE AXIS AND THE INITIAL LOCATION OF THE CYLINDRIC JOINT OF LINK 4

Figure 5 shows the C–S dyad pair in vector form. As was the case for the R–S dyad, we are given the position and orientation of the moving coordinate system at each of the three prescribed positions; this is given by eqn (5).

Again we assume the location of the spherical joint, in this case joint C, in the moving $oxyz$ system located by vector γ and embedded in the coupler link. We can now write an equation like eqn (6) to give γ_j in the fixed frame of reference:

$$\gamma_j = [R_j]\gamma, \quad j = 1, 2, 3. \tag{16}$$

Note that the $[R_j]$-s represent the prescribed orientation of the moving body, and are the same as for the R–S dyad. Thus, everything on the r.h.s. of eqn (16) is known.

At this point we must assume two additional scalar quantities in order to synthesize the C–S dyad. This can be done by either assuming the two displacements of the cylindric joint axis, namely S_2 and S_3, or by assuming the direction of the cylindric joint axis, $\hat{\mathbf{u}}_c$. Note that $\hat{\mathbf{u}}_c$, being a unit vector, is specified by giving only two of its three scalar components. The remaining component can then be found from

$$u_{cx}^2 + u_{cy}^2 + u_{cz}^2 = 1. \tag{17}$$

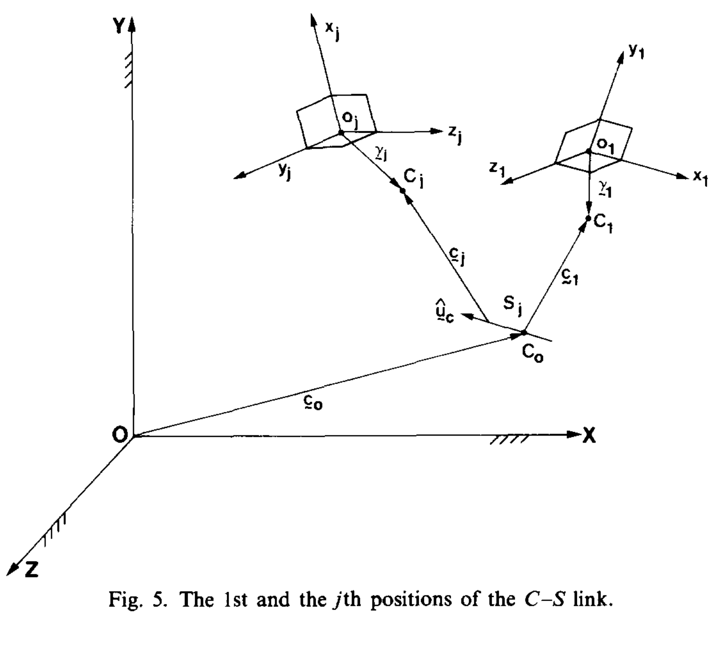

Fig. 5. The 1st and the jth positions of the C–S link.

The relationship between the vector $\hat{\mathbf{u}}_c$ and the scalar displacements S_2 and S_3 can be determined from Fig. 6 to be:

$$S_2 = \hat{\mathbf{u}}_c \cdot (\mathbf{C}_2 - \mathbf{C}_1) \tag{18}$$

$$S_3 = \hat{\mathbf{u}}_c \cdot (\mathbf{C}_3 - \mathbf{C}_1). \tag{19}$$

If we assume values of S_2 and S_3, eqns (17)–(19) form a set of three equations in the three unknown components of $\hat{\mathbf{u}}_c$. On the other hand, if we assume the direction of $\hat{\mathbf{u}}_c$ by assuming two of its components, we can directly solve eqn (17) for the third component of $\hat{\mathbf{u}}_c$. With the vector $\hat{\mathbf{u}}_c$ determined, eqns (18) and (19) can be directly solved for the displacements S_2 and S_3. In either case the problem is simple to solve. Thirdly, we may assume one component of $\hat{\mathbf{u}}_c$ and one axial displacement.

Figure 7 shows section A–A taken from Fig. 6. This is the plane defined by the unit normal $\hat{\mathbf{u}}_c$ and passing through the point C_1. The points C_2' and C_3' are the projections of points C_2 and C_3 onto this plane and the vectors C_2' and C_3' from the origin are given as follows:

$$\mathbf{C}_2' = (\hat{\mathbf{u}}_c \times ((\mathbf{C}_2 - \mathbf{C}_1) \times \hat{\mathbf{u}}_c)) + \mathbf{C}_1 \tag{20}$$

$$\mathbf{C}_3' = (\hat{\mathbf{u}}_c \times ((\mathbf{C}_3 - \mathbf{C}_1) \times \hat{\mathbf{u}}_c)) + \mathbf{C}_1. \tag{21}$$

Notice that all values on the r.h.s. of these equations are known, and that $\mathbf{c}_1' = \mathbf{c}_1$.

In the C–S dyad, the cylindric joint constrains the spheric joint to move on the surface of a cylinder whose axis is defined by $\hat{\mathbf{u}}_c$. The projections of points

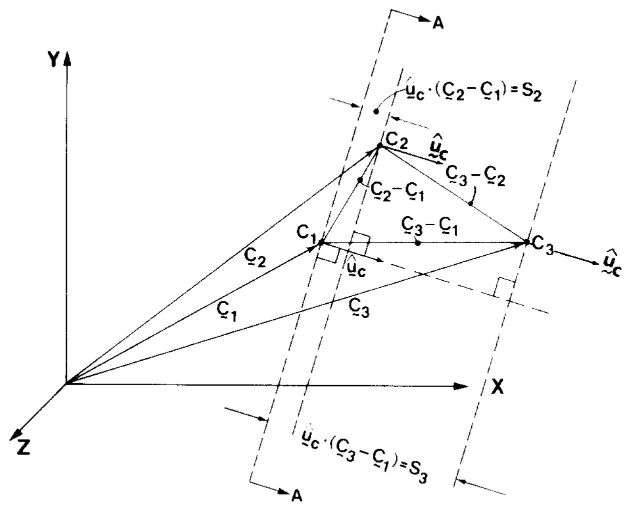

Fig. 6. The three positions of the C–S link and the associated vectors.

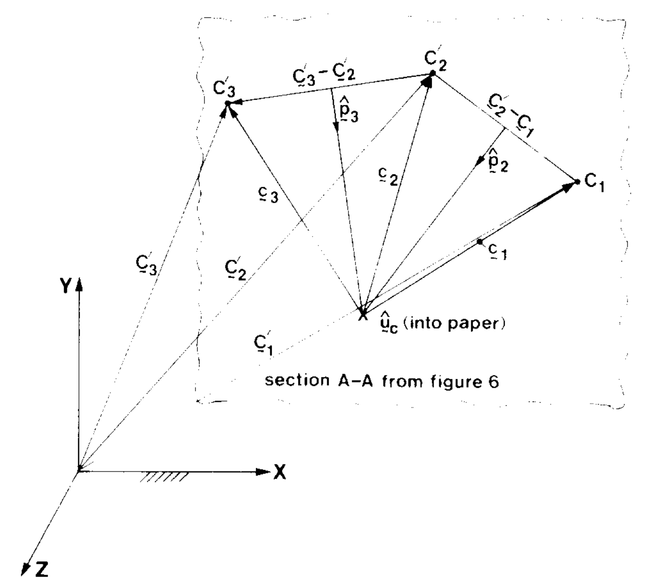

Fig. 7. Projections of the C–S link vectors onto the plane normal to $\hat{\mathbf{u}}_c$ and passing through point C_1.

on the cylinder onto a plane normal to $\hat{\mathbf{u}}_c$ will, therefore, lie on a circle. Thus, points C_1, C_2' and C_3' define a circle with its center on the line defined by $\hat{\mathbf{u}}_c$.

The problem of finding the initial location vector of the cylindric joint, $\mathbf{c}_0$, has now been reduced to the problem of finding the location C_0 of the revolute axis $\hat{\mathbf{u}}_c$ for an R–S dyad, as discussed in section 7.

For the C–S dyad, the synthesis eqns (13)–(15) of section 7 become:

$$\lambda_3 = \frac{\hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_3 \cdot \left[\hat{\mathbf{p}}_2 \times \mathbf{C}_1 + \hat{\mathbf{p}}_2 \times \dfrac{\mathbf{C}_2' - \mathbf{C}_1}{2} - \hat{\mathbf{p}}_2 \times \mathbf{C}_2' - \hat{\mathbf{p}}_2 \times \dfrac{\mathbf{C}_3' - \mathbf{C}_2'}{2} \right]}{\hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_3 \cdot \hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_3} \tag{22}$$

$$\mathbf{c}_0 = \lambda_3 \hat{\mathbf{p}}_3 + \mathbf{C}_2' + (\mathbf{C}_3' - \mathbf{C}_2')/2 \tag{23}$$

$$\mathbf{c}_1 = \mathbf{C}_1 - \mathbf{c}_0. \tag{24}$$

Thus, the dimensions of the C–S dyad are fully determined.

A numerical example illustrating the synthesis of an *RSSR–SC* spatial mechanism for motion generation is presented in the Appendix.

10. CONCLUSION

This paper presented the development of procedures for synthesis of an *RSSR–SC* five-link spatial mechanism for motion generation. The procedure of synthesis is simple and may be easily extended to include the design for multiply separated spatial positions of the rigid body. The technique is also readily adaptable for interactive computation with visual graphic display.

Since the equations of synthesis are linear, little difficulty is encountered in computation. However crank rotatability and branching problems are encountered in the computation. Since there are several arbitrary choices available to the designer (for example, α, β, γ, S_2 and S_3) a mechanism which guides the body through three prescribed positions and also satisfies additional constraints of crank rotatability is not difficult to obtain. The authors are working on development of synthesis procedures for motion generation including the constraints of crank rotatability and will present these results in forthcoming papers.

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APPENDIX

Example: Design of an *RSSR–SC* mechanism to guide a rigid body through three finitely-separated positions in space. ($o_{jz} = 0$ does not affect generality, because 3 points define a plane.)

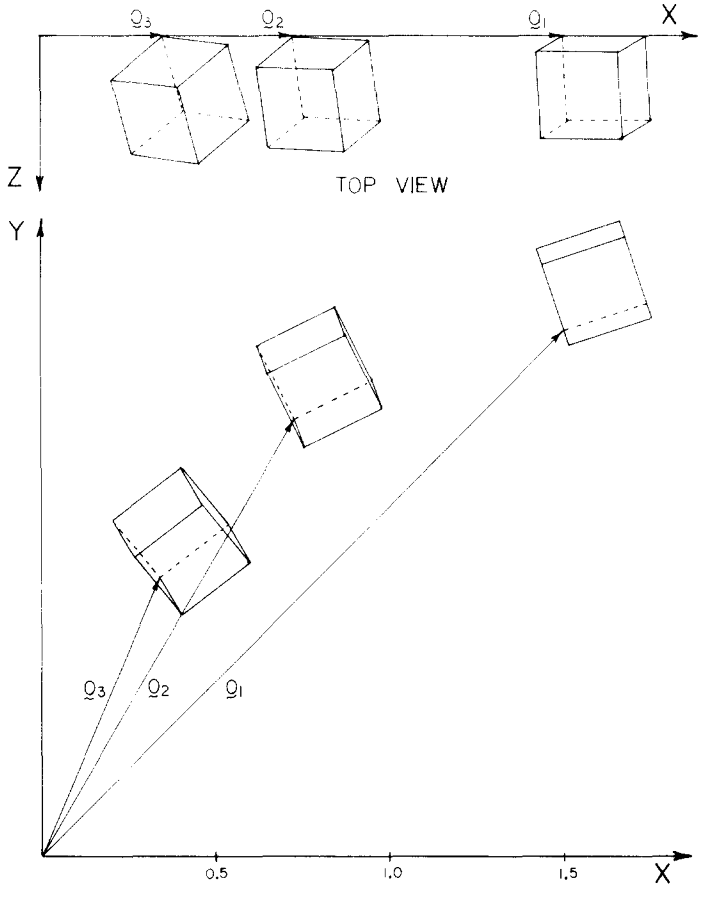

Fig. 8. The three prescribed positions used in the example.

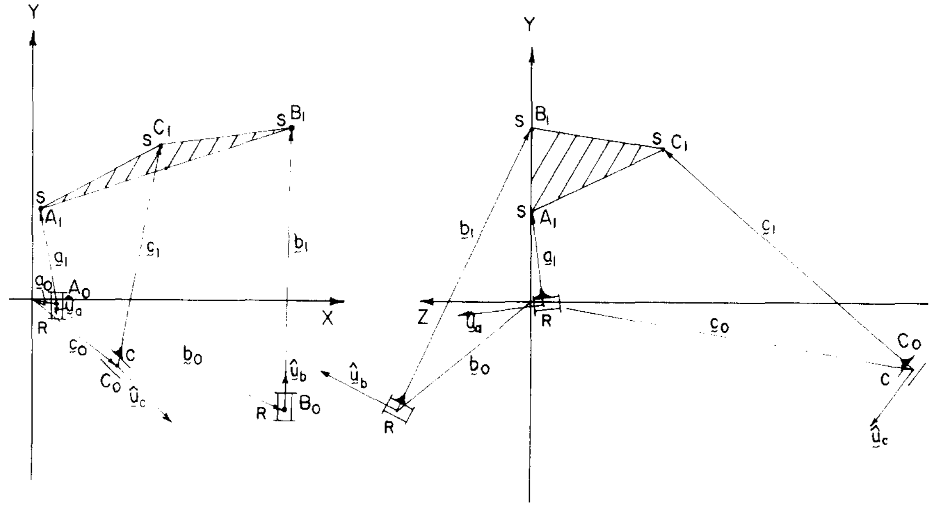

Fig. 9. Two views of the synthesized mechanism.

Input Data:
Moving origin:

$$o_j = \begin{bmatrix} 1.50 & 1.50 & 0 \\ 0.75 & 1.25 & 0 \\ 0.35 & 0.80 & 0 \end{bmatrix} = \begin{Bmatrix} \mathbf{o}_1 \\ \mathbf{o}_2 \\ \mathbf{o}_3 \end{Bmatrix}$$

Euler angles of the moving plane (in degrees)

$$\begin{matrix} 18.5 & 21 & 30 & \lambda_{1z} & \lambda_{2z} & \lambda_{3z} \\ 10 & 20 & 30 = \mu_{1x'} & \mu_{2x'} & \mu_{3x'} \\ 0 & 5 & 10 & \xi_{1z''} & \xi_{2z''} & \xi_{3z''} \end{matrix}$$

Here the first subscript denotes the position and the second subscript denotes the axis of rotation. For example, λ_{1z} is the rotation about the z axis in the initial (first) position, $\mu_{2x'}$ is the rotation about the new x' (rotated x) axis in the second position.

The positions of the moving body are illustrated in Fig. 8 by means of a set of orthographic projections of the body at each of the three prescribed positions. A cube with edges 1 unit long is used to describe the position and orientation of the moving body in the three prescribed positions. The front and the top view of the cube in the three prescribed positions are shown in Fig. 8.

Location of spheric joints A, B and C in the moving frame of reference

$$\begin{Bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \\ \boldsymbol{\gamma} \end{Bmatrix} = \begin{bmatrix} -1.5 & 0 & 0 \\ 1.5 & 0 & 0 \\ 0 & 0 & -1.5 \end{bmatrix}$$

Direction of the cylindrical joint axis, $\hat{\mathbf{u}}_c$

$$\hat{\mathbf{u}}_c = 0.632\hat{\mathbf{i}} - 0.632\hat{\mathbf{j}} + 0.447\hat{\mathbf{k}}.$$

Dimensions of the synthesized mechanism (all vectors are expressed in the fixed frame of reference.)

	$\hat{\mathbf{i}}$	$\hat{\mathbf{j}}$	$\hat{\mathbf{k}}$
$\mathbf{a}_0 =$	0.287	-0.055	-0.130
$\mathbf{a}_1 =$	-0.21	1.080	0.130
$\mathbf{b}_0 =$	2.89	-1.230	1.530
$\mathbf{b}_1 =$	0.037	3.200	-1.53
$\mathbf{c}_0 =$	0.958	-0.751	-4.360
$\mathbf{c}_1 =$	0.460	2.500	2.880
$\hat{\mathbf{u}}_a =$	0.008	-0.118	0.993
$\hat{\mathbf{u}}_b =$	0.009	0.431	0.902
$S_1 =$	-0.497	scalar quantities.	
$S_2 =$	-0.644		

The synthesized mechanism is shown in Fig. 9.

To verify the results of this synthesis, an analysis method was developed and a computer program was written and run. Additionally, a scale model was built. Both the analysis and the model verified the results of the synthesis program presented here. The method of analysis and the results of the computer program will be presented in a forthcoming paper.

LA SYNTHESE ANALYTIQUE NON-ITERATIVE D'UN GENERATEUR DE MOUVEMENT SPATIAL AVEC CINQ BARRES

G.N. Sandor, D. Kohli, C.F. Reinholtz, A. Ghosal

Résumé - Dans la synthèse des mécanismes qui produisent le mouvement, on cherche à déterminer les dimensions principales d'un modèle préconçu d'un mécanisme avec un seul degré de liberté qui guide un de ses accouplements à travers une succession de positions arbitrairement prescrites et finiment ou infiniment séparées. Dans l'espace, chaque position de précision finiment séparée peut être déterminée par le rayon-vecteur de l'origine et par les angles d'orientation d'Euler d'un système de coordonnées, fixé dans l'accouplement guidé par rapport à un système de coordonnées fixes.

Le mécanisme du générateur de mouvement considéré ici se compose de deux barres R-S mis au sol, d'une barre C-S mise au sol, un couplage S-S-S ternaire et le bâti R-R-C fixe. Le mouvement du couplage doit être prescrit. Chaque équation de synthèse est écrite en formulant la fermeture du polygone des vecteurs qui comprennent le commencement et une position déplacée de la dyade formée par un accouplement mis au sol et par le couplage. Le système résultant peut être résolu pour les vecteurs inconnus qui définissent la dyade dans sa position initiale et jusqu'à trois positions de précision sans itération.